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The Effect of Induced Velocity Variation on Helicopter Rotor Damping in Pitch or Roll

By

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ROYAL AIRCRAFT ESTABLISHMENT

The Effect Of Induced Velocity Variation On Helicopter Rotor Damping In Pitch Or Roll

by

G. J. Sissingh, Dr. Ing. habil.

SUMMARY

The present investigation is a continuation of a recent report by K. B. Amer on the aerodynamic damping of a rotor with centrally arranged flapping hinges in a steady pitching or rolling motion. It considers the effect of variation in the induced velocity due to the changes in the distribution of the thrust around the rotor disc. The results are compared with the flight measurements given in Amer's report and the agreement is good.

The rotor damping primarily depends on two quantities:-

(a) the ratio of collective pitch to the thrust coefficient ÷ solidity ratio (already found by Amer), and

(b) $\dfrac{\text{tip speed ratio} \times \text{rotor angle of attack}}{\text{collective pitch}}$

With regard to the latter, the damping is increased if this expression is positive and decreased if it is negative. This means an increase in damping in autorotation and a loss in damping for helicopter flight, especially at higher tip speed ratios or in climbing flight.

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1 Introduction

The present report is mainly concerned with the damping in roll, i.e. with the lateral tilt of the thrust vector in a rolling motion. For simplicity, the damping calculation of a rotor is generally based on the assumption that the thrust-vector-tilt is equal to the tilt of the tip path plane. Some recent investigations have shown, however, that this assumption is not quite correct and that in some cases results may be obtained which are highly misleading, see Ref.1 and 2. Ref.1 contains theoretical values for the hovering condition only; more recently, Amer has derived in Ref.2 a more general expression which includes the forward flight. The latter report also gives a comparison of the theoretical values with flight measurements. On the whole, the agreement between theory and experiment is satisfactory. However there still remains a discrepancy which cannot be explained. As the theoretical investigations of Ref.1 and 2 are based on the assumption of a uniform induced velocity, it has been supposed that the discrepancy is primarily due to changes in induced velocity because of changes in the distribution of the thrust around the rotor disc. Due to the complexity of the problem these changes have hitherto been neglected. In the present report an attempt has been made to consider the effect of induced velocity variation by a relatively simple approach.

2 Some general remarks on the induced velocity of rotary wing aircraft and on the method used to evaluate the changes in the induced velocity

As the exact theoretical calculation of the induced velocity field of a rotor with changes in the distribution of the thrust around the rotor disc would involve excessive labour, some simplified assumptions must be made.

In the present report the necessary equations are obtained from the momentum theory where the known relation between uniform induced velocity and disc loading is - in a somewhat modified version - also applied to the periodic components of the thrust.

The momentum theory states

$$A V' \rho \quad 2v = T \tag{1}$$

In this equation T/A = disc loading, v = induced velocity in the rotor disc, and $V' = V \dotplus v$, the resultant velocity near the rotor, see Fig.1. Hence

$$\begin{aligned} V' &= \sqrt{V^2 \cos^2\alpha + (V \sin\alpha - v)^2} \\ &= \sqrt{V^2 + v^2 - 2Vv \sin\alpha} \end{aligned} \tag{2}$$

Equation (1) can also be written as

$$v \sqrt{V^2 + v^2 - 2Vv \sin\alpha} = \frac{T/A}{2\rho} \tag{3}$$

which is a 4th order equation in v. It can easily be seen that for the two boundary conditions $V = 0$ and $V >> v$ the following simple expressions are obtained:-

With $V = 0$ equation (3) becomes

$$v = \sqrt{\frac{T/A}{2\rho}} \tag{4}$$

and with $V \gg v$

$$v = \frac{1}{V}\frac{T/A}{2\rho} \tag{5}$$

Comparison of the results obtained from equation (3) and equation (5) shows that for the disc loading of present day rotary wing aircraft the simplified equation (5) practically holds good for all flight conditions with approx. $V > 40$ mph. Therefore, the above result can be summarised as follows. In hovering or near hovering condition the induced velocity is proportional to the square root of the disc loading and for all velocities above 40 mph directly proportional to the disc loading.

Let us consider now the case that the thrust increases from T to $(T + \Delta T)$ and results in a corresponding increase in the induced velocity from v to $(v + \Delta v)$. For the ratio $(\Delta v/v)$ the following equations are obtained.

(a) <u>Hovering Condition</u> From equation (4) it follows

$$\frac{v + \Delta v}{v} = \sqrt{\frac{T + \Delta T}{T}} \tag{6}$$

or,

$$(1 + \Delta v/v)^2 = 1 + \Delta T/T \tag{7}$$

As in the hovering (or near hovering) condition $(\Delta v/v)^2 \ll 1$, equation (7) can be simplified to

$$2\Delta v/v = \Delta T/T \tag{8}$$

(b) <u>Forward Flight with $V > 40$ mph</u> From equation (5) it follows

$$\frac{v + \Delta v}{v} = \frac{T + \Delta T}{T}$$

which means

$$\Delta v/v = \Delta T/T \tag{10}$$

Equations (8) and (10) can be combined to

$$k(\Delta v/v) = \Delta T/T \tag{11}$$

where for the hovering condition $k = 2$ and for velocities above 40 mph $k = 1$. Similarly, intermediate conditions could be considered by using appropriate values for k.

If for any reason the lift L of a rotor blade varies with the azimuth angle ψ, corresponding changes in the induced velocity v occur. Let us assume a lift distribution of the form

$$L = L_o + L_1 \cos\psi + L_2 \sin\psi + \ldots \qquad (12)$$

with the consequent induced velocity

$$v = v_o + v_1 \cos\psi + v_2 \sin\psi + \ldots \qquad (13)$$

As already stated, an exact calculation of the induced velocity is out of the question. It is therefore assumed that equation (11) can also be applied to the periodic components of the thrust which means, for instance, that

$$k \frac{v_1}{v_o} = \frac{L_1}{L_o} \qquad (14)$$

It is agreed that this assumption represents a severe simplification of the problem but it appears justified by the fact that the discrepancy left in Ref.2 can be explained.

3 The damping in roll (or pitch) with consideration of induced velocity variation

For simplicity, a steady rolling motion with the angular velocity p about the rotor centre is investigated.* The rotor rotates anti-clockwise when viewed from above and the effect of the coning angle on the lift distribution has been neglected. The nondimensional expressions for the velocities at the blade element are:-

$$u_T = x + \mu \sin\psi \qquad (15)$$

$$u_P = \lambda + x \frac{p}{\Omega} \sin\psi - x \frac{\dot{\beta}}{\Omega} - \mu\beta \cos\psi \qquad (16)$$

where

$$\beta = - a_1 \cos\psi - b_1 \sin\psi \,.. \qquad (17)$$

and

$$\dot{\beta} = a_1 \Omega \sin\psi - b_1 \Omega \cos\psi \qquad (18)$$

As the damping in roll depends on the lateral tilt of the thrust vector i.e. on the term b_1 and thus on the $\cos\psi$ component in the moment of the air forces about the flapping hinge, the inflow ratio may be expressed in the form

---

* It can be shown that the result can equally be applied to the damping in pitch.

$$\lambda = \lambda_o + \lambda_1 \cos\psi \tag{19}$$

Inserting equations (17), (18) and (19) in equation (16) leads to

$$\begin{aligned} u_P = \lambda_o + 0.5\, \mu a_1 &+ (x \tfrac{p}{\Omega} - x a_1) \sin\psi \\ &+ (\lambda_1 + x b_1) \cos\psi \\ &+ (0.5\, \mu b_1) \sin 2\psi \\ &+ (0.5\, \mu a_1) \cos 2\psi \end{aligned} \tag{20}$$

If the induced velocity is written as

$$v = v_o + v_1 \cos\psi \tag{21}$$

it follows from the equation of definition and from equation (19) that

$$R\Omega\,(\lambda_o + \lambda_1 \cos\psi) = V \sin\alpha - (v_o + v_1 \cos\psi) \tag{22}$$

and that for small angles α

$$\frac{v_1}{v_o} = \frac{\lambda_1}{\lambda_o - \mu\alpha} \tag{23}$$

According to equation (14), the ratio v_1/v_o is assumed to be proportional to L_1/L_o where L_o and L_1 are components of the lift L of the rotor blade during its travel around the rotor disc, see equation (12). The well known equation for the lift is

$$L = R^3\Omega^2\, ca\, \frac{\rho}{2} \int_o^B u_T^2 \left(\theta + \frac{u_P}{u_T}\right) dx \tag{24}$$

where $$u_T^{\,2} = x^2 + \tfrac{1}{2}\mu^2 + 2x\mu \sin\psi - \tfrac{1}{2}\mu^2 \cos 2\psi \tag{25}$$

and $$\begin{aligned} u_T u_P = x\lambda_o + \tfrac{1}{2}\mu x p/\Omega &+ (\lambda_1 x + b_1 x^2 + \tfrac{1}{4} b_1 \mu^2) \cos\psi \\ &+ (\lambda_o \mu + x^2 \tfrac{p}{\Omega} - a_1 x^2 + \tfrac{1}{4} a_1 \mu^2) \sin\psi \\ &+ (\mu x a_1 - \tfrac{1}{2}\mu x p/\Omega) \cos 2\psi \\ &+ (\tfrac{1}{2}\mu\lambda_1 + \mu x b_1) \sin 2\psi \\ &+ (- \tfrac{1}{4}\mu^2 b_1) \cos 3\psi \\ &+ (+ \tfrac{1}{4}\mu^2 a_1) \sin 3\psi \end{aligned} \tag{26}$$

Equation (24) results in the following expressions for L_o and L_1:-

$$L_o = R^3\Omega^2 \, ca \frac{\rho}{2} \left[\left(\frac{1}{3}B^3 + \underline{\tfrac{1}{2}B\mu^2}\right)\theta + \tfrac{1}{2}B^2\lambda_o + \underline{\tfrac{1}{4}B^2\mu p/\Omega} \right] \qquad (27)$$

$$L_1 = R^3\Omega^2 \, ca \frac{\rho}{2} \left[\tfrac{1}{2}B^2\lambda_1 + \tfrac{1}{3}B^3 b_1 + \underline{\tfrac{1}{4}B\mu^2 b_1} \right] \qquad (28)$$

It can easily be seen that the underlined terms in these equations are of minor importance, they can therefore be neglected.

Inserting equations (23), (27) and (28) in equation (14) leads to

$$\frac{k\lambda_1}{\lambda_o - \mu\alpha} = \frac{\tfrac{1}{2}B^2\lambda_1 + \tfrac{1}{3}B^3 b_1}{\tfrac{1}{3}B^3\theta + \tfrac{1}{2}B^2\lambda_o} \qquad (29)$$

or, solving for λ_1:-

$$\lambda_1 = b_1 \frac{\tfrac{1}{3}B^3 (\lambda_o - \mu\alpha)}{\tfrac{1}{3}B^3 k\theta + \tfrac{1}{2}B^2\lambda_o (k - 1) + \tfrac{1}{2}B^2\mu\alpha} \qquad (30)$$

From the known equation for the thrust coefficient (terms with μ^2 are again neglected)

$$C_T = a\sigma \left(\tfrac{1}{6}B^3\theta + \tfrac{1}{4}B^2\lambda_o\right) \qquad (31)$$

it follows

$$\lambda_o = \frac{4C_T/\sigma}{B^2 a} - \tfrac{2}{3}B\theta \qquad (32)$$

Inserting equation (32) in equation (30) finally leads to

$$\lambda_1 = Bb_1 \frac{\tfrac{4}{3}(1 - f) - 2f\frac{\mu\alpha}{\theta B}}{2(k - 1) + 2f\left(1 + \frac{1.5\mu\alpha}{\theta B}\right)} \qquad (33)$$

where

$$f = \frac{B^3 a}{6} \times \frac{\theta}{C_T/\sigma} \qquad (34)$$

Equation (33) represents an equation for the two unknowns λ_1 and b_1. Another relation can readily be obtained from the equation of motion of the flapping of the blades. It can be shown (the deduction of this equation is not given here) that with consideration of the λ_1-term in the inflow ratio λ, see equation (19), the corresponding equation of the flapping motion is given by

$$- 2\frac{p}{\Omega} = \frac{\gamma}{2}\left[b_1\left(\tfrac{1}{4}B^4 + \underline{\tfrac{1}{8}B^2\mu^2}\right) + \tfrac{1}{3}B^3\lambda_1\right] \tag{35}$$

where in the first approximation the underlined term with μ^2 can again be neglected. Without consideration of λ_1 the above equation becomes the well known relation

$$b_{1o} = -\frac{16}{\gamma B^4}\frac{p}{\Omega} \tag{36}$$

with b_{1o} as given in equation (36), equation (35) can also be written as

$$b_1 - b_{1o} + \frac{4\lambda_1}{3B} = 0 \tag{37}$$

or

$$\lambda_1 = \tfrac{3}{4}B(b_{1o} - b_1) \tag{38}$$

Equating equations (30) and (38) leads to the following ratio for the lateral tilt of the tip path plane with and without consideration of changes in the induced velocity

$$\frac{b_1}{b_{1o}} = \frac{2(k - 1) + 2f\left(1 + \frac{1.5\mu\alpha}{\theta B}\right)}{2\left(k - \frac{1}{9}\right) + f\left(\frac{2}{9} + \frac{\mu\alpha}{3\theta B}\right)} \tag{39}$$

The above expression can be interpreted as a corrective factor for the term

$$\frac{\Delta b'}{\Delta b_1} = 1.5 - \frac{B^3 a}{12} \times \frac{\theta}{C_T/\sigma} = 1.5 - \frac{f}{2} \tag{40}$$

calculated by Amer in Ref.2.

With consideration of this factor that takes into account changes in the induced velocity, the ratio (rotor-force-vector tilt)/(tilt of the rotor disc as given by equation (36)) finally becomes

$$\left(\frac{\Delta b'}{\Delta b_1}\right)_{CIV} = \frac{2(k - 1) + 2f\left(1 + \frac{1.5\mu\alpha}{\theta B}\right)}{2\left(k - \frac{1}{9}\right) + f\left(\frac{2}{9} + \frac{\mu\alpha}{3\theta B}\right)} \times \left(1.5 - \frac{f}{2}\right) \tag{41}$$

It may be of interest to show that for the hovering condition Amer's term $\Delta b'/\Delta b_1$, see equation (40), is identical with the coefficient

$$\eta = 1 - \frac{\sigma a B^2}{8\sqrt{k_T}} \tag{42}$$

This coefficient was derived in earlier work at RAE and is plotted in Fig.21 of Ref.1. For the hovering condition

$$\lambda = -\sqrt{\frac{C_T}{2}} \tag{43}$$

and

$$C_T/\sigma = a(\tfrac{1}{4}B^2\lambda + \tfrac{1}{6}B^3\theta) \tag{44}$$

from equations (43) and (44) it follows

$$\frac{1}{12} B^3 a \frac{\theta}{C_T/\sigma} = \frac{1}{2} + \frac{B^2 a\sigma}{8\sqrt{2C_T}} \tag{45}$$

Inserting equation (45) in equation (40) leads to the expression

$$\frac{\Delta b'}{\Delta b_1} = 1 - \frac{\sigma a B^2}{8\sqrt{2C_T}} \tag{46}$$

which (since $k_T = 2C_T$) is identical with equation (42).

4 Comparison with flight measurements

The result given by equation (41) is shown in Figs.2,3,4 for k = 2, 1.5 and 1 respectively. As found in chapter 2, k = 2 refers to a hovering or near hovering condition and k = 1 to flight velocities > 40 mph. The value k = 1.5 is used to cover the range between these boundary conditions. In each of the curves $\frac{\mu\alpha}{\theta} = +0.8$; 0 and -0.4 have been calculated. This range corresponds approximately to the normal flight conditions of present day rotary wing aircraft, the positive values covering autorotation and the negative values covering helicopter operating conditions.

Moreover, in Figs.3 and 4 Amer's quantity $\frac{\Delta b'}{\Delta b_1}$ and the flight measurements given in Ref.2 are shown. In accordance with theory, the helicopter flights lie below and the autorotation flights above the curve $\frac{\mu\alpha}{\theta} = 0$. Even the fact that the measurements C

level flight $\mu = 0.25$

climb $\mu = 0.15$

lie farther below the curve mentioned above than the measurements B (level flight = 0.17) can be fully explained. The tests C have either a higher tip speed ratio or (in climbing flight) a greater forward tilt of the thrust vector i.e. a larger negative rotor angle of attack and by that a larger negative value $\frac{\mu\alpha}{\theta}$.

Both theory and flight measurements indicate that the rotor damping decreases in helicopter flight, especially for large values $\frac{\theta}{C_T/\sigma}$ and/or large negative values $\frac{\mu\alpha}{\theta}$. The latter means either high tip speed ratios or climbing flight.

5 Conclusions

5.1 A method has been derived for evaluating helicopter rotor damping in pitch or roll including the effect of induced velocity variation.

5.2 The limited flight test data available appears to substantiate the theory.

LIST OF SYMBOLS

R	rotor radius, ft
A	rotor disc area, ft^2, $A = \pi R^2$
c	blade chord, ft
σ	rotor solidity
Ω	rotor angular velocity, rad/s
θ	Collective pitch, rad.
V	velocity of flight, ft/s
v	induced velocity in the plane of the rotor disc, ft/s $v = v_o + v_1 \cos\psi$
V'	resultant velocity in the rotor disc, ft/s
α	rotor angle of attack, angle between flight path and plane perpendicular to axis of no feathering, positive when axis is pointing rearward, rad.
μ	tip speed ratio, $\mu = (V \cos\alpha)/R\Omega$
γ	inertia number of blade
a	lift curve slope
ψ	blade azimuth angle measured from downwind position in direction of rotation, rad.
B	tip loss factor, $B \approx 0.97$

β	flapping angle, rad. $$\beta = -a_1 \cos\psi - b_1 \sin\psi$$
λ	inflow ratio $$\lambda = \lambda_o + \lambda_1 \cos\psi = \frac{V \sin\alpha - v}{R\Omega}$$
x	radius of blade element divided by R
u_T	nondimensional expression for the velocity U_T perpendicular to blade-span axis and to axis of no feathering, $$u_T = U_T/R\Omega$$
u_P	nondimensional expression for the velocity U_P perpendicular both to blade-span axis and U_T, $$u_P = U_P/R\Omega$$
ρ	density of air, lb $s^2 ft^{-4}$
p	angular velocity in roll, positive for right roll, rad/s
T	rotor thrust, lb
L	lift of the rotor blade, lb $$L = L_o + L_1 \cos\psi + L_2 \sin\psi + ..$$
C_T, k_T	thrust coefficients, $$T = C_T A(\Omega R)^2 \rho$$ $$T = k_T A(\Omega R)^2 \rho/2 \qquad \text{i.e. } 2C_T = k_T$$
f	nondimensional quantity, $f = \frac{1}{6} B^3 a \frac{\theta}{C_T/\sigma}$
k	factor depending on velocity of flight
η	nondimensional aerodynamic damping coefficient of Ref.1
$\frac{\Delta b'}{\Delta b_1}$	quantity taken from Ref.2, ratio (rotor-force-vector tilt)/(calculated tip-path-plane tilt) during steady rolling motion
$\left(\frac{\Delta b'}{\Delta b_1}\right)_{CIV}$	the same as above but with consideration of changes in the induced velocity because of changes in the lift distribution

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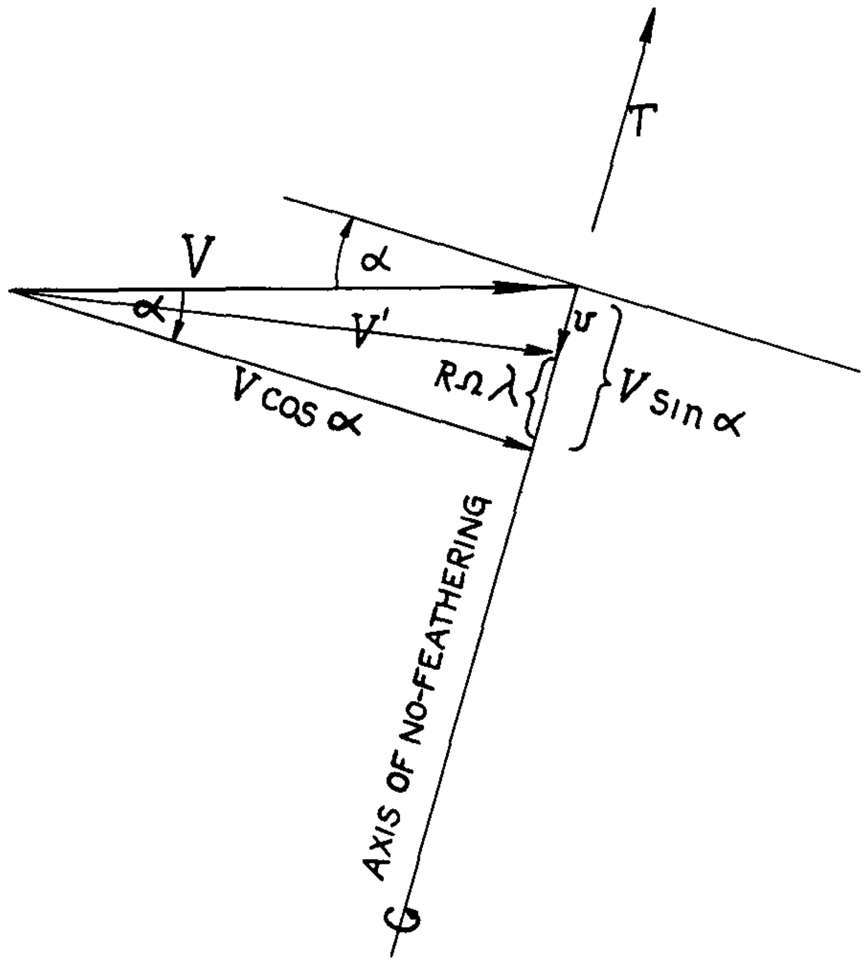

FIG. I. NOTATION OF VELOCITIES PARALLEL AND PERPENDICULAR TO AXIS OF NO-FEATHERING.

FIG.2.

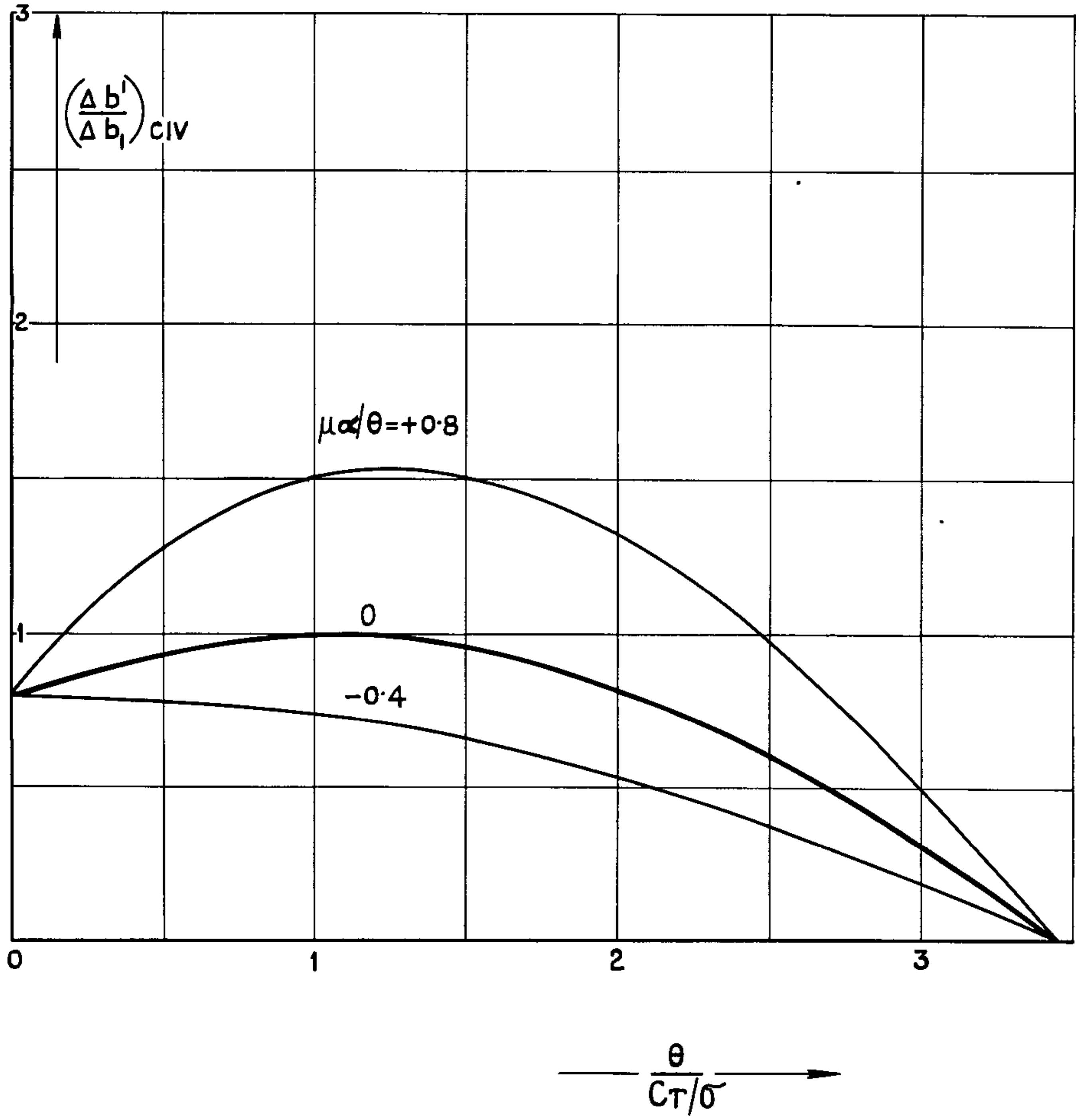

FIG. 2. RATIO OF ROTOR-FORCE-VECTOR TILT TO DISC TILT DURING STEADY ROLLING VELOCITY.
($k = 2$, i.e. HOVERING OR NEAR HOVERING CONDITION.)

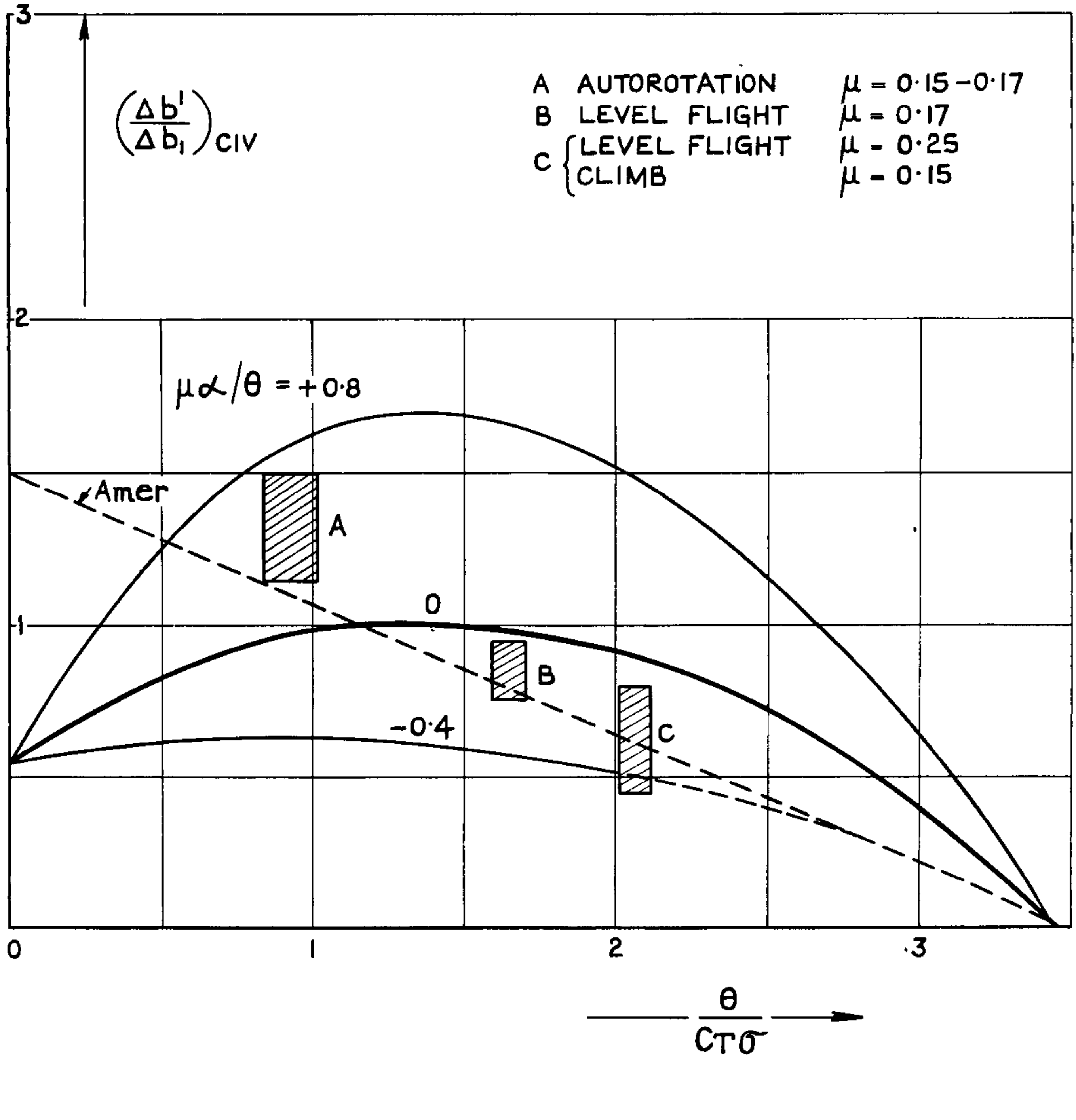

FIG.3. RATIO OF ROTOR-FORCE-VECTOR TILT TO DISC TILT DURING STEADY ROLLING VELOCITY. (k = 1·5 i.e. APPROX. $0 < V < 40$ mph)

FIG.4.

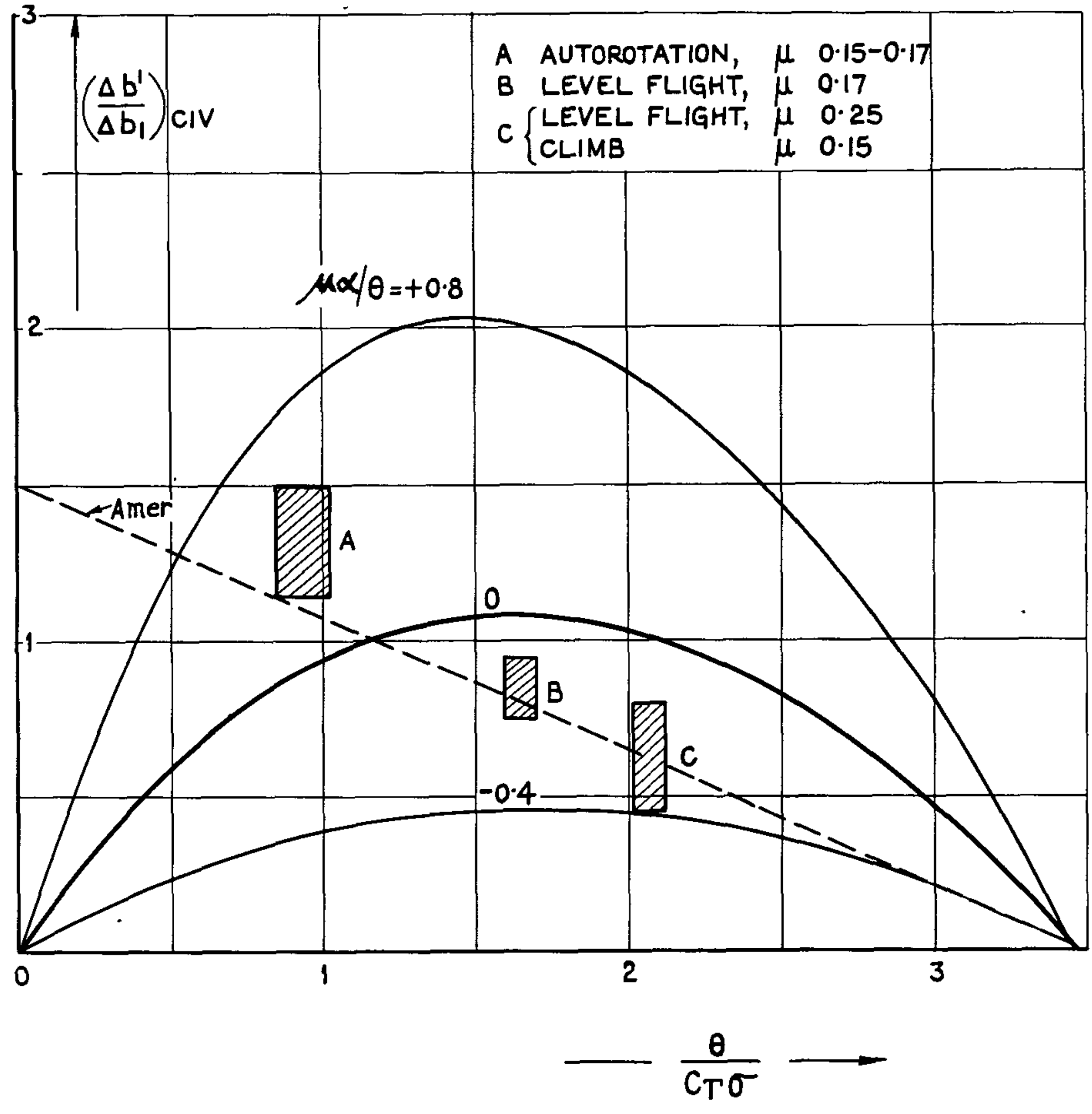

FIG. 4. RATIO OF ROTOR-FORCE-VECTOR TILT TO DISC TILT DURING STEADY ROLLING VELOCITY. (k - 1, i.e. APPROX. V > 40 m.p.h.)

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