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An Investigation of the Scatter in Variable-Amplitude Fatigue-Test Results of 2024 and 7075 Materials

by

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SUMMARY

The scatter in the fatigue lives of identical simple laboratory specimens, which have been tested under variable amplitude sinusoidal loading, is estimated, using a log-normal distribution of fatigue lives as a basis, from the fatigue test results of a number of experimenters. These estimates of the scatter are analyzed with a view to studying the effect of various test loading parameters on the magnitude of the scatter produced and the results of this analysis are then discussed in terms of a simplified model of the mechanism of fatigue failure.

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1 INTRODUCTION

The amplitude of the sinusoidally alternating load experienced by most components during their practical life is by no means constant. The severity of gusts, the amount of movement of the control surfaces, the changes in torque or speed of the drive shafts, all these are variable quantities and so the amplitude of the loads produced by these factors will also be subject to variations. Thus, as the ultimate aim of the fatigue test is an accurate forecast of the fatigue life of an item in service, an attempt must be made to represent these variations in alternating load amplitude in laboratory tests of fatigue specimens.

A great deal of work has been, and is being, conducted all over the world on the subject of cumulative damage laws for relating the fatigue damage incurred under variable amplitude loading to that expected from the constituent constant amplitude cycles. As yet, however, none of the hypotheses propounded has proved capable of forecasting the lives of all types of specimens under all loading conditions and, for this reason, the aircraft industry still uses Miner's law, with one or two refinements for residual stress and such phenomena.

Fatigue tests, especially those on full scale structures, are expensive, and with the inadequacies of Miner's law known but not understood the resulting questions, as to the validity of any service life calculated from constant amplitude test data, makes the economics of constant amplitude testing for the prediction of service life doubtful. The trend in modern fatigue testing of specimens for practical use is thus towards as accurate as possible a representation of the loads that will be encountered in service, by the use of variable amplitude fatigue testing.

Many simplified forms of variable amplitude loading have been employed by experimenters in the past with the aim of reducing the number of load interaction effects present. The various forms will be considered in this Report in the order of increasingly accurate representation of the loads experienced by an aircraft structure in service, but the data on each form of testing is not as copious as that on constant amplitude testing analyzed in Ref.1. Not only have few experimenters carried out variable amplitude tests, but also each experimenter has tested very few specimens under each set of conditions. Thus a statistical analysis of the available data is difficult and for a trend in this analysis to be statistically significant the trend itself must be most marked. For this reason the analysis presented below is in terms of apparent but not statistically significant trends.

2 A REVIEW OF PREVIOUSLY USED FATIGUE FAILURE DISTRIBUTIONS

Ref.1 contains a short synopsis of the findings of some of the many investigations that have been made into the form of the distribution function of the fatigue lives obtained from macroscopically similar specimens tested under identical constant amplitude loading conditions. Few investigations have been carried out into the distribution of the number of cycles to fatigue failure under variable amplitude loading, but some experimenters, who were not interested in studying the form of this distribution themselves, present their results in a manner which makes an examination of their data in these terms possible. However, whereas in constant amplitude loading the meaning of the expression 'identical loading conditions' is self-evident, interpretation of this term under some forms of variable amplitude testing is more difficult. When the order of application of the various load levels is programmed and the frequency of application of each load level is governed by a load spectral density function, then specimens tested under the same load spectrum must also be tested under the same sequence of load levels for the loading conditions to be described as identical. However, if the order of application of the load levels is random, only the load spectral density function need be the same for the two sets of conditions to be identical.

The most simple form of variable amplitude fatigue test, the one step test, subjects the specimen initially to a fixed number of cycles at one constant amplitude level (the initial stress) after which a different constant amplitude stress level (the runout stress) is applied until failure occurs, Fig.1a. Swanson[2], Schijve and Jacobs[3] and Webber and Levy[4] are amongst those that have conducted fatigue tests to this pattern of loading and whilst quite large deviations from a log-normal distribution, possibly caused by failures under the initial stress, occur in Ref.4 the results for those tests of Ref.2, in which the initial stress was high ($S_a > 34$ ksi) and was applied for only a few cycles, were well represented by a log-normal distribution. Swanson's tests in which the initial stress was lower than the runout stress were not sufficiently numerous to define the distribution with any assurance but this data and that from Ref.3, presented in Fig.2 of this Report, conform approximately to the log-normal form. It is of interest that a slightly better agreement with this distribution than that given in Figs.16-20 of Ref.2 is obtained if the distribution of the total cycles to failure, rather than that of just the runout cycles, is considered, e.g. Figs.3 and 4.

Besides the results for one step tests mentioned above, Refs.3 and 4 include data on two level repeated block fatigue tests in which two levels of constant amplitude alternating stress are applied in turn for a fixed number of cycles each, Fig.1b. Results of similar tests are also presented in Ref.5 and in all three references the log-normal distribution appears to give a reasonable fit to the observed values, e.g. Fig.5.

The number of stress levels employed in tests of the distinct load levels type has varied from 2 in the cases mentioned above, to 3 in Ref.4, to 4, 5 and 6 in Refs.6, 7, 8, 9, 10, 11 and 12 and to 22 in Ref.13. This designation of test type includes all block testing whether of random or of programmed sequence and all testing in which the load cycles applied must be at one of a set of distinct load levels Figs.1a-d. The analyses in Refs.7, 9, 10, 11 and 12 are all in terms of the Weibull-Gumbel 3 parameter extreme value distribution which from figures in those references can be seen to provide a good fit with the data for tests to both gust and manoeuvre spectra. Figs.6-15 present some of the data from these references plotted in terms of the log-normal distribution which also appears to represent the observed values from these references quite closely, whilst Figs.16, 17 and 18 show good agreement between a log-normal representation and some of the results of Refs.6, 8 and 13 respectively.

Fatigue tests under sinusoidally alternating stress, the amplitude of which was varied continuously but to a periodic pattern, are reported in Refs.14 and 15. Three types of test were conducted, the repeating patterns which the amplitude of the alternating stress followed being respectively a sinusoidal variation, an exponential increase to a limiting value followed by an exponential decrease back to the original value, and a gust frequency distribution of load amplitudes applied in the order of increasing load amplitudes. The amplitude of the applied load in Ref.16 is also continuously variable but as the stress history is derived from a white noise source the sequence of load amplitudes is random, Fig.1f. The data obtained from these references are plotted in Figs.19-21 and again give good agreement with a log-normal distribution, as do the results from Ref.17 for tests under random amplitude loads to a Rayleigh distribution of peaks, Fig.22. However, the results from this last reference for tests conducted with a random amplitude initial stressing for a fixed number of cycles followed by a constant amplitude runout exhibit a degree of truncation, which is reduced if the lives plotted include the number of cycles at the initial stress as well as the number of cycles under the runout stress, e.g. Fig.23.

3 SELECTION OF A DISTRIBUTION

The review, presented in the previous section, of the representation of a limited amount of variable amplitude fatigue data in terms of the log-normal distribution of lives to failure indicates that, in the range of probabilities covered, this form of distribution fits the observed values adequately. The data available are not sufficiently numerous to define the parent distribution with any confidence, but, for the reasons quoted in Ref.1 in support of analyzing the constant amplitude data in terms of a log-normal distribution of lives, this form of distribution function was also adopted in this Report for an analysis of the variable amplitude fatigue data. The arguments of Ref.1 in favour of the adoption of a log-normal distribution of life are reproduced below for ease of reference, namely:-

(1) The discrepancies between practice and the log-normal distribution appear to be small in the range of probabilities of failure $0.90 > P > 0.10$. (This range is smaller than that quoted for constant amplitude tests because the available variable amplitude data are more limited in number.)

(2) The mean and variance of a log-normal distribution provide the basis for a readily comprehensible comparison of the positions and extents of the distributions obtained under different loading conditions using various forms of specimen.

(3) The use of a normal form of distribution, rather than any other basic type, considerably eases the analysis of the data without impairing the validity of the conclusions.

(4) The analysis in terms of a log-normal distribution of life involves no graphical procedures and thus is exactly reproducible.

4 DEFINITION AND DERIVATION OF TERMS

The normal distribution is defined by the probability density function

$$p(x)\,dx = \frac{1}{\sqrt{2\pi\,\sigma^2}} \exp - \tfrac{1}{2}\left(\frac{\mu - x}{\sigma}\right)^2 dx$$

where $p(x)\,dx$ is the probability of occurrence of the event between conditions x and $x + dx$, and μ and σ are the population mean and standard deviation respectively. When a log-normal distribution of cycles to failure is used, the above expression must be replaced by

$$p(\log L)\, d(\log L) = \frac{1}{\sqrt{2\pi\, \sigma^2}} \exp - \tfrac{1}{2}\left(\frac{\mu - \log L}{\sigma}\right)^2 d(\log L)$$

where μ and σ are now the mean and standard deviation of log (life) and L is the number of cycles to failure of a randomly selected specimen.

The best estimates of μ and σ that can be obtained from a sample of c test specimens are given by

$$\hat{\mu} = \overline{\log (L)} = \frac{\sum_{r=1}^{c} \log L_r}{c}$$

and

$$\hat{\sigma} = \left\{ \frac{\sum_{r=1}^{c} (\log L_r)^2 - c \left(\frac{\sum_{r=1}^{c} \log L_r}{c} \right)^2}{(c - 1)} \right\}^{\frac{1}{2}}$$

where it is to be noted that the unbiassed estimate of the standard deviation is used, as the number of specimens tested under one set of condition is generally small. These two parameters are enough by themselves to define the normal distribution, but sometimes another parameter, the coefficient of variation v, is used. Any two of these three parameters μ, σ and v are sufficient to define the normal distribution, for the coefficient of variation is defined by the expression

$$v = \frac{\sigma}{\mu} \quad .$$

Comparisons between the scatter, or standard deviation, obtained under different conditions are conducted using the variance ratio F, where F is defined as $\hat{\sigma}_1^2/\hat{\sigma}_2^2$, $\hat{\sigma}_1$ and $\hat{\sigma}_2$ being the two sample estimated standard deviations being compared. The null hypothesis tested is that the two samples from which s_1 and s_2 were calculated were drawn at random from normal populations of equal variance. Table 4 of Ref.18 gives values of F at various levels of significance such that, if the appropriate value of F is exceeded, the null hypothesis is contradicted and the two samples of which the standard deviations are being compared can be said, at the appropriate level of significance, to be taken from different parent populations.

5 THE EFFECTS OF VARIOUS PARAMETERS ON THE SCATTER IN VARIABLE AMPLITUDE FATIGUE TESTS

A large amount of data exists on the effect of various test parameters on the mean lives obtained in variable amplitude fatigue tests, but relatively few studies have been made into the effect of these parameters on the scatter produced in the lives of sets of the specimens tested under identical conditions. The aim of this Report is to consider these effects in the various forms of variable amplitude fatigue testing and to attempt an explanation of any trends that are noticed. The presentation and analysis of the data collected are described in the Appendices and Figs.24-59, whilst an edited account of the form of this analysis and of the major trends noted follows.

The variable amplitude fatigue data available are sorted according to four somewhat artificial subdivisions, namely

(1)	One step tests	{section 5.1}
(2)	Two level repeated block tests	{section 5.2}
(3)	Multi level block tests	{section 5.3}
(4)	Continuously variable amplitude tests	{section 5.4} .

The loading patterns experienced in each of these forms of testing are indicated diagrammatically in Figs.1a-f and defined here in terms of the necessary parameters.

(1) A one step test consists of n_1 cycles of a constant amplitude sinusoidally alternating initial stress S_1 followed by application of cycles of a different constant amplitude stress S_2, the runout stress, until failure. (3 defining parameters only, n_1, S_1 and S_2.)

(2) A two level repeated block test consists of the alternate application of two sinusoidally alternating constant amplitude stresses S_1 and S_2 in blocks of cycles of lengths n_1 and n_2 respectively, until failure. (4 basic parameters n_1, n_2, S_1 and S_2 and in addition the stress level to be applied first must be defined.)

(3) A multi level block test consists of the repeated application until specimen failure of a series of blocks of cycles. Each block is made up of a number of cycles of a particular constant amplitude sinusoidally alternating stress level, the number of blocks within the repeated series and the stress level and the number of cycles in each block being decided before the test is

started. The sequence of application of the blocks within the repeated series is an important parameter of a multi level block test and is defined in terms of the magnitude of the stress level employed in each block, conforming generally to one of the self explanatory patterns:- high-low; low-high; high-low-high; low-high-low or random. Thus a two level repeated block test is a particular form of the multi level block test. (A large number of parameters.)

(4) A continuously variable amplitude test consists of the application of sinusoidal cycles of stress of an amplitude that can take any value within the range of loads applied in the test but that is governed by some controlling mechanism. (A very large number of parameters.)

The amount of data available on any of these forms of testing is scanty, and with so many parameters subject to variation the supporting evidence for any of the various effects noted is still more scanty. A synopsis of the analysis in Appendices A, B, C and D is presented below.

5.1 One step tests

The analysis of one step test data (Appendix A and Figs.24-30) was conducted in terms of a specimen life defined as the sum of the cycles at the initial stress and the cycles at the runout stress, as it appeared (section 2) that this variable provides a better fit with the log-normal distribution than that given by a life defined solely as the cycles at the runout stress. The general band of scatter covered by these tests is quite large and shows no overall correlation between mean life and coefficient of variation, whilst the effect of the number of cycles at the initial stress appears to depend on the relative and absolute magnitudes of the two stress levels used. The number of cycles of an initial stress which is considerably below the fatigue limit shows no effect on the scatter, whilst if the initial stress is at or about the fatigue limit, or noticeably lower than the runout stress level, an increase in the number of cycles at the initial stress seems to increase the scatter. The evidence on the effect of increasing the number of cycles of an initial stress that is higher than the runout stress is confused but it seems probable that an increase in this number will either leave the scatter unaltered or slightly decrease it, if the initial stress is high enough to produce a constant amplitude failure in the region of 10^4 - 10^5 cycles on its own; whilst if both stresses are low, constant amplitude lives in the range 10^5 and upwards, the mean life will be long and the scatter will be subject to large variation as is experienced at low stresses in constant amplitude tests.

Besides the various effects mentioned above of the different magnitudes of the initial stress, a slight increase in scatter seems to be associated with a decrease in the initial stress level whilst an increase in runout stress level when only a few initial stress cycles are applied causes a decrease in the scatter.

5.2 Two level repeated block tests

The introduction of the possibility of changing from the second stress level that is applied to the specimen back to the first stress and so on carries with it the need for a definition of the time at which such a change should occur. Thus a need arises for the additional parameter n_2, the number of cycles per block of the second stress level, and a study of the effect of this parameter must be included in an analysis of the scatter on two level repeated block tests, such as that given in Appendix B and Figs.31-37.

The values of coefficient of variation obtained in the analysis of the data from this type of test in general appear to be low except for cases when both stress levels are low enough to give constant amplitude lives above about 10^5 cycles. A decrease in the scatter, from the value that would be obtained by constant amplitude tests at the lower stress level, accompanies the introduction of the higher stress level and any further increase, above about 5%, in the proportion of the life which is spent at the higher stress level does not appear to produce any result in the final scatter, provided this stress level is reasonably high.

However, a decrease in the magnitude of the higher stress level seems to increase the scatter, whereas variations of the lower stress level appear to leave the scatter unaltered, although when both stress levels are low the scatter is high. Neither changes in the length of the block size nor changes in the order of application of the load levels give any discernible effect for either the 2024 or the 7075 materials, which exhibit the same behaviour in scatter as one another. Overall it seems likely that the higher stress level of the two applied in this form of testing defines the amount of scatter produced in the tests, provided the block size is not too large and that the higher stress level is applied sufficiently frequently. If, however, both stress levels are low the lower stress level may well influence the degree of scatter obtained.

5.3 Multi level block tests

An increase in the number of discrete amplitude levels at which the stress experienced by the specimen may be applied facilitates the representation of service loading spectra in tests on laboratory specimens. The number of load levels used varies from experimenter to experimenter, being as high as 22 in Ref.13, whilst the shape of the overall service loading spectra to be represented also varies from one laboratory to another. In general however aircraft spectra may be grouped according to the following basic classification.

(1) Gust spectra (loads caused by atmospheric turbulence).

(2) Manoeuvre spectra (pilot induced loads resulting from turns, rolls, etc.).

(3) Other spectra (including composite spectra with varying proportions of gust and manoeuvre spectra added together).

In general the loads experienced by an aircraft in turbulence (gust spectrum) are approximately symmetrical about the level flight condition, that is a nearly equal number of positive and negative loads of any magnitude occur, whereas a manoeuvre load spectrum is usually asymmetrical, many more positive than negative manoeuvres been carried out. Also in the particular case of fighter aircraft, which carry out rapid manoeuvres, much higher loads are introduced by manoeuvres than are caused by gusts. Because of this distinction between gust and manoeuvre spectra, these two types are considered separately in each section of the analysis (Appendix C) of the results of fatigue test results under multi level block loading (Figs.38-51).

Each section in that appendix considers the effect of a change in one particular parameter on the scatter, determined on the basis of a log-normal distribution of life, obtained in fatigue test results to these two types of testing. The sparse fatigue test data from tests under one or two other types of loading which do not comply with the definition of either pure gust or pure manoeuvre spectra given above are analyzed as a group in section (C.13) of Appendix C. However, as no distinction is apparent between the effects of any parameters on the different groups of spectra the synopsis below deals with all types together.

The first point to notice in all the results obtained is that the scatter under this multi level block testing was low, being about that produced by fatigue tests under high constant amplitude loads (V = 0.02 or less), and that no effect of the mean life on the coefficient of variation was noticeable.

The overall impression created by the analysis is that the governing feature, as far as scatter is concerned, is the magnitude of the higher load levels in the test and the frequency of application of those levels. Provided some load level, that would produce a constant amplitude life of below 10^5 cycles with the associated small scatter, is applied sufficiently frequently then the scatter appears to be independent of the spectrum shape. The size of the block, the load level sequence and the introduction of ground-air-cycles had no noticeable effect on the scatter under either gust or manoeuvre spectra, whilst a gust spectrum gave most scatter when represented as a random $\frac{1}{2}$ cycle unrestrained test (Appendix C, section C.11), the other representations giving indistinguishable scatter.

5.4 Continuously variable amplitude tests

When the fatigue testing machine can apply any load level required, within a range of stresses, to the specimen in the course of any one test, the different possibilities as to the form the loading takes are innumerable. However in general two basic types of test are carried out on this type of machine namely:-

(1) those tests in which the stress history to be applied to the specimen follows a regular cyclic predetermined pattern, and

(2) those tests in which the stress history to be applied to the specimen conforms to a prescribed distribution of the frequency of occurrence of every stress but within which the sequence of the loads is random.

This latter type of test is probably the best approximation that can be made to the service loading likely to be experienced by an aircraft in the future and is thus of great interest in full scale testing, although unfortunately very few experimenters have yet conducted tests of this form. For this reason the number of results analyzed in Appendix D, which are related to this continuously variable amplitude type of testing, are few and present only a very vague outline of the scatter likely to be met when this method of testing is adopted, Figs.52-57. As in all the other appendices the assumption of a log-normal distribution is basic to the analysis presented, a synopsis of which is given below. The amount of scatter produced in tests under sine wave loading whose amplitude is varied continuously according either to a sinusoidal or to a repeated, exponentially increasing-decreasing pattern appeared to follow the general band governed by the constant amplitude data of Ref.1, exhibiting a tendency to give a good deal of scatter at the longer mean lives, Figs.52 and 53, whilst tests in which the amplitude followed a pattern that provided more cycles at the higher load levels, e.g. the gust frequency data of Fig.53 and the data of Fig.54, produced low scatter more consistent with the multi level gust data of the previous section.

In comparison, the tests conducted under a random load sequence when the input to the testing machine was provided by a white noise generator appear to give scatter that is independent of mean life, whilst tests in which three different severities of random testing are applied in repeated block form consistently give low scatter, Fig.55, although the mean lives are long. However a random loading used as the initial stressing to a constant amplitude runout stress provides results which are scattered to extents that appear to depend on the relative severity of the two loading portions and on the number of cycles of initial stress applied, with no obvious consistent trend.

6 DISCUSSIONS

In Ref.1 the concept of fatigue crack nuclei or initiation sites was applied to the interpretation of the analysis, presented in that report, of the scatter produced in constant amplitude fatigue tests. An active site was defined to be a location in a metal at which cyclic slip, caused by dislocation or vacancy movement, produces the extrusion of thin tongues or the intrusion of hollows on the surface of the metal. These formations act as sites from which fatigue cracks may propagate. The variation of the strengths and orientations of grains within a specimen leads to a distribution of active sites over the surface of the metal and also to a distribution of the relative effectiveness of these sites, where the effectiveness is to be taken as a measure of the quickness of formation of a crack. For each apparently identical specimen these two distributions will be different and the fatigue life of these specimens will thus exhibit scatter.

A possible correlation between the average number of nuclei per specimen, observed on the fracture surface at the end of a constant amplitude fatigue test, and the amount of scatter exhibited by tests of that form is shown in Fig.58, which is a reproduction of Fig.47 of Ref.1. This figure indicates that in general high scatter is associated with a scarcity of fatigue crack nuclei, such that some specimens may form two or more nuclei whilst others under the same conditions have only one site, which may be less effective than any of the more numerous sites in another specimen.

The available data on the average number of crack nuclei per specimen observed under variable amplitude loading is even more scarce than that for constant amplitude loading, but the trend shown in Fig.59, which presents this variable amplitude data, is of very much the same form as that of Fig.58 for the constant amplitude tests. High scatter is again associated with a scarcity of nuclei. Unfortunately the types of variable amplitude testing for which such data are available are limited but, in view of the similarity of Figs.58 and 59, it seems likely that this apparent correlation between the number of nuclei per specimen and the scatter exhibited is valid for all forms of loading. The discussion that follows, which considers the effect of various test parameters on the amount of scatter obtained from laboratory specimens tested under variable amplitude fatigue loadings, is based on this correlation. The four most common divisions of this type of loading, as defined in section 5 above, are treated separately in the order of increasingly realistic representation of service conditions.

6.1 One step tests

Unfortunately no data for any material are available on the number of active sites present per specimen at the end of one step tests and so no direct evidence exists that the correlation of Fig.59 applies to this form of testing. However, this difficulty is overcome by separating the one step test into two distinct portions of constant amplitude testing and by considering the effect of each portion on the number of nuclei present, bearing in mind that Ref.1 showed that high constant amplitude loading produces several nuclei per specimen with low associated scatter, whilst low constant amplitude loading produces few nuclei with correspondingly high scatter.

If a high initial stress is used in a one step test and applied for long enough for several nuclei to be formed, then it seems quite likely that the overall scatter will be unaffected by the runout stress level for the rates of crack propagation from all the nuclei will be similarly affected by this stress level. Thus the scatter expected would be that for a constant amplitude test at the high stress level. However, if the high initial stress is applied for a limited number of cycles such that the formation of all the active sites which would be created in a constant amplitude test at that stress level is not complete by the time the runout stress level is applied, then the runout stress will affect the scatter. A few cycles of a high initial stress will not be enough to form any sites and the scatter will be wholly governed by the runout stress level, whilst more cycles of the initial stress will create some nuclei, but not the full complement, and so the scatter could take any value depending on the length of application of the initial stress.

By similar reasoning, if a low initial stress is applied for a time long enough for the one or two nuclei associated with that stress level in constant amplitude testing to be developed, then it seems likely that ultimate failure will be caused by a crack from one of these nuclei rather than from any other nucleus formed under the runout stress, whether high or low. In this case then the scatter would be expected to be high whereas application of the low initial stress for a shorter period would mean that the crack from any nucleus formed would not have developed very far by the time the change of stress level occurred and so failure might occur from a nucleus formed under the runout stress level, which would then govern the scatter to a certain extent. If both stress levels are high or both low the scatter will clearly be low or high respectively.

Relating the picture developed in the previous paragraphs to the practical data on one step tests is difficult, as the times to the formation of nuclei under the various stress levels are not known. However the wide range of scatter encountered in this form of testing is explained by this model, and the majority of the trends, noted in Appendix A and summarized in section 5.1, can be explained with suitable assumptions as to the time to nucleus formation under each of the stress levels.

6.2 Two level repeated block testing

Two level repeated block testing, for which again no data is available on the number of nuclei per specimen, can also be considered in terms of the effects of two separate constant amplitude portions. In general the number of cycles of each stress applied in each block of loading is limited and the higher of the two stress levels is applied to the specimen fairly soon after the start of the test, even if the lower stress level is applied first, i.e. even if the sequence is low-high. Thus provided the period under which the specimen was loaded at the lower stress level is short enough for the formation of any nuclei to have advanced only slightly, the initiation stage of the fatigue failure of the specimen will for the most part take place under the higher stress level and a corresponding number of sites will be formed. If both stress levels are low, clearly the number of nuclei will be lower and the scatter higher, whereas if one of the stresses is reasonably high and applied in a block long enough to form nuclei then the scatter will be low.

It seems likely that, if a test was conducted in low-high sequence in which the first low level block was of such a length as to enable nuclei to form under the low stress conditions, then, however high the other stress was, the scatter would be high. Otherwise the block size would not be expected to affect the amount of scatter obtained so long as the majority of the initiation stage of the life of the specimen is under the higher stress level. Similarly the magnitude of the higher and lower stresses would not be expected to affect the scatter, provided that the higher stress is sufficiently high to give a constant amplitude life of below about 10^5 cycles, i.e. to cause the formation of several nuclei per specimen.

6.3 Multi level block testing

Similarly it seems likely that, whatever the load level sequence in multi level block testing provided the higher loads, which would produce several nuclei per specimen under constant amplitude loading, are applied sufficiently frequently, the scatter will be low. For in such cases the majority of the nucleus formation will take place at the higher stress levels even though they are less frequent than the lower levels, because the time to initiation is so short at these levels. Thus the number of nuclei formed will correspond to the higher constant amplitude stress level cases and the resulting scatter will be low. Omission of the highest load level will only affect the scatter if the highest of the remaining stress levels is not high enough to produce several nuclei, whilst omission of the lowest level will probably not affect the scatter at all. Changes in block size, load level sequence, basic spectrum shape and spectrum severity similarly would not be expected to alter the scatter if the higher stress levels still occur sufficiently frequently after the changes.

6.4 Continuously variable amplitude tests

Some data on the average number of nuclei per specimen, observed after various forms of tests involving continuously variable amplitude loading, are available from Ref.17 and from some unpublished work at the R.A.E. and these results are presented on Fig.59, from which it is interesting to notice that those test series which gave high scatter are tests in which a random initial loading, applied for a fixed number of cycles, was followed by a constant amplitude runout stress. However, all those tests in which the loading was random throughout gave low scatter, whilst the tests conducted under a sinusoidally alternating stress whose amplitude was continuously variable in a repeated pattern gave low scatter only when the number of cycles at the high stresses was appreciable, i.e. for the gust loading representation, but not for the exponential or sinusoidal variation of the amplitude. Thus in this case again the amount of scatter obtained can be explained in terms of the percentage time spent at the high stress levels and so in terms of the number of nuclei that would be expected.

The data on the number of damage nuclei observed in the specimens of Ref.17, the results for which are presented in Fig.56, fit the general trend of the scatter-number of nuclei relationship of Fig.59, but one or two odd points are apparent in Fig.56, e.g. when the random alternating initial stress had a root-mean-square value of 16 ksi, those specimens which were subjected to this initial stress for 90000 cycles gave more scatter apparently than those subjected for only 45000 cycles. If the random initial stress applied for 45000 cycles was sufficient to nucleate cracks, as appears likely from the shortness of the life under the low runout stresses after the initial stressing, then the scatter when the initial stress was applied for 90000 cycles should be as low as when applied for 45000 cycles. However this is not apparent from Fig.56. Similarly no reason can be given for the scatter being high when the runout stress level was high. If nucleation was complete under the initial stress, no effect of the runout stress level would be expected, whilst if incomplete the highest runout stress would be expected to give least scatter. The only explanation that can be suggested for either of these apparent trends is that in fact the formation of nuclei is not complete under any of the initial stresses and that the scatter is thus likely to be high, the disposition of the points of Fig.56 in the apparent trends noted above being a fortuitous event. This view is supported by the fact that the random stresses of 16 ksi and 13.5 ksi rms which are used as initial stresses in the tests considered here gave respectively 7 and $3\frac{1}{2}$ nuclei per specimen on average, whilst the highest average number of nuclei per specimen in the random initial stress tests was 3.

7 CONCLUSIONS

The fatigue lives, obtained from identical simple laboratory specimens under variable amplitude fatigue loading, exhibit a scatter which, in all cases but one, appears to correspond to the scatter that would be obtained from constant amplitude fatigue tests conducted under the maximum, frequently occurring load level in the variable amplitude loading. The one exception is the one-step test which consists of cycles of an initial constant amplitude sinusoidally alternating stress followed to failure by cycles of a second constant amplitude stress. In this type of test the scatter in fatigue lives depends not only on the magnitude of the two loads involved, but also on the order of their application and on the number of cycles of each load experienced by the specimens. No differences in the behaviour of 2024 and 7075 aluminium alloys under the variable amplitude loading can be distinguished and no test parameter other than the maximum, frequently occurring load level appears to affect significantly the amount of scatter exhibited.

Appendix A

ONE STEP TESTS

A study of the scatter produced in one step fatigue tests, as defined in section 5 of this Report, is made in this appendix and the effects of the three parameters n_1, S_1 and S_2 are considered. The life of a specimen in a one step test is defined as the sum of the number of cycles at the initial stress and the number of cycles at the runout stress, for it appears that the life defined this way gives the best fit with a log-normal distribution (section 2).

A.1 The effect of the number of cycles of initial stress (n_1)

Only three reports could be found which presented sufficient results, for either 7075 and 2024 materials, that related to the effect of changing the number of cycles of the initial stress applied, keeping S_1 and S_2 unaltered. Swanson[2] tested unnotched 2024-T4 specimens while Schijve and Jacobs used unnotched and notched ($K_T = 2.85$) sheet[3] and riveted lap joints[19], all of 2024 material under axial loading. The scatter produced by the unnotched specimens of Ref.3 appeared to be significantly affected by the number of cycles at the initial stress, an increase in n_1 increasing the scatter, and this effect seemed more pronounced when the initial stress was greater than the run-out stress Fig.24. An opposite trend occurred for the change from $n_1/N_1 = 0.05$ to $n_1/N_1 = 0.10$ in both sets of results, i.e. when $S_1 > S_2$ and when $S_2 > S_1$, this trend being significant at the 10% but not at the 5% level for the case when $S_2 > S_1$. However, both the stress levels applied in these tests are relatively low, giving constant amplitude lives of 2.0×10^5 and 1.3×10^6 cycles. Swanson's results[2] for unnotched materials with a manually applied high initial stress followed by a lower runout stress, 34 ksi for all Swanson's results, did not exhibit this increase of scatter with a greater number of cycles of the initial stress and the scatter appears to be independent of n_1, Fig.26. The range considered, 0-100 cycles only, was very small and may not be enough to effect the scatter. Similarly Swanson's tests with an initial stress of 12 ksi (below the fatigue limit) showed no effect on scatter of a change of n_1 in the range 0-35000 of cycles, except for that set of tests with only 10 cycles of the initial stress applied which exhibited greater (but not significantly so) scatter Fig.27. At an initial stress of 16 ksi, just about the fatigue limit, the scatter appeared (variance ratio significant at the 5% level) to be increasing with an increase in the number of cycles at the initial stress, Fig.27, thus following the apparent trend of the unnotched results of

Ref.3, whilst the notched results of the same reference (Fig.25) showed no apparent effect of the number of cycles at the initial stress when $S_2 > S_1$. No results for the case of $S_1 > S_2$ were suitable for analysis as several specimens ran out without failure.

Two other comparisons as to the effect of the number of cycles of the initial stress were possible in Ref.2, both of which (changes from 10-50 kc at 24 ksi and 10-30 kc at 28 ksi) showed a slight increase in scatter with an increase in the number of cycles at the initial stress which was lower than the runout stress. The riveted joints of Ref.19 showed no apparent effect with n_1 when $S_1 > S_2$ Fig.28, but when $S_1 < S_2$ the scatter was very low and appeared to decrease with an increase in n_1, reaching values much lower than those associated with either S_1 or S_2 under constant amplitude loading Fig.29.

A.2 Effect of the magnitude of the initial stress

Only the results of Swanson[2], for unnotched 2024 with a runout stress of 34 ksi, are sufficiently numerous to provide a comparison between the scatter produced by various initial stresses at constant runout stress, Figs.26 and 30. There is no well defined effect of increasing the magnitude of the initial stress in the range of stresses (48-68 ksi) and cycles of the initial stress (0-100 cycles) used in this reference, although in general the lower initial stresses appear to occur towards the top of the scatter bands shown, indicating possibly an increase in scatter with a decrease in initial stress. Eight different initial stresses were applied for 10 kc in the vibrophore tests and of these stresses 44 ksi and 40 ksi gave significantly more scatter than stresses of 32, 28 and 16 ksi and more, though not significantly so, than 24, 20 and 12 ksi with a runout stress of 34 ksi. The coefficients of variation for the initial stresses 12, 16, 20, 24, 28 and 32 ksi are respectively 0.041, 0.021, 0.034, 0.032, 0.028 and 0.021 which indicate an increase in scatter with a decrease in stress level when the initial stress has been applied for a reasonable number of cycles.

A.3 Effect of the magnitude of the runout stress

Again Ref.2 provides the only direct comparisons of the scatter involved in conditions identical but for the runout stress level. The cases considered in this reference on unnotched 2024 material were

(a) S_1 = 56 ksi for 20 cycles; S_2 = 20 and 40 ksi

(b) S_1 = 64 ksi for 10 cycles; S_2 = 20 and 40 ksi

(c) S_1 = 34 ksi for 10 kc; S_2 = 24 and 44 ksi .

All three comparisons gave significantly less scatter for the higher runout stress level than for the lower level, and it is to be noted that n_1/N_1 was small in all cases, for the mean life at 34 ksi was about 10^5 cycles.

Appendix B

TWO LEVEL REPEATED BLOCK TESTS

Whereas in a one step test a change from one stress level to the other is irreversible, a two-level repeated block test makes possible the alternate application of two stress levels throughout the life of the specimen,Fig.1b. A definition of this form of testing is given in section 5 of the main text whilst this appendix presents an analysis of the scatter obtained in such tests, the basis of this analysis being the assumption of a log-normal distribution of life under variable amplitude loading conditions.

B.1 Effect of the percentage of life at the higher of the two stress levels

The simplest measure of the percentage life at the higher stress level can be taken, if a large number of blocks are applied before failure, as the ratio of the number of cycles in a block of the higher stress level to the sum of the number of cycles in a low level and a high level block together. Refs.5, 20 and 21 combined provide results for both 2024 and 7075 unnotched specimens in rotating bending and they all show a reduction in scatter (and coefficient of variation) as soon as the higher stress level is introduced, a reduction, that is, from the constant amplitude scatter obtained at the lower stress level. Examples of this reduction are given in Figs.31, 32 and 33 in which the constant amplitude data at the lower stress has been plotted on the 0.1% line as the scale is logarithmic and a zero line cannot be plotted. The coefficient of variation appears to be sensibly independent of the percentage life at the higher stress level,once this level is introduced at all.

The results from Ref.22, by Smith, Howard and Smith, Figs.34, 35 and 36, for axially load sheet to 24S-T3 and 7075-T6 specifications, show no noticeable trend with percentage life at the higher stress level as is the case for the riveted lap joints of 2024 material from Ref.19 by Schijve and Jacobs Fig.37. In the former report only four specimens on average were tested under each condition and in all cases the higher stress was applied for a considerable percentage of the time, whilst in the latter report no absolute comparisons were possible as the overall combined block size was altered at the same time. Six comparisons of the effect of increasing the percentage of life at the higher stress level (50 ksi) from 4% to 10% for lower stresses of 20, 30 and 40 are given by Ref.23 and of these three gave higher coefficients of variation and three lower coefficients with the increase in percentage life, indicating again a lack of effect when the higher stress is present for any reasonable number of cycles.

B.2 Effect of the magnitudes of the higher stress level

Both the results of Corten, Sinclair and Dolan[5], Fig.32, and Liu and Corten[20] showed less scatter for the greater of two higher stress levels, keeping the lower stress constant. The effect was less obvious in Ref.20 than in Ref.5 possibly because the overall magnitude of the scatter was smaller, whilst in Ref.22 no effect with a change in higher stress level between 30, 40 and 60 ksi, with lower stresses of 16 and 30 ksi, was noticeable although when the two stresses applied were 16 and 17 ksi the scatter was greater than the general level.

B.3 Effect of the magnitude of the lower stress level

No significant or obvious effect of the magnitude of the lower stress level on the scatter in life was noticeable either in the rotating bending results of Refs.5, 20, 21 or 23, e.g. Fig.33, or in the axial sheet tests of Ref.22. This lack of any effect of the lower stress level combined with the general trend noted in section 6.2 may indicate that, provided the higher stress level is sufficiently large and sufficiently frequent in occurrence, then it is the higher stress level that determines to a large extent the amount of scatter presented by the test results.

B.4 Effect of the sequence of load level application

Two level repeated block tests using the same two stresses and the same numbers of cycles per block can be started either by applying the higher or the lower of the stress levels first. Six comparisons of Ref.23 gave 3 with a lower and 3 with a higher scatter for a test starting at the higher stress level, and 2 comparisons in Ref.3 also split equally, whilst of about 50 comparisons from Ref.22 in which only a few specimens were tested at each condition the same result was obtained. These data indicate that there is probably no sequence effect on scatter provided that the number of blocks of each stress level applied is sufficiently large. However, had only one or two blocks of each stress level been applied before failure, it seems likely that an appreciable sequence effect would have been observed, particularly if failure in general occurred towards the beginning of a block at the higher stress level.

B.5 The effect of the size of the blocks

The block size effect was studied in both Ref.20 and Ref.22, e.g. Fig.35 but in neither case was there a clear indication of a trend in scatter with block size. In some cases, however, the situation could arise that a crack is present in the specimen which would result in failure under a single cycle of the higher stress level, but which does not break under the remainder of the lower stress cycles. This situation can result in a greatly reduced scatter, if the crack only achieved the critical length on average towards the end of a low cycle block, or in a greatly increased scatter if the crack on average achieved this length towards the end of a high stress block. Obviously the length of the block would be likely to affect the scatter obtained in these cases.

B.6 The effects of mean stress, sheet thickness and material

There was no apparent effect on the scatter of a change in mean stress from 0 ksi to 20 ksi [22] Figs.34 and 36 in which the scatter was already low, or in Ref.19 Fig.37 in which no direct comparisons could be made. Similarly no effect of the sheet thickness on the amount of scatter present could be seen in the results from Ref.22, whilst overall there did not appear to be any distinction as to scatter in these two level block tests between the 2024 and the 7075 materials.

B.7 General

Perhaps the most obvious observation on the amount of scatter involved in two level repeated block tests is that the scatter is generally small ($v < 0.02$) unless both stress levels are low. The scatter in the one step tests is on the whole higher for the same stress levels, Fig.24, especially if the lower stress level is the initial stress in the one step tests. Possibly the scatter in two level repeated block tests is largely governed by the higher of the two stress levels present if the blocks are not too long.

Appendix C

MULTI LEVEL BLOCK TESTS

The addition of further load levels at which stresses can be applied to the specimen enables the experimenter to represent aircraft flight loading more accurately. When several stress levels are used in a test the possible variations in the order of application of the different magnitudes of stress level become numerous, but of these orders some are more frequently used than others. That sequence in which the highest load block is applied first and the stress level thereafter decreased to the end of the block is designated high-low, Fig.1c, whilst if the lowest stress level is applied first and the stress thereafter increased the designation is low-high. Similarly such nomenclature as low-high-low and high-low-high is self explanatory, whilst the final of the more common sequences is termed random. In this form the relative frequency of application of each load level block is governed by the load frequency distribution desired but the sequence of application of these load levels is random, Fig.1d.

In general, for fatigue tests relating to aircraft usage, two basic types of overall loading spectra are used, namely gust spectra (loads caused by atmospheric turbulence) and manoeuvre spectra (pilot induced loads resulting from turns, rolls etc.). The loads in the manoeuvre spectra are generally higher than those in the gust spectra, in the case of fighter aircraft which carry out rapid manoeuvres, and are asymmetric about the level flight condition, whilst gust loads are usually symmetric about this condition. These two types of multi level tests are considered separately within each section below. However in both cases the analysis is based on the assumption of a log-normal distribution of lives under variable amplitude loading of all forms.

In section C.13 of this appendix the available fatigue test results conducted under some other basic spectra, such as an exponential spectrum which can be used to simulate both a gust and a manoeuvre spectrum, are analyzed. These tests however also belong to the multi level block family and are analyzed in terms of the log-normal distribution of life as well.

C.1 Gust spectra-sequence effect

Several references provide data that enable a comparison to be made between different sequences of application of the same load spectra and among them is Ref.6 in which Schijve and Jacobs present results for 7075 and 2024 riveted joints. The amounts of scatter in the 7075 material for low-high,

high-low and low-high-low sequences are very similar, whilst for the 2024 material low-high and high-low sequences give significantly smaller scatter than for the low-high-low tests, although even for these latter specimens the standard deviation was only 0.154, Figs.39 and 40. No significant effect of sequence on scatter can be found in Refs.24, 25, 26, 27 and 28 which together provide 26 valid comparisons between low-high, high-low, high-low-high, low-high-low and random orders for both 2024 and 7075 material, e.g. Fig.41, and similarly no sequence effect was visible in the results of Refs.7 and 11, Figs.45 and 46, for which the coefficient of variation was unusually high, nor in Ref.29 for a comparison between programme and random tests at 12 stress levels on a 2L.65 material.

Indirect comparisons only, as other parameters also change, are possible from the results of Refs.30 and 31, Fig.38, for 7075 material, but here again there is no obvious distinction in the magnitude of the scatter between low-high and random sequences. Thus of more than 30 comparisons made as to the effect of sequence on scatter only one was statistically significant and this was related to abnormally low rather than abnormally high scatter.

Manoeuvre spectra

Crichlaw et al.[30] showed no distinction between the scatter in a set of low-high multi level block testing and that in a set of tests of random sequence block testing, whilst Corbin and Naumann[26] similarly showed no effect of the type of test on the scatter, comparing random cycle and block testing to a manoeuvre spectrum.

C.2 The effect of the size of the block

Gust spectra

Twelve valid comparisons can be made from the references quoted in the previous section as to the effect of the number of cycles per block used in the tests, e.g. Fig.41. Of these six gave more scatter to the larger block size whilst six gave less scatter, but none of the comparisons was statistically significant. As in the case of the two level repeated block test, it appears that provided enough blocks are applied to the specimen before failure, then the scatter is independent of the block size, but it would also seem likely that if only one or two blocks were sufficient to cause failure then the time of failure within a block might become important and affect the scatter.

Manoeuvre spectra

Naumann and Schott[25] studied 7075 material to a manoeuvre spectrum, and their results indicated, Fig.47, that a programme with 15000 cycles per block gave significantly less scatter than a programme with either 300 or 3000 cycles per block. The 15000 cycles per block also gave less but not significantly less scatter than the 1000 cycles per block programme, whilst the mean life appears to increase slightly with the size of the block.

C.3 The effect of the number of load levels used

Gust spectra

One comparison was possible from Ref.25 by Naumann and Schott between an 8-level and a 4-level representation of the same spectrum, and the 8-level test gave slightly greater scatter, whilst from Ref.30 although no strict comparisons were possible, as other parameters altered as well, the greatest scatter was given by the 4- and 5-level representations. With more levels than this no effect was discernible from these results.

Manoeuvre spectra

Ref.30 showed no effect of the number of load levels employed in the spectrum representation whilst Ref.25 showed a significant decrease in scatter with change in mean when the number of load levels was reduced from 8 to 4 in a manoeuvre spectrum representation.

C.4 The effect of the basic spectrum shape

Gust spectra

The effects on the fatigue lives of specimens tested under two basic gust frequency spectra are compared in Ref.24. Spectrum B, with more cycles at the lowest stress level but fewer at all the higher levels, appeared to give longer mean lives in general, Fig.42, and perhaps slightly more scatter than spectrum A although no direct comparisons were possible. In Refs.7 and 11 nine different spectra made up from the same six stress levels in various proportions were used on 2024 and 7075 materials, but in this case no effect of the spectrum shape on the scatter is visible Figs.45 and 46, even though three different proportions of the maximum load level were used.

C.5 The effect of the omission of low load levels

Gust spectra

In Refs.28 and 32 the effect on 2024 and 7075 materials of omitting the lowest of eight stress levels just below the fatigue limit both in the presence and in the absence of ground-air-cycles is seen to be a reduction in mean life in terms of cycles but without any change of the scatter. Similarly the effect of omitting the next lowest stress level as well, just above the fatigue limit, and the next two lowest stress levels is tested in Ref.32 and a further reduction in mean, leaving the scatter unaltered, is noted, Fig.43. The same effect is also shown for both materials by Ref.28, Figs.39 and 40, on omission of the lowest and the two lowest load levels, both of which were below the fatigue limit.

Manoeuvre spectra

Omission of the lowest of eight load levels in a manoeuvre spectrum representation was found by Naumann and Schott[25] to leave the scatter sensibly unaltered whilst reducing the mean slightly. Naumann[32] Fig.47 found the same effect for 7075 with a slight reduction in mean but with the scatter unaltered. However, in the same report, when considering only a 4-, rather than an 8-, load level representation of the same manoeuvre spectrum, omission of the lowest load level increased the scatter appreciably, reduced the mean and so resulted in a reasonable increase in the coefficient of variation. When the two lowest load levels were omitted from the 8-level representation Fig.37 the mean is further decreased but the coefficient of variation is sensibly unaltered.

C.6 The effect of omission of high load levels

Gust spectra

Ref.8, Fig.44, and Refs.6 and 32 show no appreciable effect on the scatter of omitting the highest load level or of applying it less frequently, although all these changes resulted in a reduction in the mean number of cycles to failure. Similarly Schijve and Jacobs[6] omitted the highest load level but increased the number of the next highest load cycles threefold and achieved a reduction in mean leaving the scatter unaffected. However, when in Refs.6 and 32 the second highest load level was omitted as well as the highest level, the coefficient of variation increased appreciably with little effect on the mean.

Manoeuvre spectra

The inclusion of two infrequently occurring high loads in an 8-level manoeuvre spectrum representation[25] seemed to have little effect on either the mean or the scatter, whilst the addition of two high negative load factors in the basic spectrum appeared to reduce the scatter significantly and the mean slightly. Together the effect of the two negative and the two positive high load factors gave a reduction in scatter, mean and coefficient of variation.

Corbin and Naumann[26] observed the reverse trends; the addition of two high positive load levels increased the scatter slightly, leaving the mean unaffected whilst two negative load factors decreased the mean but left the scatter unaltered. The experimenters of Ref.32 ran two series of tests; in one the number of cycles at the highest stress level was decreased and in the other the number of cycles was increased. The former showed little or no change in mean or scatter, whilst the latter set of tests appeared to decrease the mean and to increase the scatter considerably.

C.7 The effect of a preload and of periodic high loads

Gust spectra

A decrease in scatter in the results for the riveted joint of Ref.8, Fig.44, and of Ref.6, Fig.39, was obtained when in the former case a positive half-cycle preload was applied and when in the latter case of a positive half-cycle preload was applied omitting the highest stress level of the spectrum. Periodic half cycles of high positive load applied after every low-high stress sequence left the scatter unaltered, whilst stopping the application of these high half cycles after 50 blocks reduced the mean and the coefficient of variation slightly and applying the high half cycles after every second block resulted in a significant increase in the scatter, Fig.39. When the load sequence was changed to high-low and periodic high half cycles were applied after each block, the mean life was lower than for the low-high sequence, and the scatter significantly greater, but the introduction of an infrequently occurring high load in Ref.8 did not noticeably affect the scatter, although increasing the mean slightly.

C.8 The effect of ground-air-cycles

The effect on the scatter in fatigue life of introducing ground-air-cycles into the spectrum representation has been studied in Refs.6, 13 and 28 in all of which the mean life was reduced but the scatter was sensibly unaltered. Preceding or following the ground-air-cycle by a high positive half cycle in Ref.6 increased the mean life especially in the latter case but again left the scatter apparently unaltered, whilst increasing the frequency of application or the magnitude of the ground-air-cycles in Ref.28 had the same effect.

C.9 The effect of spectrum severity

Gust spectra

The effect on the scatter in fatigue life of changing the severity of a load spectrum, keeping the basic shape constant, as given by Refs.24, 25, 26 and 28 is a change in mean life with no obvious effect on the scatter, although a change in mean stress from 10 or 20 ksi to 0 ksi increased the scatter by a factor of 2 for 7075 material.

Cricklaw et al.[30] showed no noticeable effect of the mean stress on scatter, whilst Freudenthal and Heller[7,11] who varied the severity of their basic spectra in several ways, namely by proportionally diminishing each stress level $A \rightarrow A_o$, $B \rightarrow B_o$, $C \rightarrow C_o$, Figs.45 and 46, by reducing the range of stress levels employed, (in A_L, B_L and C_L all the stress levels are low, whilst in A_H, B_H and C_H all the stress levels are high), and by subtracting a fixed stress from each stress level, also found no obvious trend in scatter though the mean life was affected by the changes.

Manoeuvre spectra

A proportionate reduction in the stress levels of the spectrum by 20%, increased the mean and reduced the scatter slightly[26]. The coefficient of variation decreased slightly. This reduced spectrum was applied in a set of tests to specimens to which the original spectrum had been applied for various percentages of the expected life of the specimens. The scatter increased significantly as the percentage life, for which the original spectrum was applied, increased, the mean decreased and so the coefficient of variation increased. When the reduced spectrum was applied, before the original spectrum, for 50% of the specimens' expected life, the scatter was reduced to its original value, Fig.48.

C.10 Load location

Gust spectra

The effect of the location of loads within a test was studied for both 2024 and 7075 materials in Ref.28 by Naumann, using a gust spectrum in random cycle block testing. No effect on scatter was observed of the position of starting within the random sequence of loads, except when a high load occurred at a time such that the crack had reached a critical size. The scatter was then reduced by a factor of 2.5 to 3 with little change in mean. No steady trend with the position of the highest load was observed, but those tests which did not last until the highest load cycle was applied had the largest coefficient of variation.

C.11 Programming techniques

Gust spectra

Four methods of application of the same load levels and frequencies of occurrence were compared in Ref.28, being

(1) Block test.

(2) Random cycle (each positive ½ cycle followed by an equal negative ½ cycle).

(3) Random ½ cycle restrained (negative ½ cycle follows a positive ½ cycle).

(4) Random ½ cycle unrestrained (no limitations on each successive ½ cycle).

It was found that block programming and random half cycle restrained gave significantly less scatter than random half cycle unrestrained, the other comparisons being non-significant, although random cycle programming also gave less scatter than random half cycle unrestrained. The mean lives under the different programming techniques were found to increase in the order (1) → (2) → (3) → (4) with (1) being considerably less than the others.

Manoeuvre spectra

Representing the spectrum in terms of a random cycle test spectrum rather than in terms of a random level sequence block test spectrum resulted in no change in the coefficient of variation, whilst the life under the random cycle test tended to be shorter than under the random block test, especially when negative load factors were included[26], Fig.48.

C.12 Miscellanea

Gust spectra

Schijve and Jacobs[33] tested specimens of 2024-T3 sheet 5 mm thick in the form of lug specimens ($K_T = 3.5$), the object being to study the effect of the size of the holes through which the lugs were loaded. The arrangements tried were

(a) Standard lug.

(b) Slotted hole.

(c) 3% expanded hole.

(d) 1.5% expanded hole.

The loading was a six stress level, low-high-low sequence to a basic gust spectrum. The mean lives obtained increased in the order (b), (a), (c) and (d) whilst the scatter values obtained, in terms of the coefficient of variation, were (a) 0.026, (b) 0.0315, (c) 0.0135 and (d) 0.0151, which show that the expanded holes, with their associated relief of fretting phenomena, gave reduced scatter and longer lives than the standard lug and slotted hole specimens.

C.13 Some other load spectra

Cricklaw et al.[30] provide a few results for four different loading spectra, being

(a) Ground loading.

(b) Fighter manoeuvres.

(c) Composite manoeuvres.

(d) Composite gust.

In all cases the scatter involved was small, the coefficient of variation being less than 0.03, with no obvious trends as to load sequence and block size.

Freudenthal[12] conducted tests, to a random cycle sequence of 6-load levels. The input load spectrum was an exponential spectrum which is known to represent fairly well the manoeuvre and gust loadings of aircraft in operation and the effect of increasing the severity of the spectrum (i.e. increasing the percentage of cycles at the higher load levels) was to reduce the mean but to leave the coefficient of variation unaltered. The maximum load level was utilized in all the four different severities of spectrum applied, the fatigue life for this load level at constant amplitude loading being 5×10^4 cycles. The scatter in the test results was higher than expected ($0.028 < v < 0.04$) but there are no constant amplitude tests with which to compare the scatter obtained.

Refs.9 and 10 also contain results for rotating beam tests to a random cycle sequence of 6-load levels to an exponential spectrum. Three different spectral shapes were considered with exponential slopes $h = 17.3$, 22.9 and 34.3, where an increase in h increases the number of cycles at the lower stress levels considered. The effect of a bodily shift of the spectrum so that all the stresses in the spectrum increased by the same amount was also studied. Figs.49 and 50 present the results from Ref.9, from which the value of h, i.e. the spectral shape, appears to have little or no effect on the magnitude of the scatter. However the shifting of the whole spectrum, i.e. the increase of each load level by a fixed increment $A \rightarrow B \rightarrow C \rightarrow D$ tends to decrease the mean life and perhaps to decrease the scatter as well. In spectra A and B for any value of h the frequency of occurrence of the top two load levels is very low indeed, the maximum frequency for the one top load level being 1 in 1000 cycles in spectrum B of $h = 17.3$ which gives smaller scatter than any of the others for 2024, Fig.49. Moving on to spectra C and D, the frequency of occurrence of the high loads has increased and the scatter is lower again.

Refs.7 and 11 give data on random cycle sequenced six level loading in rotating bending. Spectra D[11] are exponential distributions and represent, reasonably well, manoeuvre spectra. It is to be noticed from Figs.45 and 46 that in all cases the coefficient of variation under spectra D is such that $0.02 < v < 0.03$. In Fig.46 twenty specimens of 7075 material were tested under the same spectrum D represented as two differently ordered random sequences. The results can be seen to agree very well with one another both as to scatter and as to mean. The mean life for 2024 material is much the same, Fig.45a, and when the stress levels are all reduced by ΔS, the stress level interval, the mean life achieved increased but the scatter remained very much as before ($D \rightarrow D(6)$) Fig.45.

Naumann[27] investigated the effect of the method of load history counting and the type of load programming on the fatigue properties of edge notched 2024-T3 aluminium alloy sheet. The loads were applied axially to the specimens by a machine that could be programmed at 55 distinct load levels. The following four load spectra inputs were studied:-

(A) White noise.

(B) Atmospheric turbulence.

(C) Single degree of freedom.

(D) Bimodal power spectrum.

These basic time histories were converted to peak histories and then the peak histories were reduced to sets of statistics by several different methods of counting, e.g. means and amplitudes, maximum peaks, level crossings, etc. The statistics thus obtained were then used to program variable amplitude fatigue tests of which three basic types were compared.

(a) Random cycle tests.

(b) Random load level sequence block tests at constant mean.

(c) Random load level sequence block tests at varied mean.

All in all ten methods of representing each basic spectrum were obtained and tests carried out to these representations. As can be seen from Fig.57 the scatter in all cases is low and no obvious trend with scatter can be distinguished between any results, although an increase in the design limit load of the white noise time history (i.e. a decrease in severity) reduces the mean and may possibly reduce the scatter slightly.

Significantly less scatter was given for spectrum (A) by program (1), in which the peaks were programmed randomly in accordance with the filtered time history, than by the other methods of counting and representation of the white noise spectrum. This was not the case for the other spectra (B), (C) and (D) and so this was thought to be most likely a fortuitous event, whilst program (6), whereby the amplitudes used in the test were obtained from the 'means and amplitudes' counting method and the test was of constant mean random load level sequence block, gave longer mean lives than the other tests but the coefficient of variation was unaltered. It is also noteworthy that the atmospheric turbulence tests gave very low coefficients of variation which were at least as low as for any of the other spectra employed here.

Fretting

In Ref.34 Gassner presents the results for tests he conducted into the effect of fretting on the fatigue life of a specimen constructed of a 2024 type aluminium alloy. The load spectrum, intended to represent a combination of gust and manoeuvre loads, was applied at 8 discrete levels in a low-high-low program of repeated block sequence. The specimen was a notched, flat specimen, 5 mm thick, with two asymmetric notches, which consisted of a large circular hole (4 mm radius) with a smaller offset circular hole overlapping (see Fig.51). Two such notches were put in each specimen in an attempt to reduce the scatter of the results. In the lower, smaller drilled hole a steel pin was inserted, in those tests involving fretting, and connected to a lever system which applied a constant load on the pin perpendicular to the hole surface. The results for these tests (Fig.51) show that fretting reduces the mean life considerably whilst the standard deviation is sensibly unaltered (the two plots are parallel).

Appendix D

CONTINUOUSLY VARIABLE AMPLITUDE TESTS

Some testing machines do not restrict the number of load levels that can be used to a discrete quantity but allow the application of any load amplitude within a defined range. Tests conducted on such machines must be governed by some control mechanism, usually either mechanical or electrical, which defines the overall spectrum of loads to which the specimen is subjected, although in some cases the load sequence to be applied cannot be forecast. Such a case arises when the load control mechanism is an electronic white noise source, whether filtered or not, which gives an output that is definable in terms of averages but not in terms of instantaneous values. Thus two basic types of continuously variable amplitude testing are possible

(i) those tests in which the load history to be applied to the specimen is known, e.g. Fig.1e;

(ii) those tests in which the load history to be applied to the specimen is not known, but in which the overall spectrum of loads is defined, e.g. Fig.1f;

and the analysis that follows considers these two groups separately, in terms of the log-normal distribution of lives under identical loading conditions.

D.1 Fixed sequence tests

Hardrath and Utley[14] and Hardrath, Utley and Guthrie[15] consider rotating beam fatigue tests of 2024 and 7075-T6 materials respectively, in which the load applied to the specimens was a sinusoidally alternating load whose amplitude was varied according to a mechanically applied signal. The variables from test group to test group were

(a) the applied signal (which affects the frequency distribution of the amplitude of the alternating load);

(b) the amplitude or range of the signal, and

(c) the mean of the signal.

From Figs.52 and 53 it can be seen that the coefficient of variation for these tests covers a wide range of values, especially for the 7075-T6 material, with no obvious distinction in the magnitude of the scatter for the two types of signal, e.g. sinusoidal and exponential variation, though for the same mean stress and range of stress the sinusoidal signal tends to give a lower mean life than the exponential signal. However, in Ref.15 Hardrath, Utley and Guthrie conducted three sets of tests to a signal representing a gust spectrum of loads, for which the coefficients of variation were all less than 0.02, Fig.53, but the gust spectrum contained cycles at higher stress levels than were applied in any of the tests under either the sinusoidal or exponential signals, and fewer low stresses as well.

The effect of the mean of the sine wave signal input can be assessed from both Refs.14 and 15. For 2024 material[14], Fig.52, there is no clear trend in scatter with the mean stress of the signal at all, but the mean life clearly is shorter for the higher mean stresses. In Ref.15 however the scatter for unnotched 7075-T6 material seems to become smaller for the larger mean stresses with six statistically significant comparisons for the unnotched sinusoidal signal and three significant comparisons for the unnotched exponential signal. Sinusoidal signal tests for 7075-T6 notched material provide statistically significant comparisons, giving less scatter for the higher mean stress of the signal in seven direct comparisons, whilst only one direct comparison is possible for the notched exponential signal which is not significant although the scatter decreases with a decrease in mean (contrary to above).

Spitzer and Corten[23] also tested specimens under a sinusoidally alternating load but in these tests the variation of the amplitude was as shown in Fig.54 and the effect of altering the position of starting within this load sequence was studied. Four different positions were investigated and the results from all four were almost identical both as to scatter and as to mean, Fig.54, the scatter being very low for all starting positions.

D.2 Random cycle sequence tests

An amplified white noise source was used by both Head and Hooke[16] and Kirkby and Edwards[35] as the load control input to their fatigue machines, which in the case of Ref.35 resulted in a test under a spectrum of loads that could be considered as a randomly modulated sine wave having a distribution of peaks tending to a Rayleigh distribution. Specimens were tested under this form of loading for several different values of the root-mean-square stress and the results of the scatter analysis of this data are plotted in Fig.55, from which it can be seen that in 3 cases the scatter was low, whilst the fourth case indicated greater scatter. Omitting one test result from this last group reduced the scatter considerably (Fig.55) to the value shown by the other 3 groups. Repeated block program tests in which 3 levels of random loading were applied in low-high sequence all produced small scatter, Fig.55, the overall spectrum of applied loads representing a gust spectrum. Increasing the severity of the spectrum A → B → C while decreasing the mean left the scatter sensibly unaltered, whilst the coefficient of variation for high-low tests to the spectrum was higher than for low-high tests to the same spectrum, but not significantly so. In spectrum B+ the maximum root-mean-square stress level block was omitted, producing a decrease in mean with no apparent change in scatter. Only two sets of tests were carried out in Ref.16 under the white noise spectrum stabilized to give a controllable average peak stress output. The average peak stresses chosen were 11820 psi and 12900 psi, for which the mean log lives were respectively 6.321 and 6.059 and the coefficients of variation 0.039 and 0.025 which are rather higher than those obtained in Ref.35.

Swanson[17] found that the effect of increasing the peak root-mean-square value of a random loading conforming to a Rayleigh density distribution, of peaks, as above, at constant mean stress was to decrease the mean life leaving the coefficient of variation sensibly constant, Fig.56. Similarly, using a two degree of freedom loading system he obtained a combined Rayleigh-Gaussian density distribution of peaks and found that an increase in root-mean-square level of the peaks decreased the mean life but left the coefficient of variation sensibly unaltered, being indistinguishable from that for the single degree of freedom system above (centred between $0.02 < v < 0.03$).

Swanson also carried out tests in which a random single degree of freedom initial stress was applied for a definite time and was followed by a constant amplitude runout stress. Fig.56 shows that the coefficient of variation obtained in these tests covers a wide range ($0.01 < v < 0.09$) and whereas for the random initial stress of 16 ksi rms applied for 90000 cycles and various runout levels the scatter was always high $0.06 < v$, for 45000 cycles of initial stress at the same level the scatter is high for a runout stress of 40 ksi but for the lower runout levels the scatter is low $v > 0.02$. When the initial stress level was 13.5 ksi rms, the scatter was high for 180 000 cycles of application and high runout stresses but low for the lower runout stresses, whilst for 90000 cycles of initial stress the scatter is high with a runout level of 40 ksi but low for the other lower runout levels except perhaps for 28 ksi where the coefficient of variation is 0.047 (which is between the high and the low scatter values).

SYMBOLS

c	number of test specimens in a sample
F	ratio of the estimated variances of two samples ($F \geq 1$)
h	experimental slope of a spectral shape
K_T	notch acuity
L	number of cycles to failure (life) of a randomly selected specimen
L_r	number of cycles to failure (life) of the r^{th} specimen (r, 1,2 ... c)
M	number of different alternating stress amplitudes used in a test
n_i	number of cycles per block of alternating stress amplitude S_i
N_i	number of cycles to failure for a specimen tested under constant amplitude alternating stress level S_i alone
$p(x)\,dx$	probability of occurrence of an event between conditions x and $x + dx$
P	probability of failure by the time considered
S_a	amplitude of a sinusoidally alternating stress (= $\frac{1}{2}$ stress range)
S_i	amplitude of the i^{th} sinusoidally alternating stress ($i = 1,2 \ldots M$)
v	coefficient of variation $\left(= \frac{\sigma}{\mu}\right)$
μ	population mean
$\hat{\mu}$	best estimate of the population parameter μ from a sample of magnitude c
σ	population standard deviation
$\hat{\sigma}$	best estimate of the population parameter σ from a sample of magnitude c
$\hat{\sigma}_1, \hat{\sigma}_2$	two best estimates of σ from two samples

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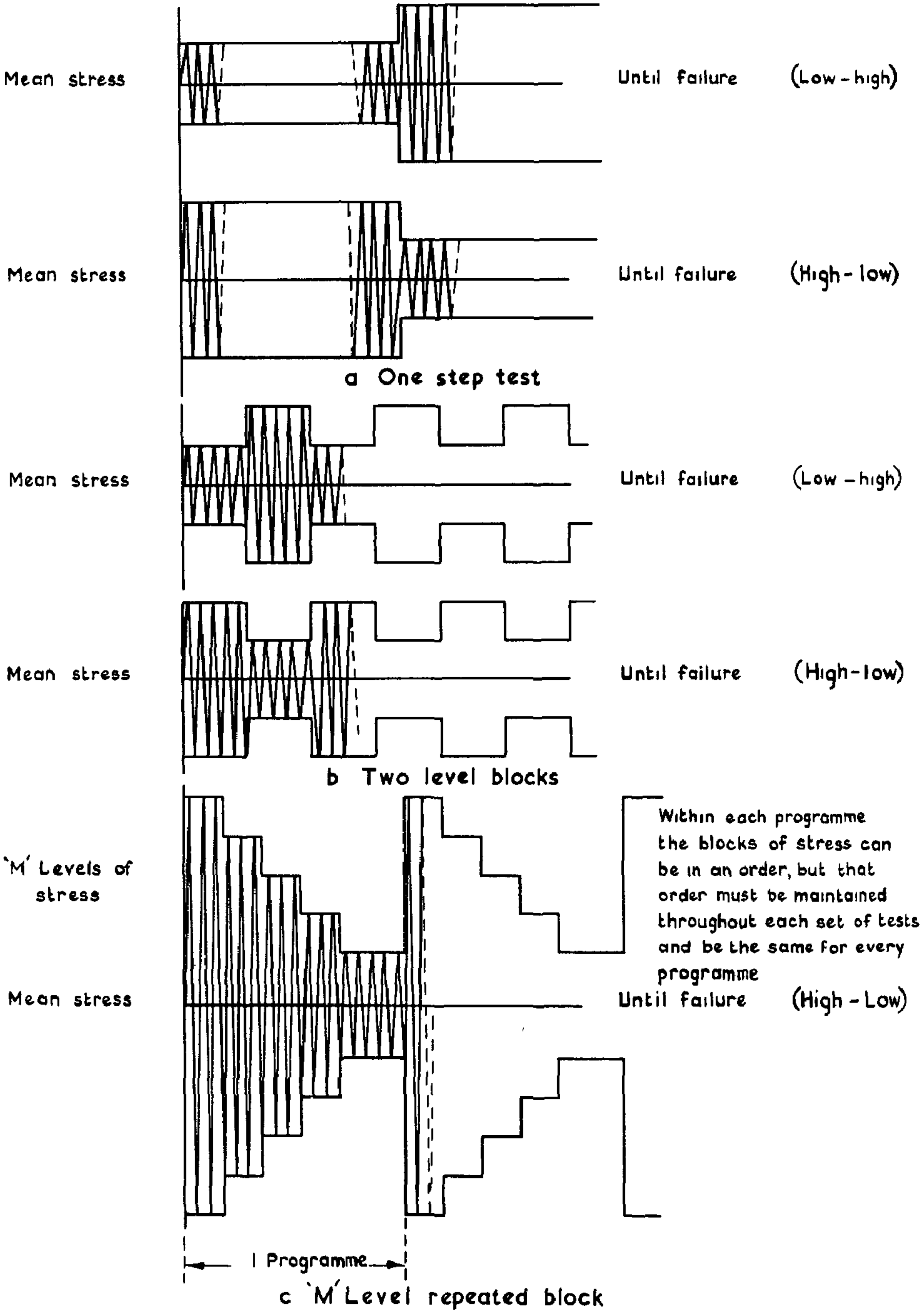
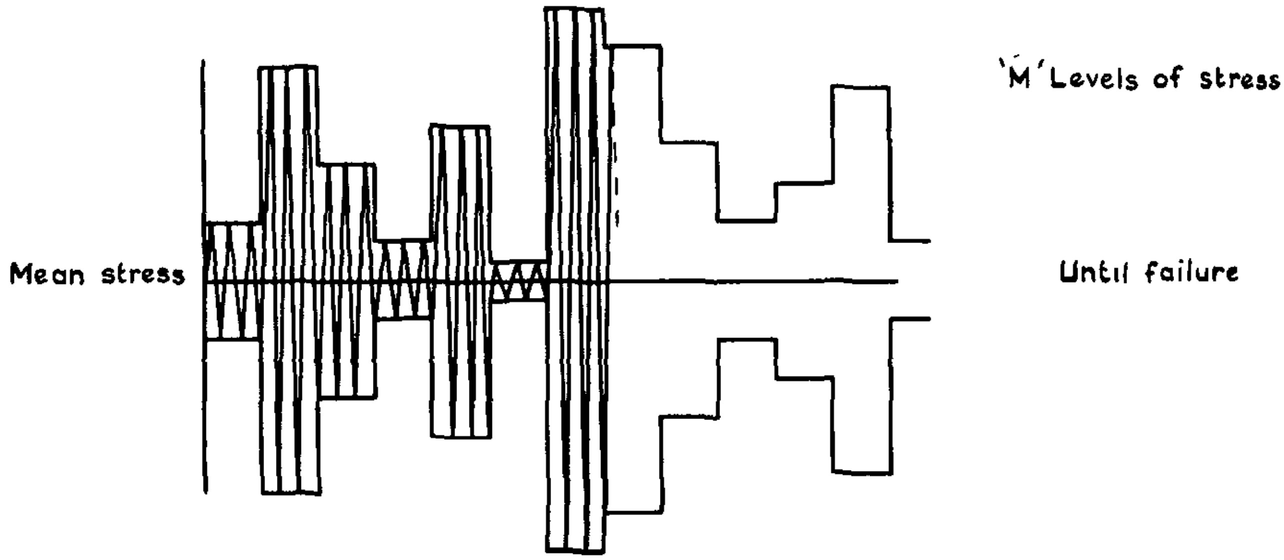
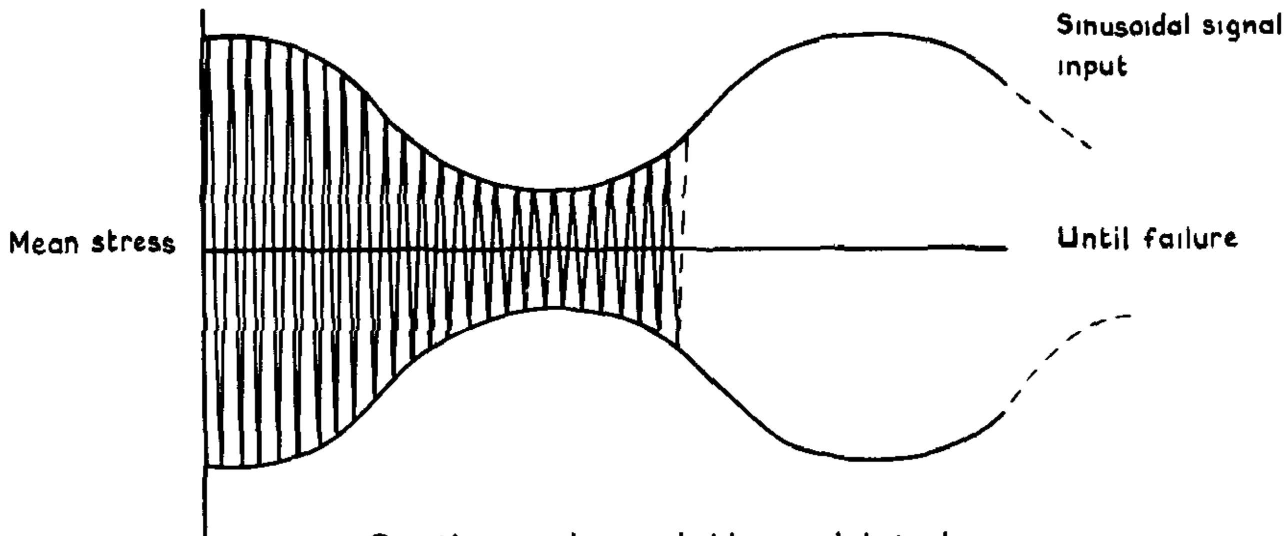


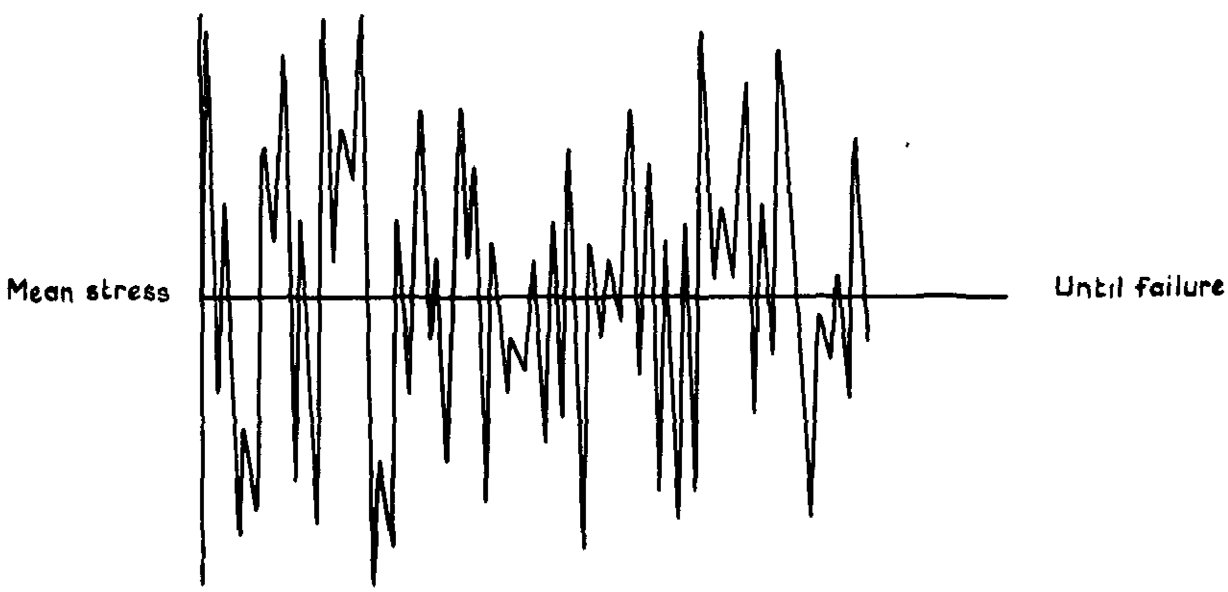
Fig 1a-c The various types of variable amplitude fatigue loading



d Random sequence block



e Continuously variable modulated



f Random cycle

Fig 1d-f The various types of variable amplitude fatigue loading

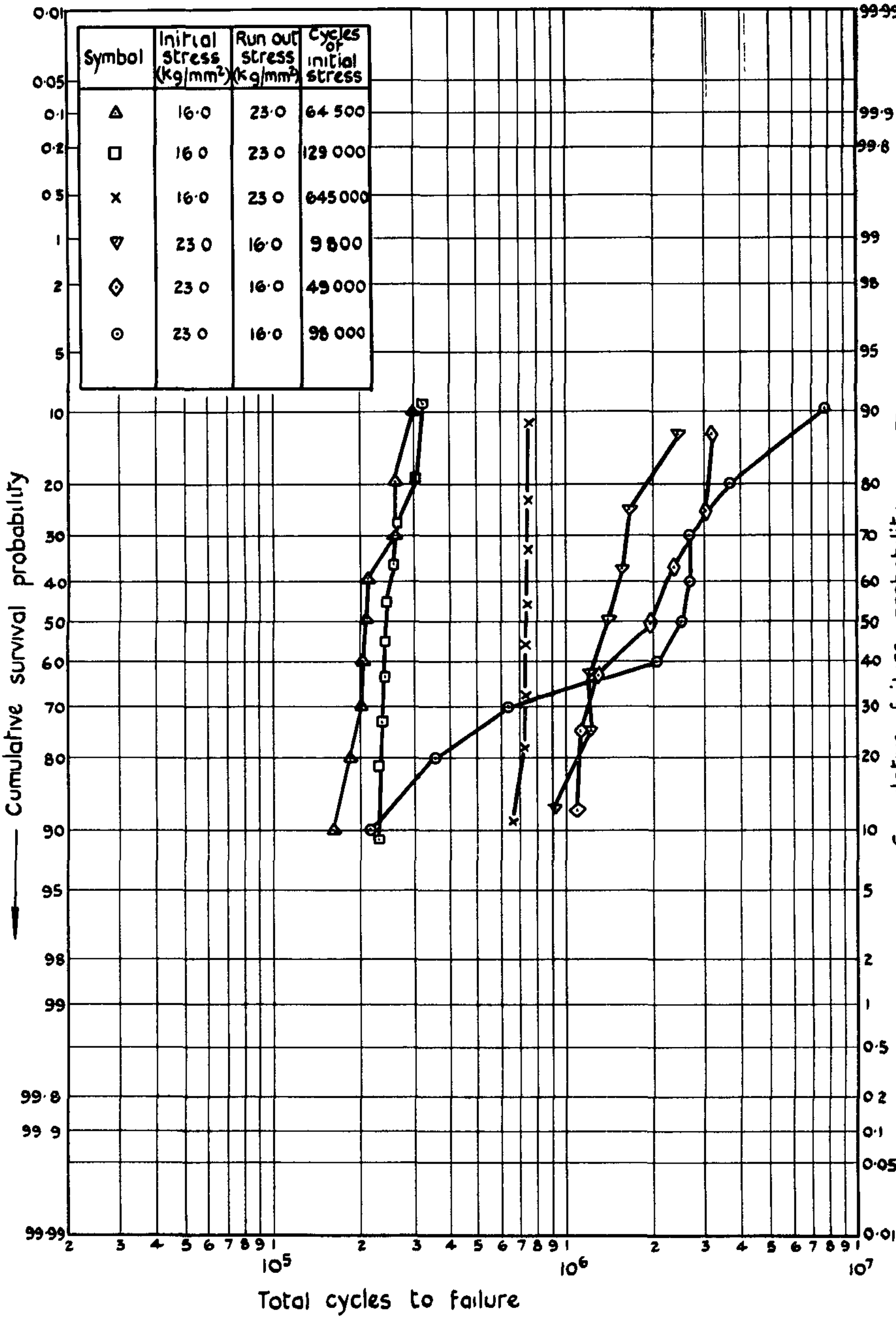


Fig. 2 Unnotched, 24 S-T sheet specimens. One-step results from Ref (3)

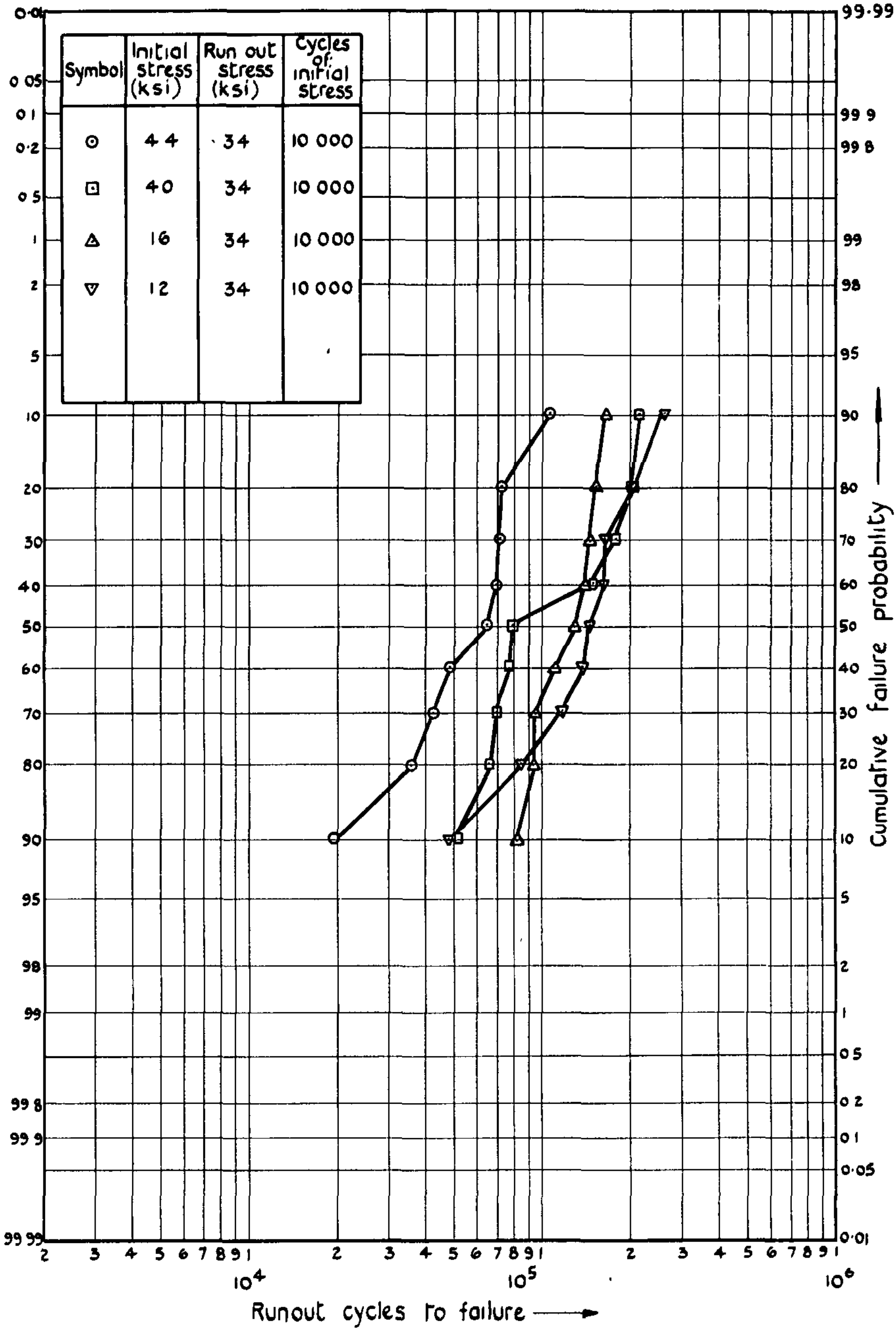


Fig. 3 Unnotched. 2024 bar specimens.
Vibrophore 'one-step' results from Ref(2)

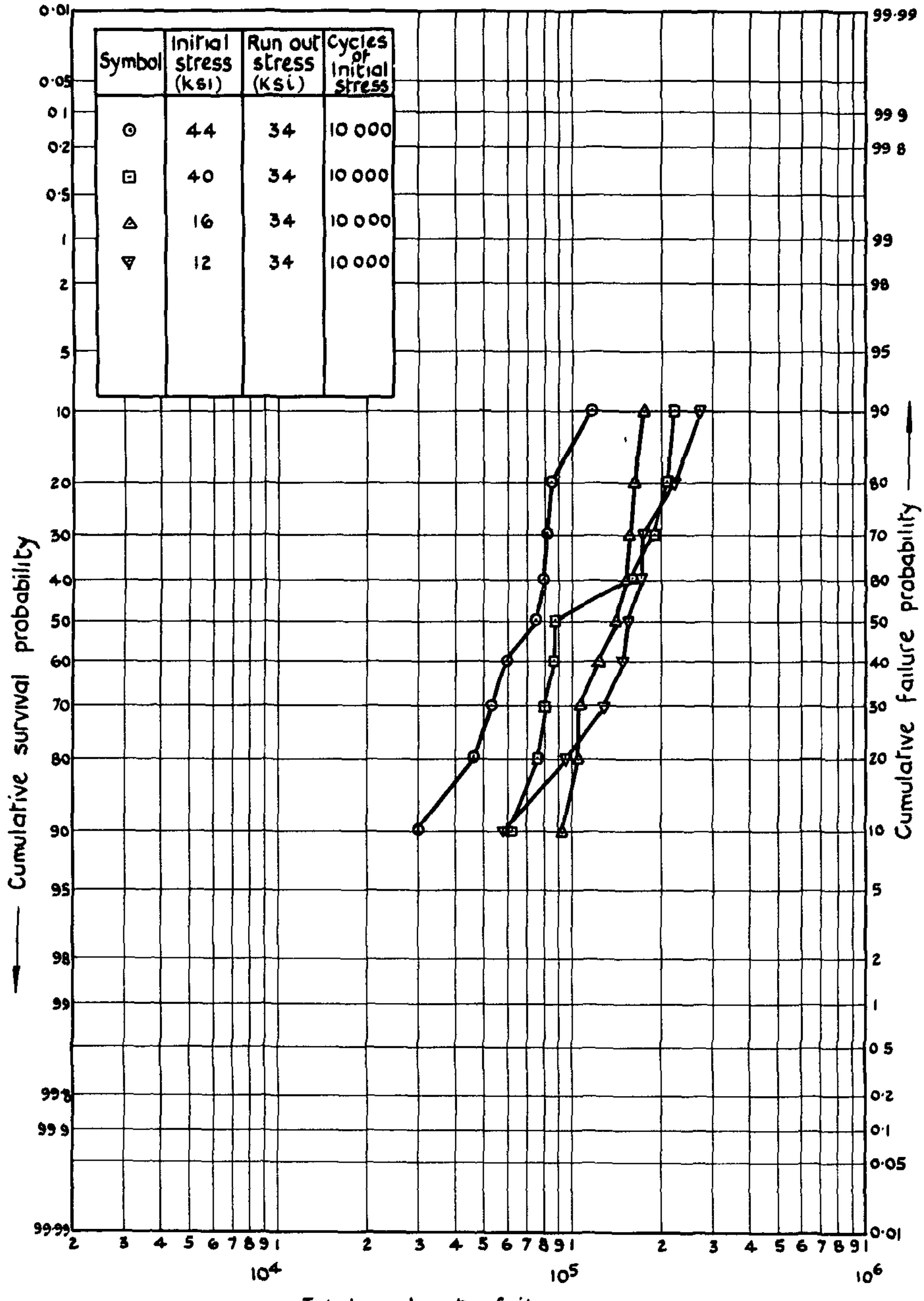
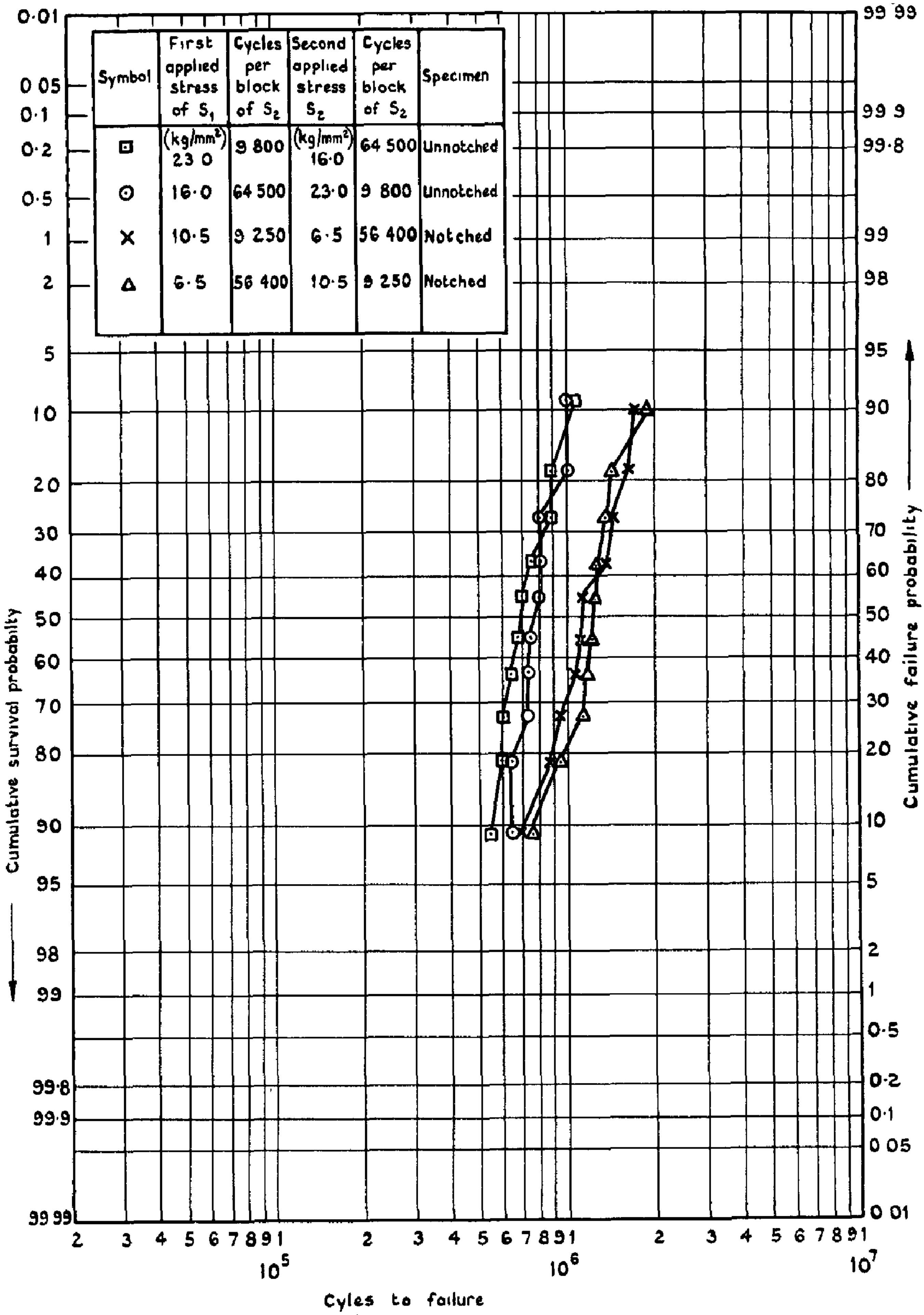


Fig. 4 Unnotched, 2024 bar specimens Vibrophore one-step results from Ref(2)



Symbol	First applied stress of S_1	Cycles per block of S_2	Second applied stress S_2	Cycles per block of S_2	Specimen
⊡	(kg/mm²) 23 0	9 800	(kg/mm²) 16·0	64 500	Unnotched
⊙	16·0	64 500	23·0	9 800	Unnotched
×	10·5	9 250	6·5	56 400	Notched
△	6·5	56 400	10·5	9 250	Notched

Fig. 5 Notched $(K_t = 2\ 85)$ and unnotched 24S-T sheet specimens. Two level repeated block results from Ref (3)

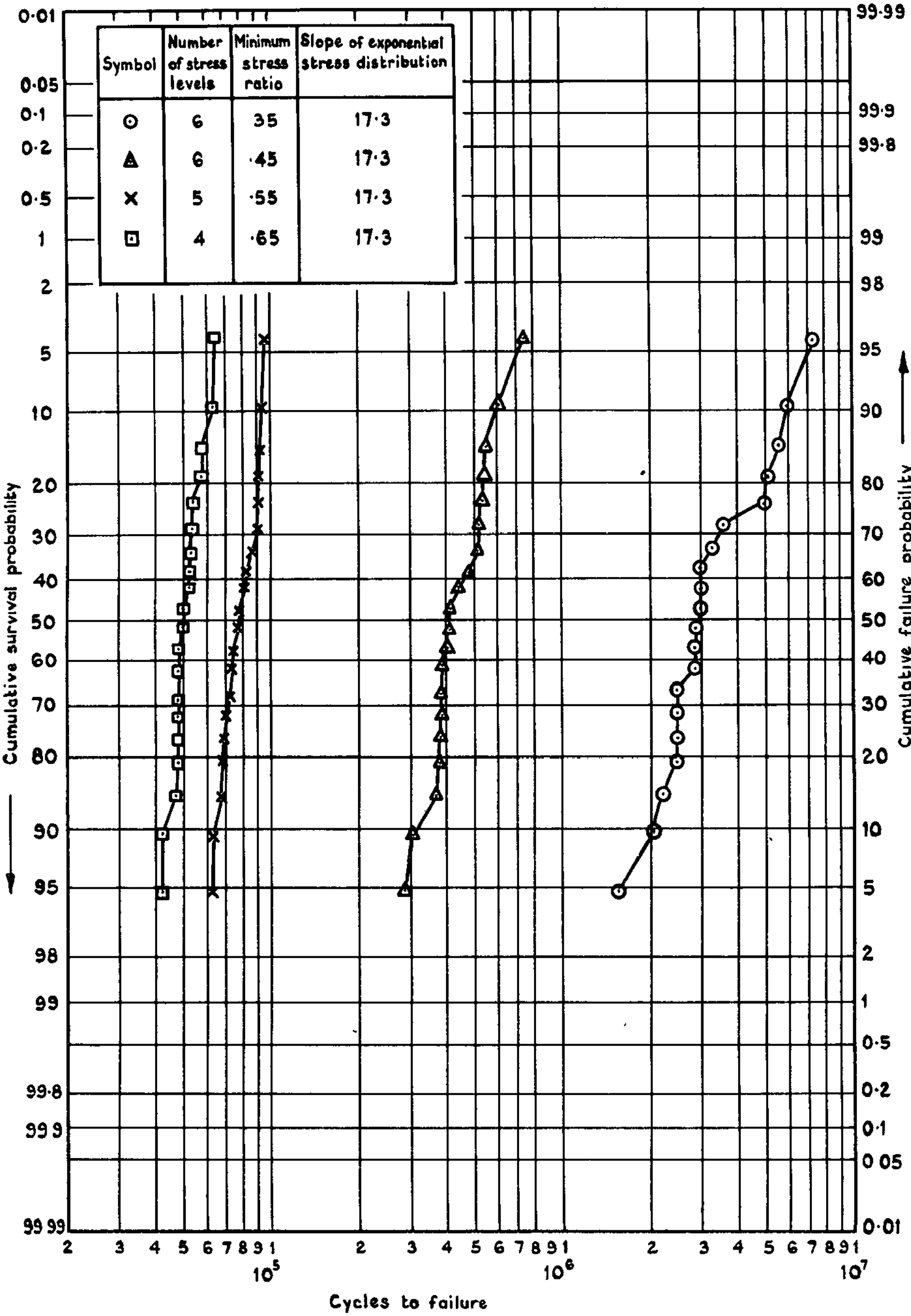


Fig. 6 Unnotched, rotating beam specimens of 2024 alloy, Ref (9). Multi-level block loading to an exponential stress distribution

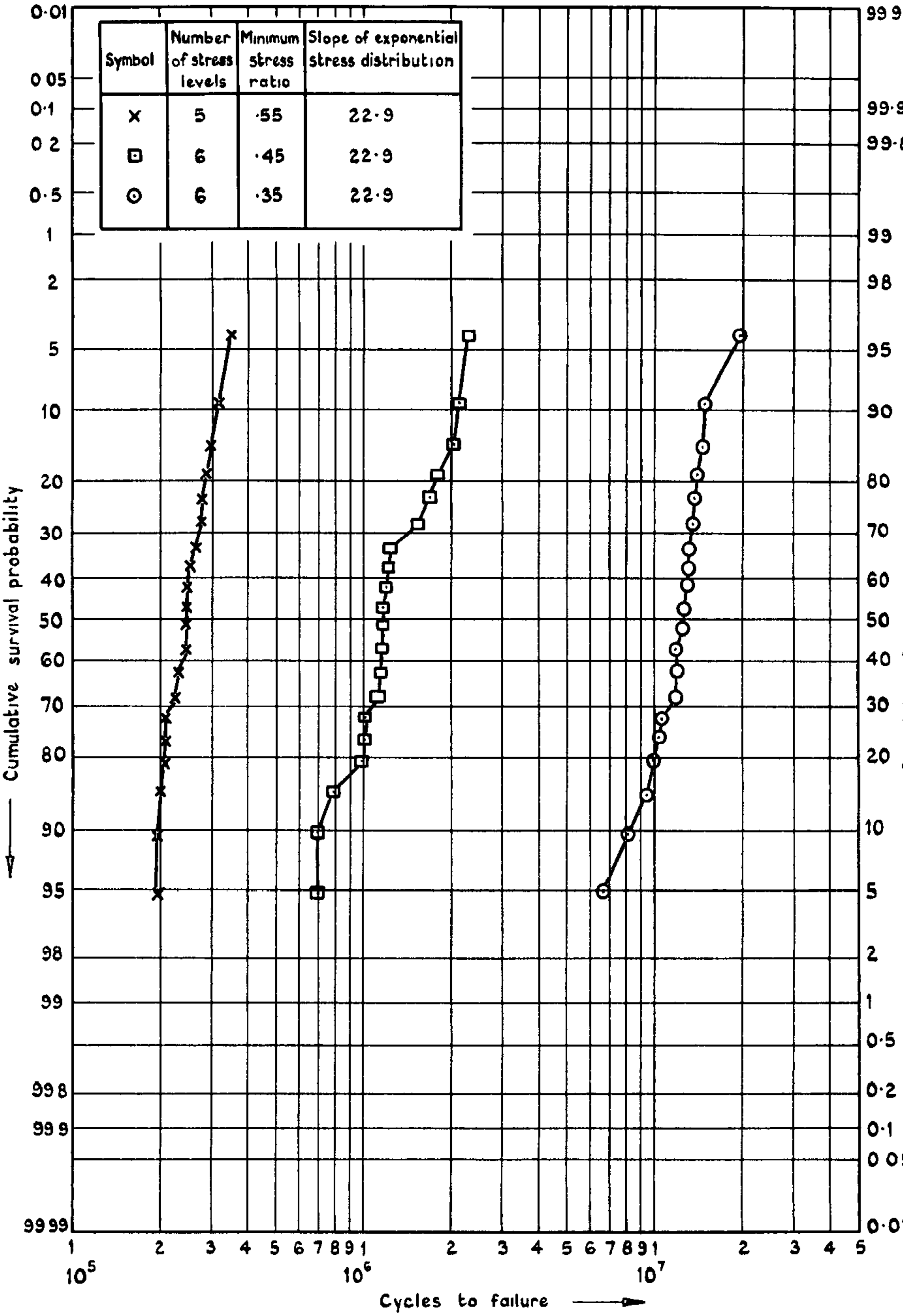


Fig 7 Unnotched, rotating beam specimens of 2024 alloy, Ref (9).
Multi-level block loading to an exponential stress distribution

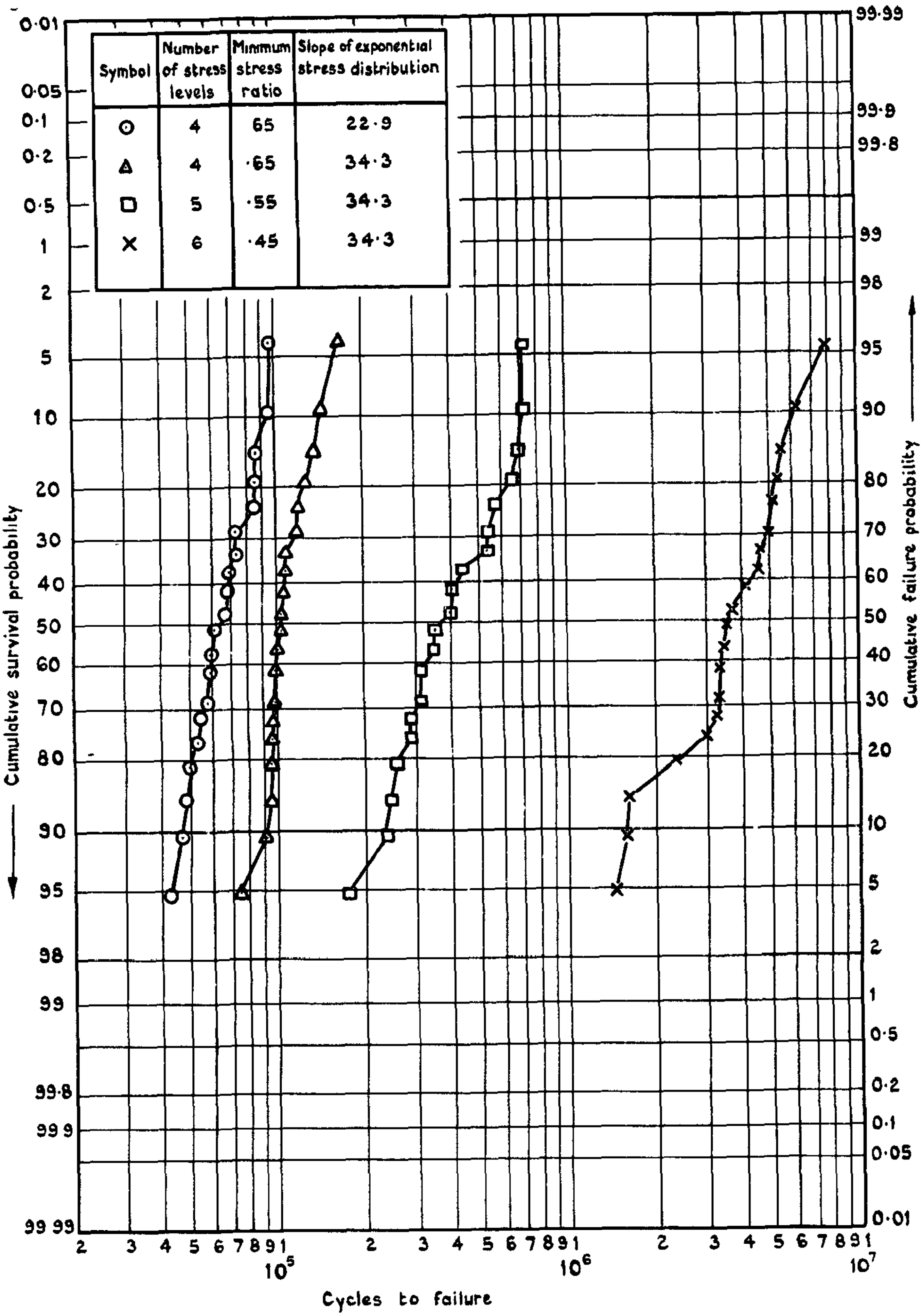


Fig. 8 Unnotched, rotating beam specimens of 2024 alloy, Ref (9).
Multi-level block loading to an exponential stress distribution

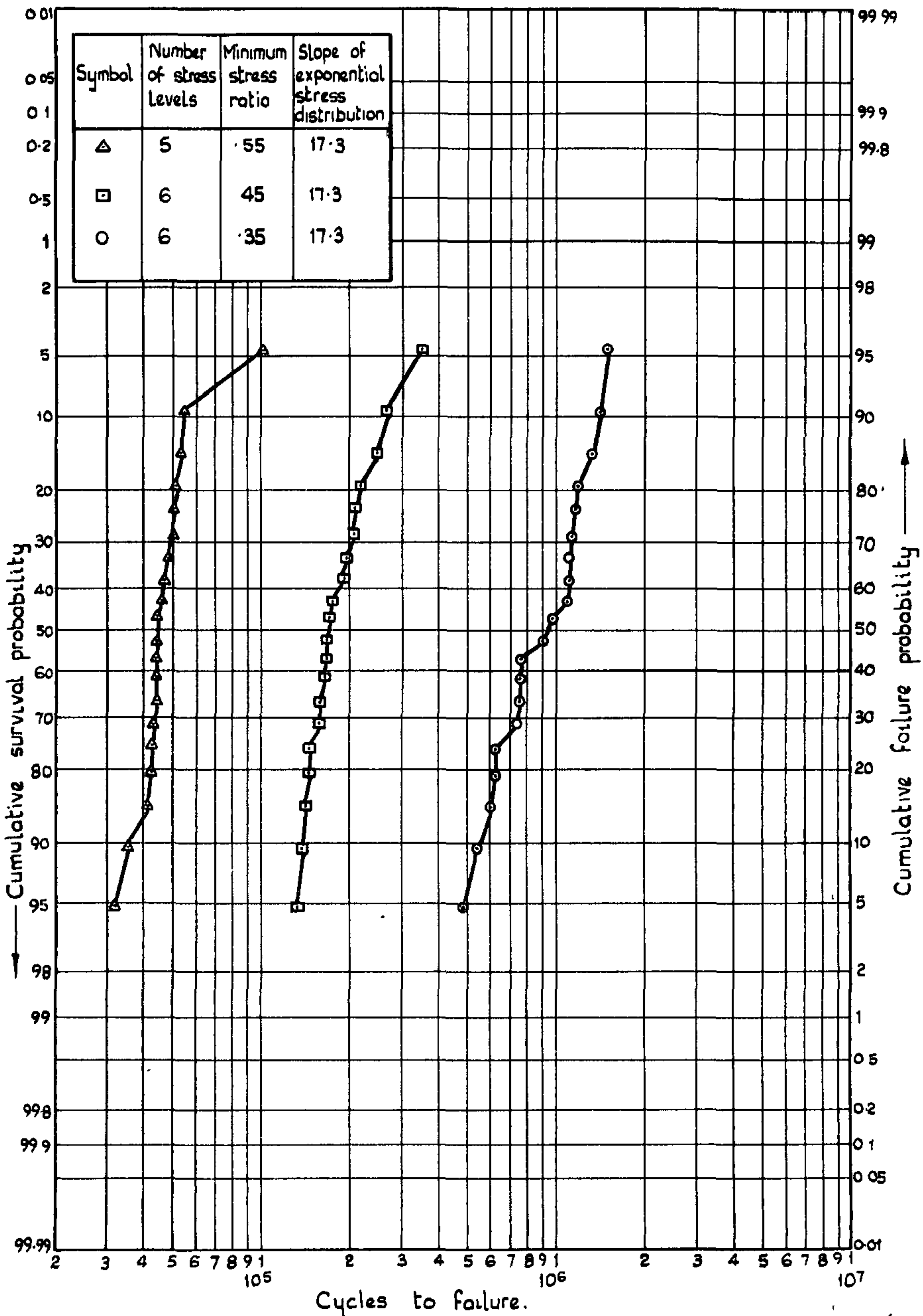


Fig.9 Unnotched, rotating beam specimens of 7075 alloy, Ref(10).
Multi-level block loading to an exponential stress distribution

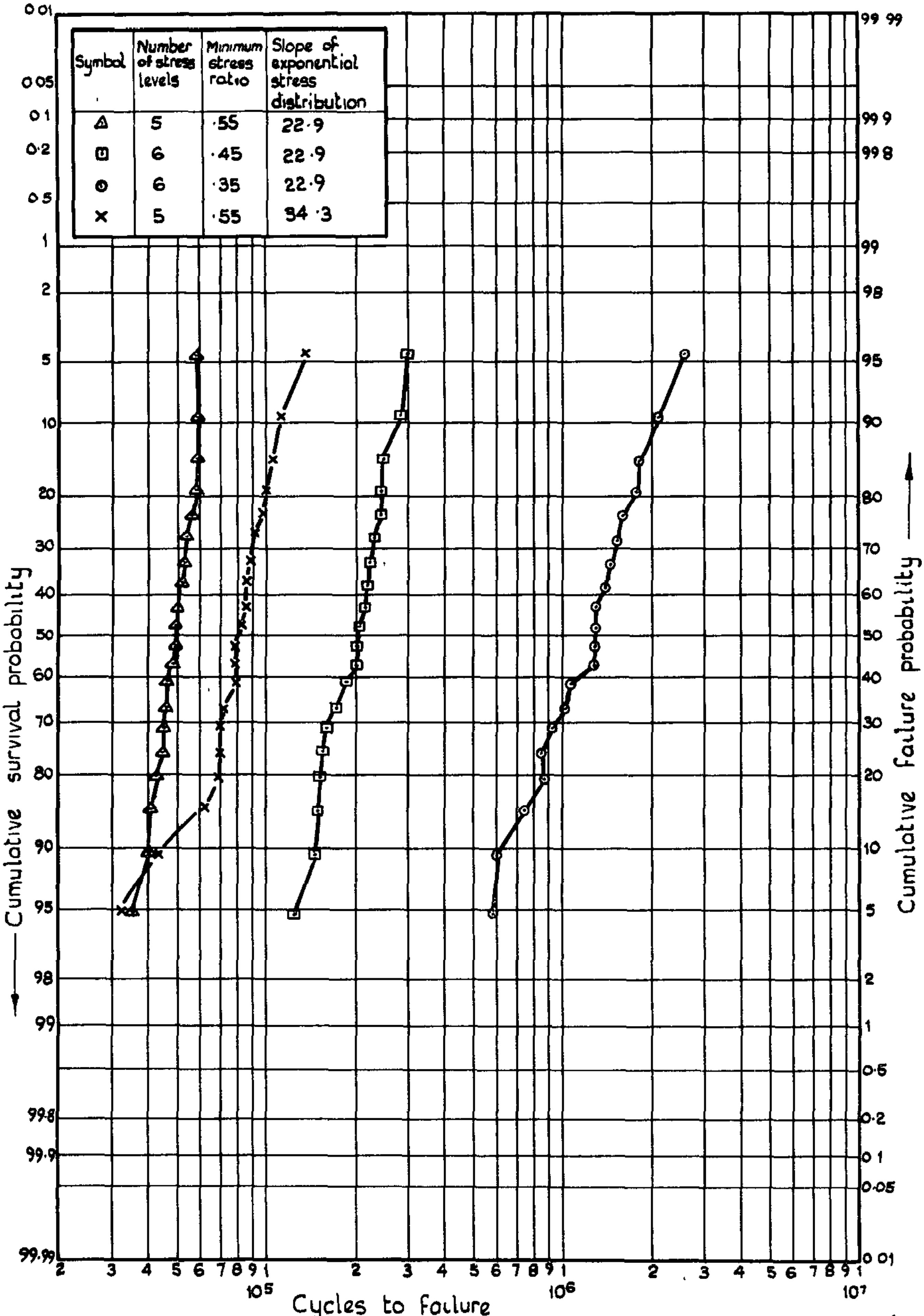


Fig. 10 Unnotched, rotating beam specimens of 7075 alloy, Ref (10).
Multi-level block loading to an exponential stress distribution

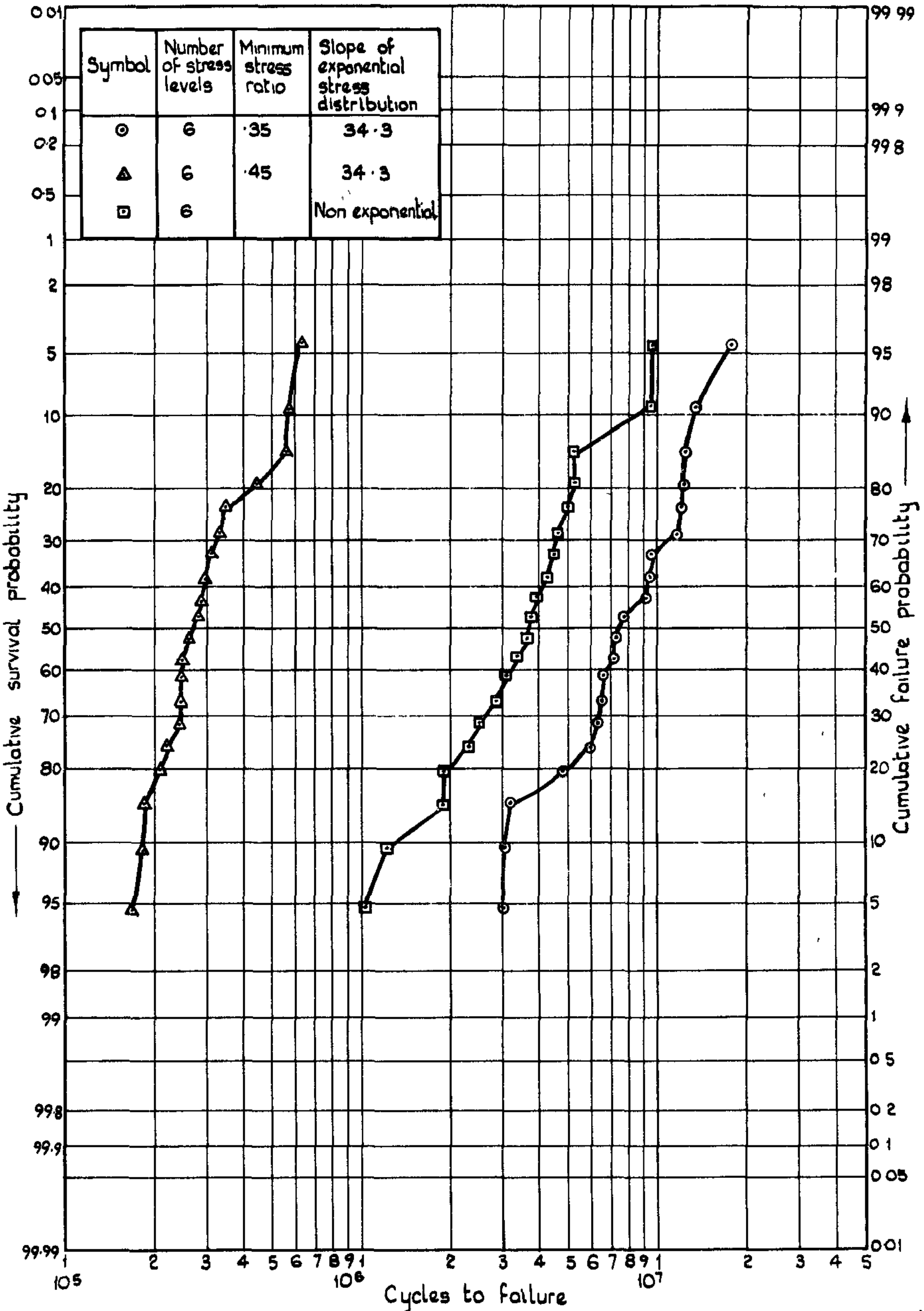


Fig. 11 Unnotched, rotating beam specimens of 7075 alloy Ref(10). Multi-level block loading to an exponential and non-exponential stress distribution

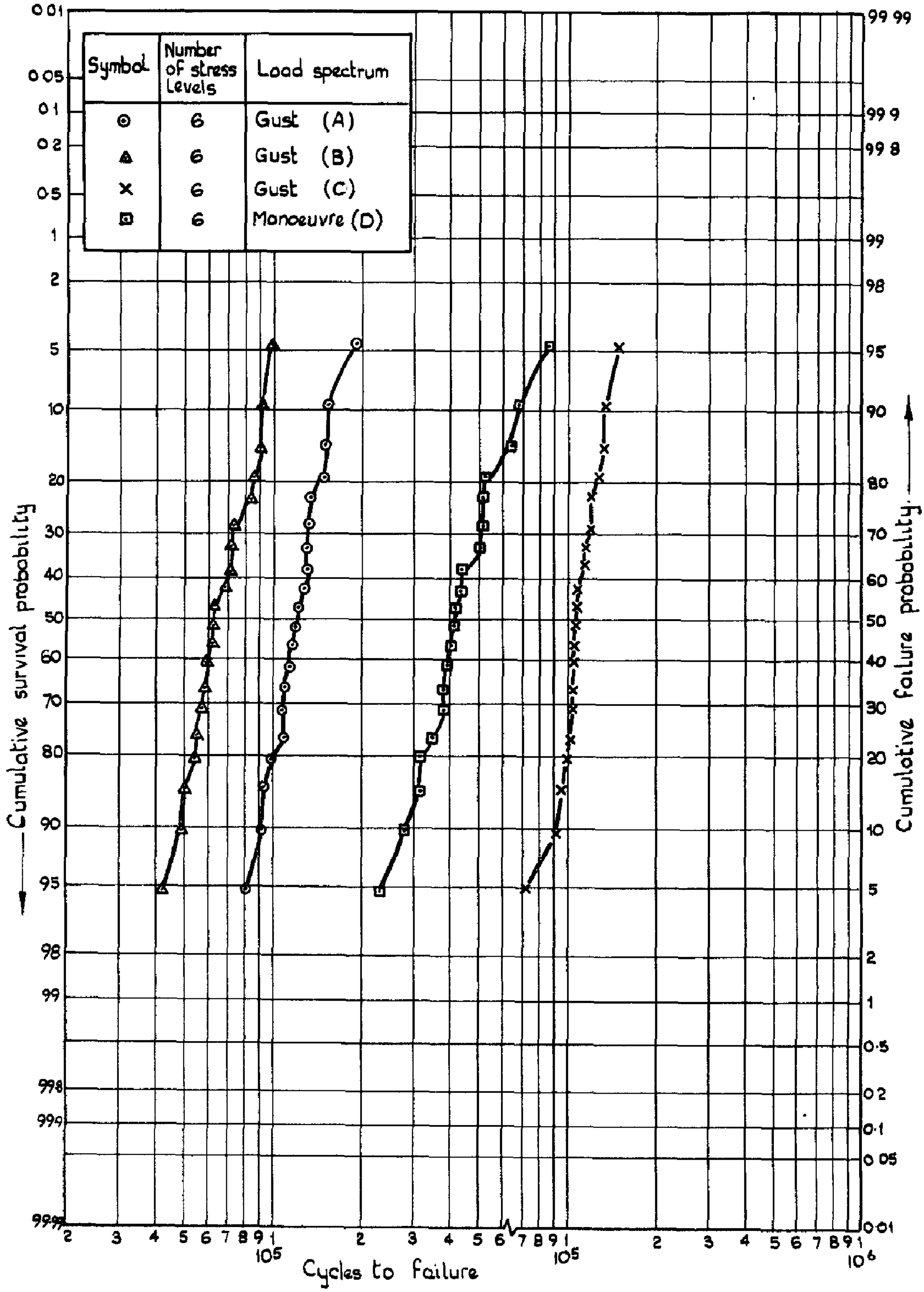


Fig 12 Unnotched, rotating beam specimens of 2024 alloy, Ref (7). Multi-level block loading to gust and manoeuvre spectra

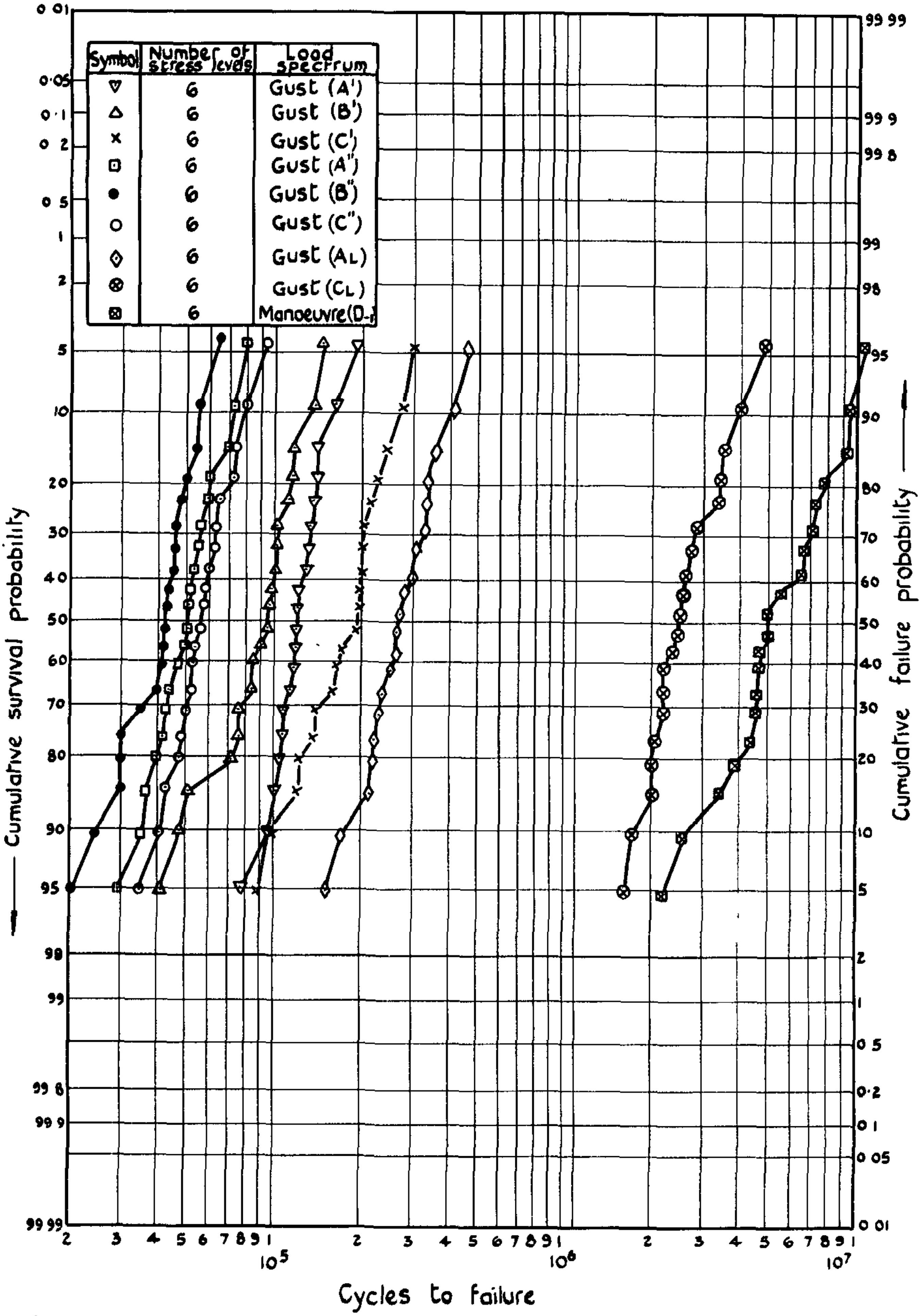


Fig. 13 Unnotched, rotating beam specimens of 2024 alloy, Ref(11). Multi-level block loading to gust and manoeuvre spectra

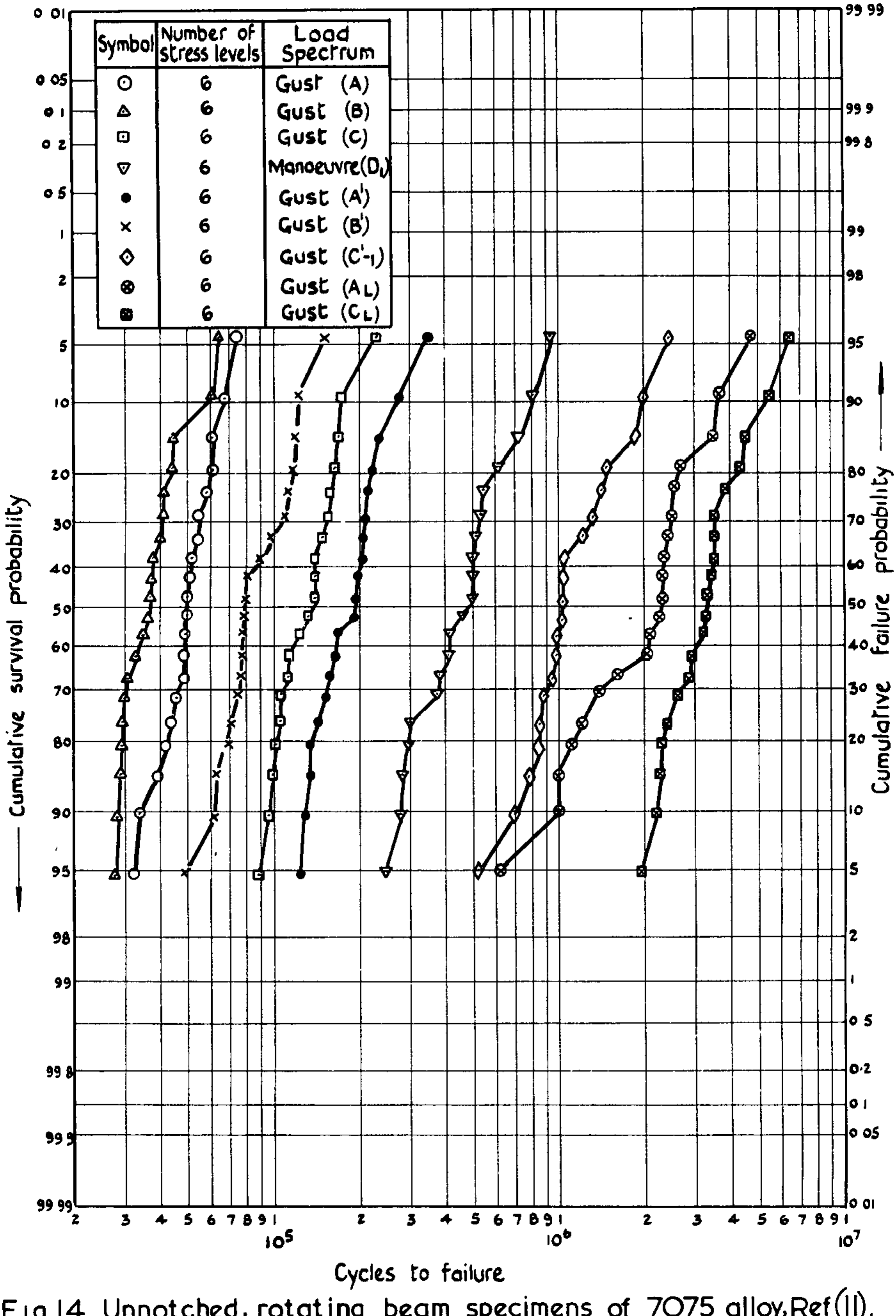


Fig 14 Unnotched, rotating beam specimens of 7075 alloy, Ref (11). Multi-level block loading to gust and manoeuvre spectra

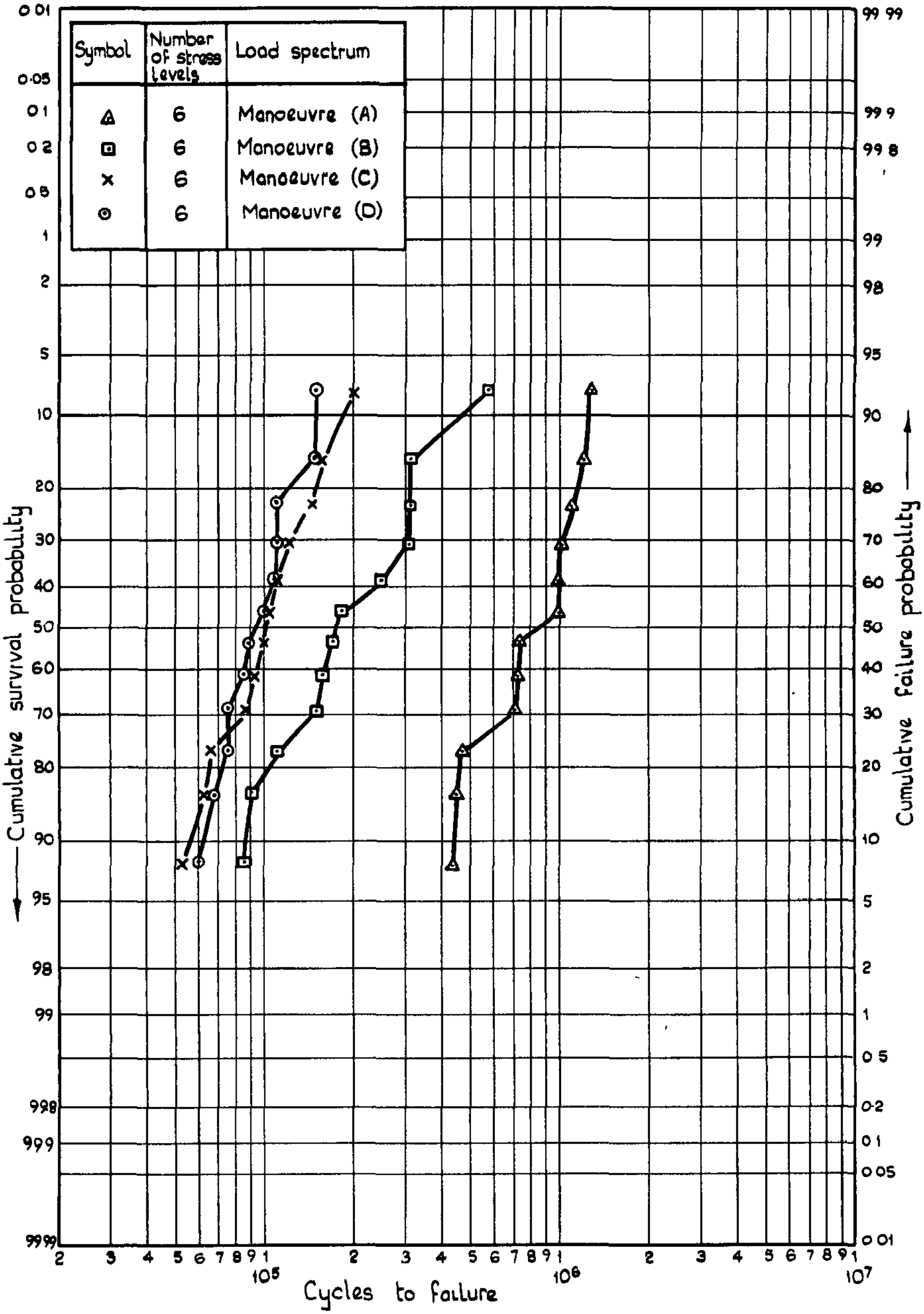


Fig. 15 Unnotched, rotating beam specimens of 7075 alloy, Ref.(12) Multi-level block loading to an exponential load distribution.

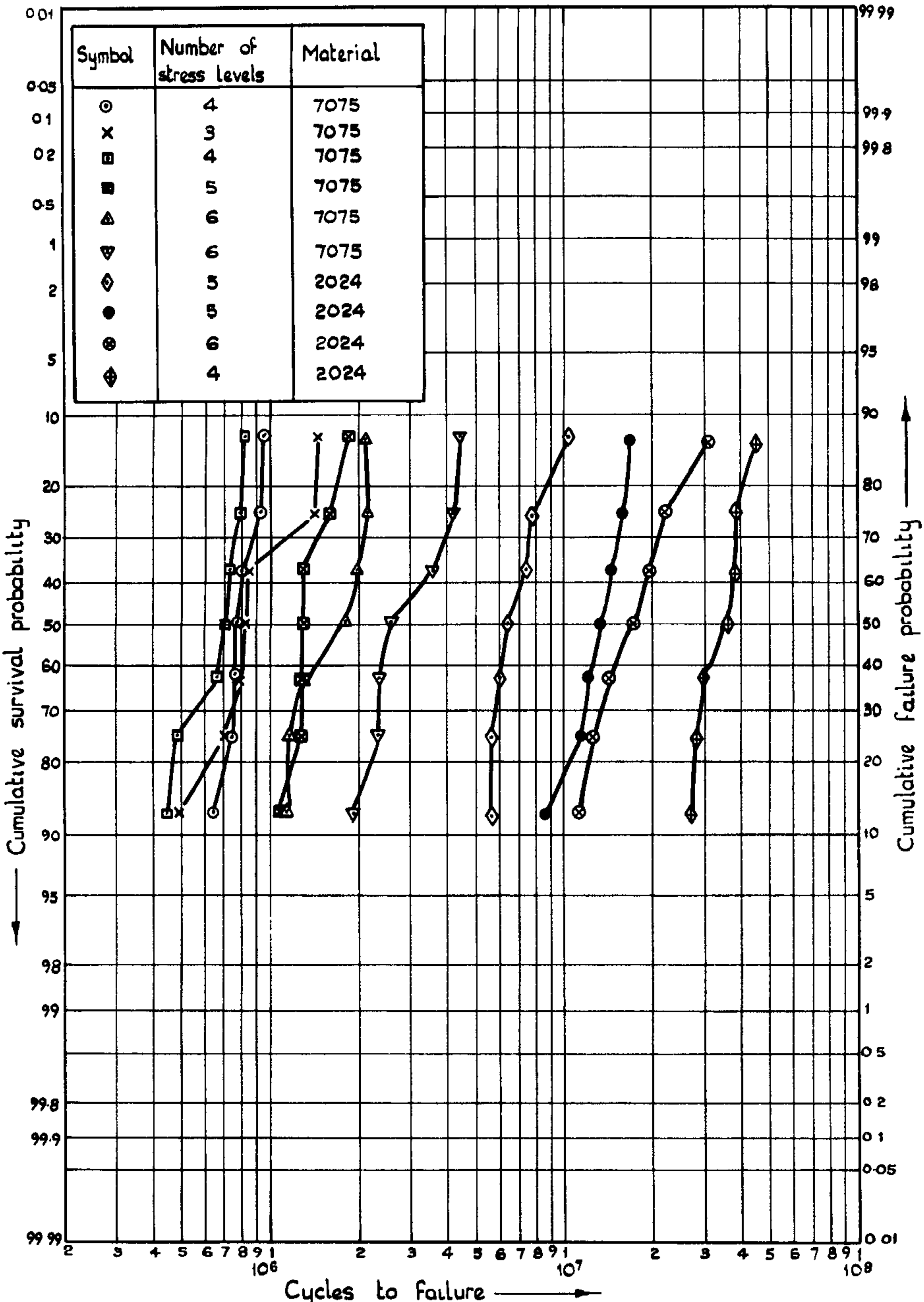


Fig. 16 Rivetted joints of 2024 and 7075 alloy, Ref (6).
Multi-level block programme loading.

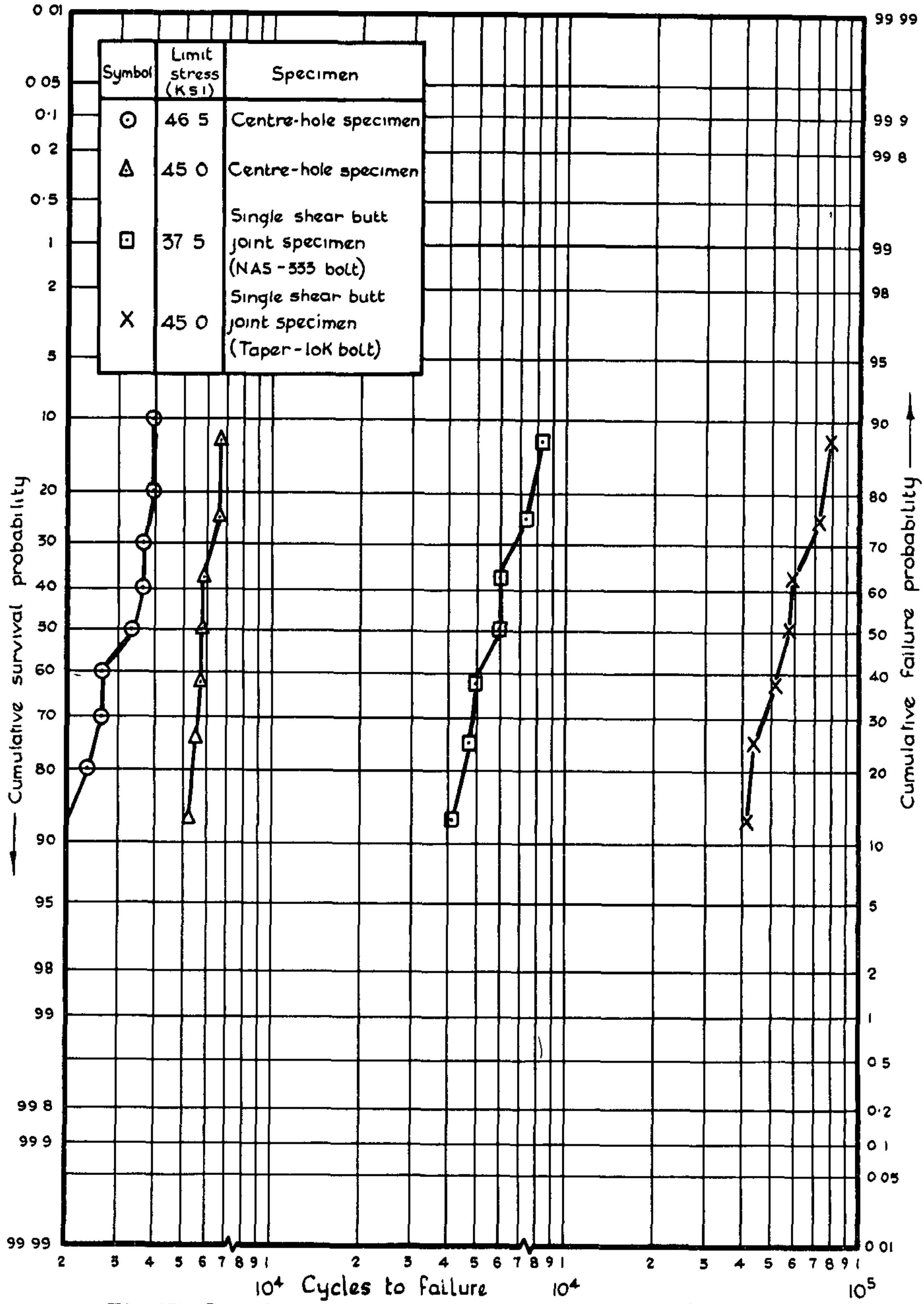


Fig.17 Specimens of 7075 alloy, Ref (8).
Multi-level block loading to a service load spectrum.

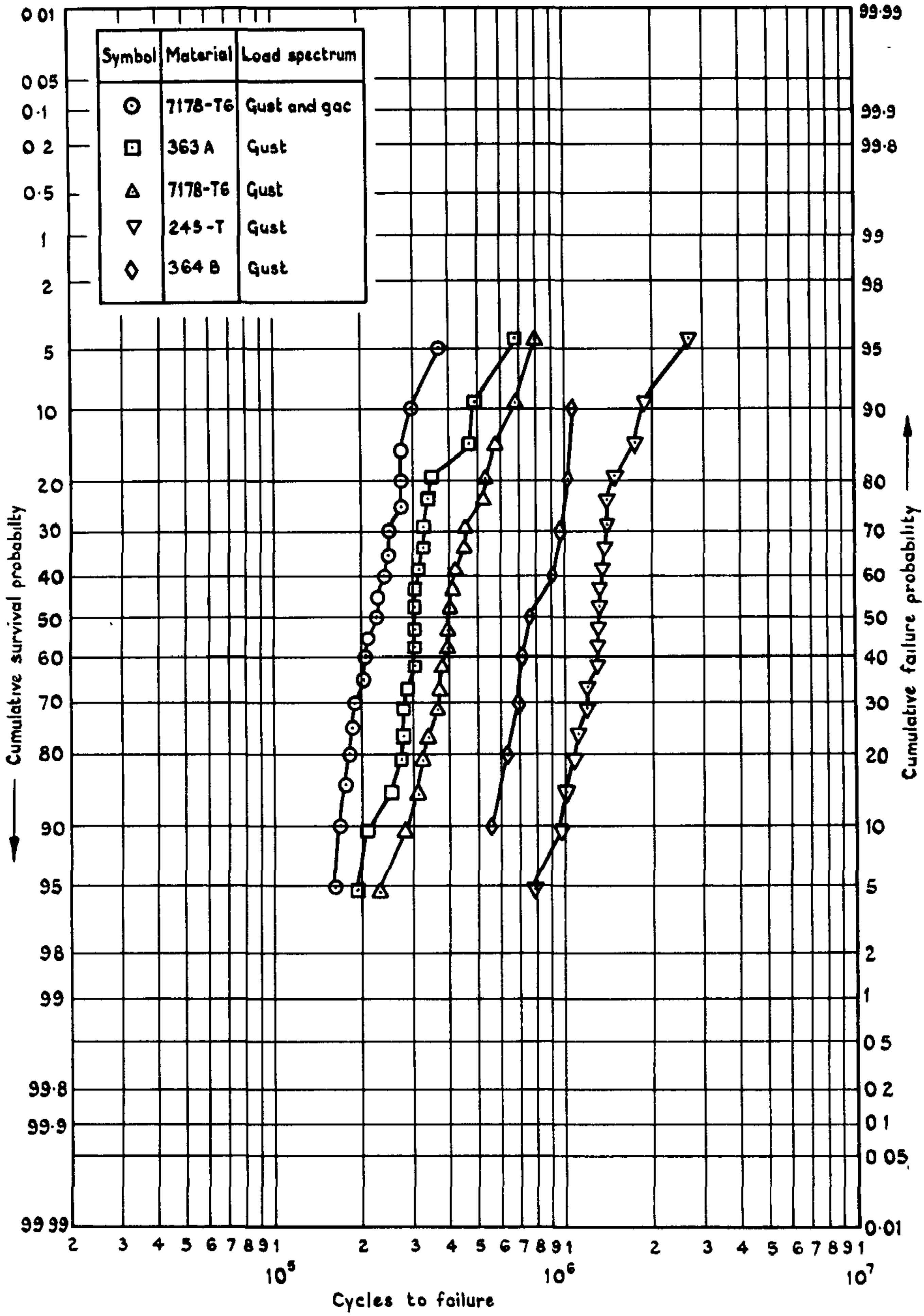


Fig 18 Notched specimens ($K_t = 7 \cdot 0$) of 4 different materials, Ref (13). Multi-level block loading to a gust spectrum

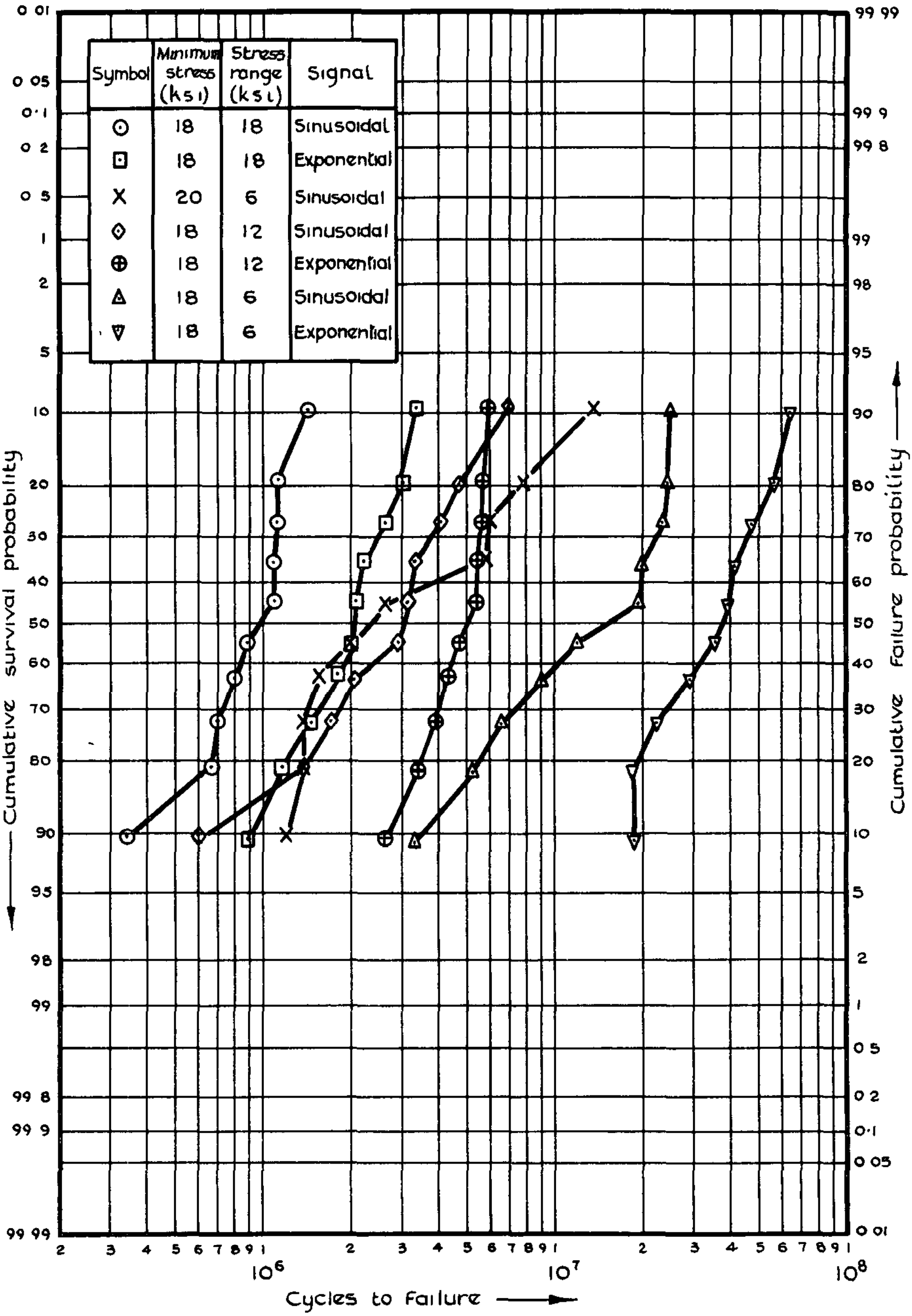


Fig. 19 Rotating beam specimens of 2024 alloy, Ref (14). Continuously variable amplitude varying to a repeated signal pattern

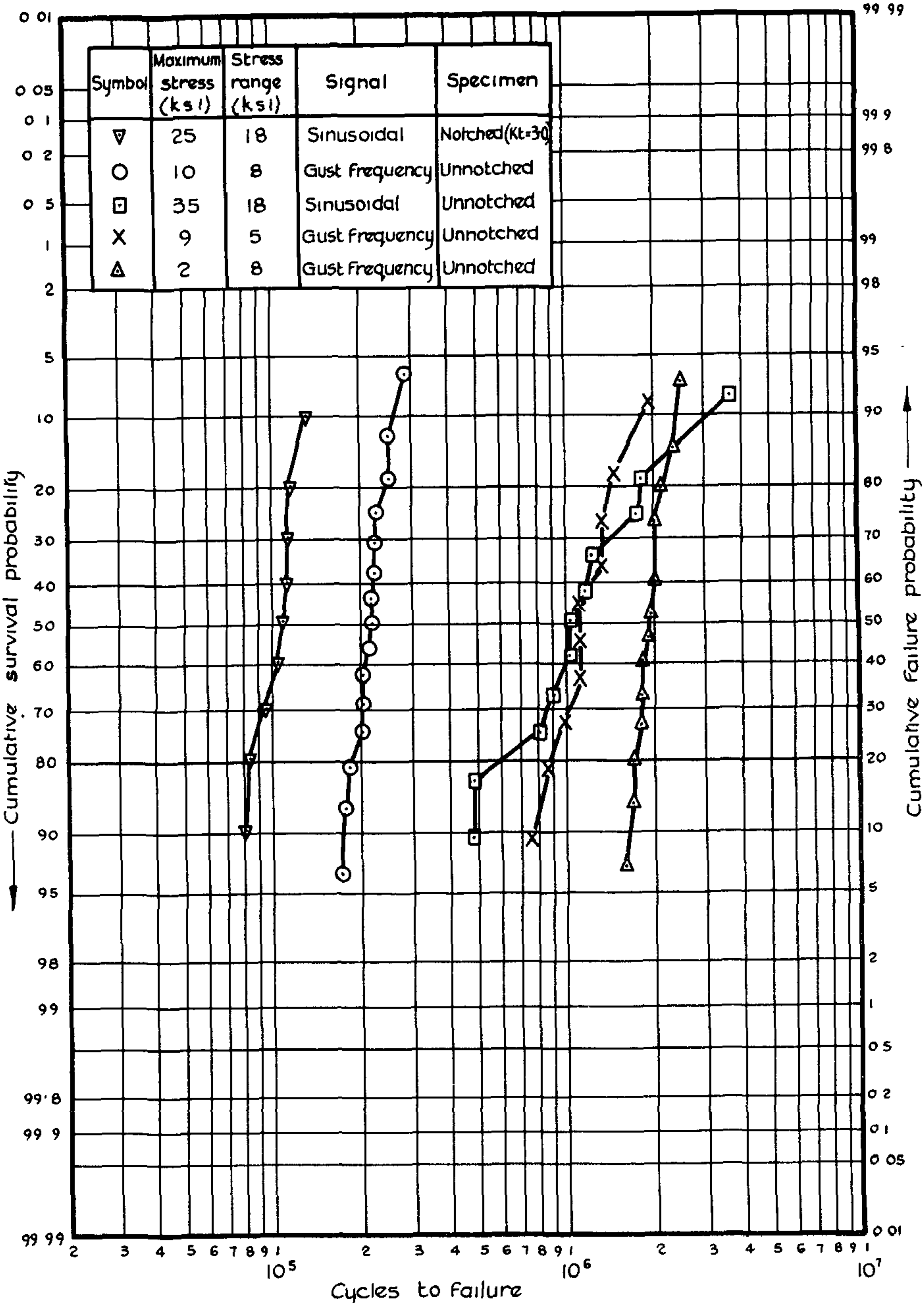


Fig 20 Notched and unnotched specimens of 7075-T6 alloy Ref (15). Sinusoidal loading of amplitude varying to a repeated signal pattern

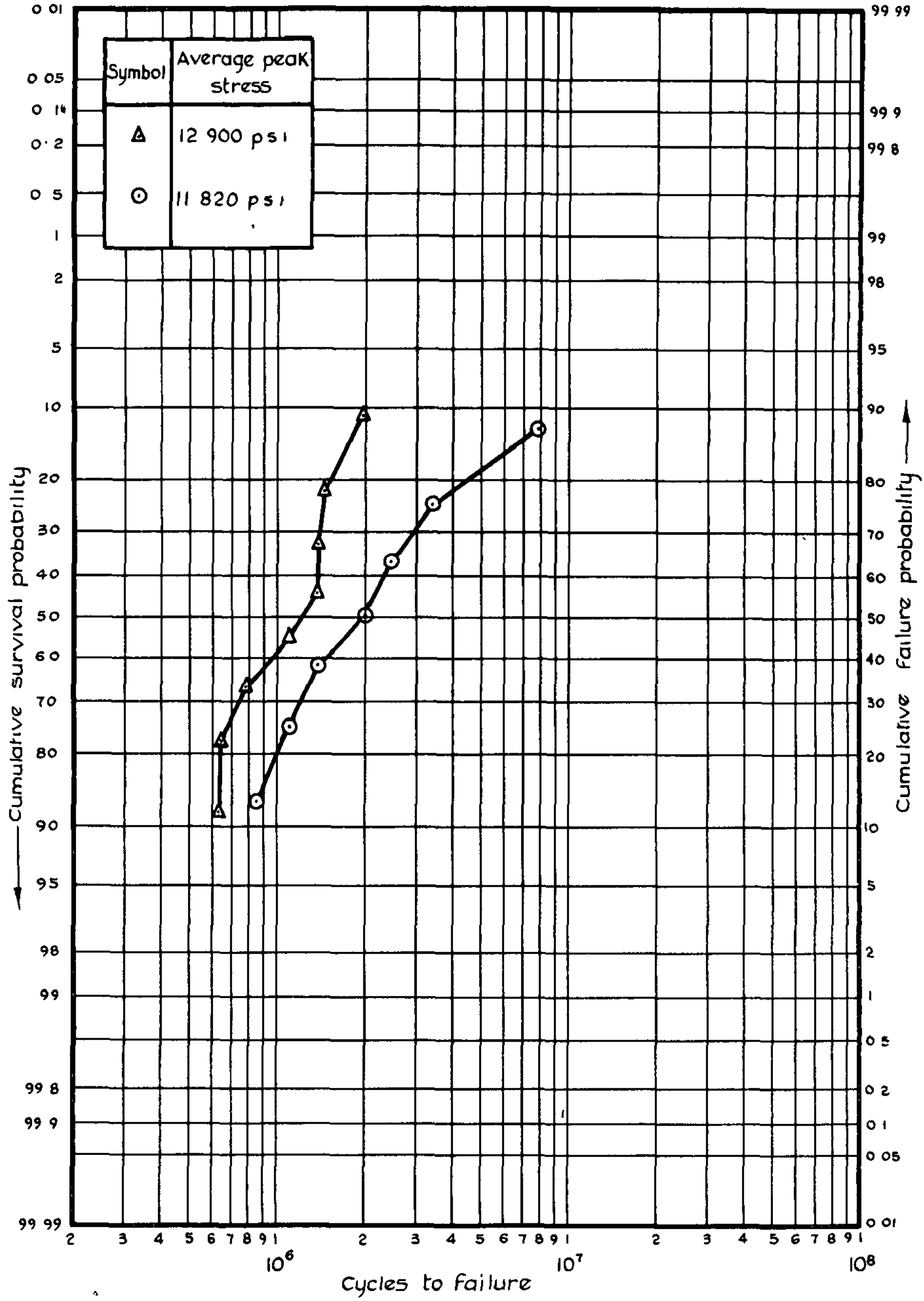


Fig. 21 Specimens of 24 S-T alloy, Ref (16).
Continuously variable random noise fatigue loads

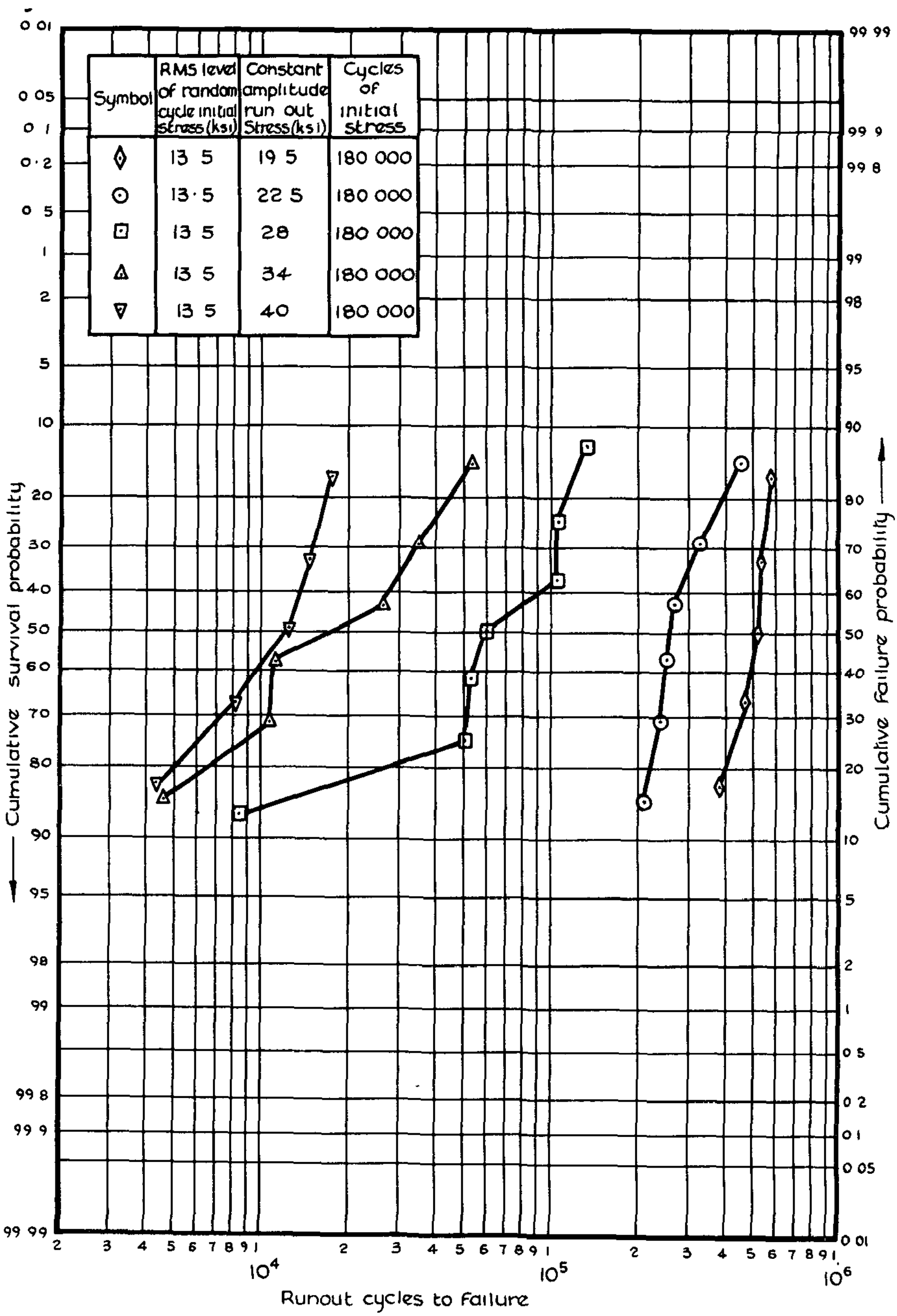


Fig. 22 Unnotched, axial bar specimens of 2024 material Ref(17). Random cycle initial stressing, followed by constant amplitude runout stressing

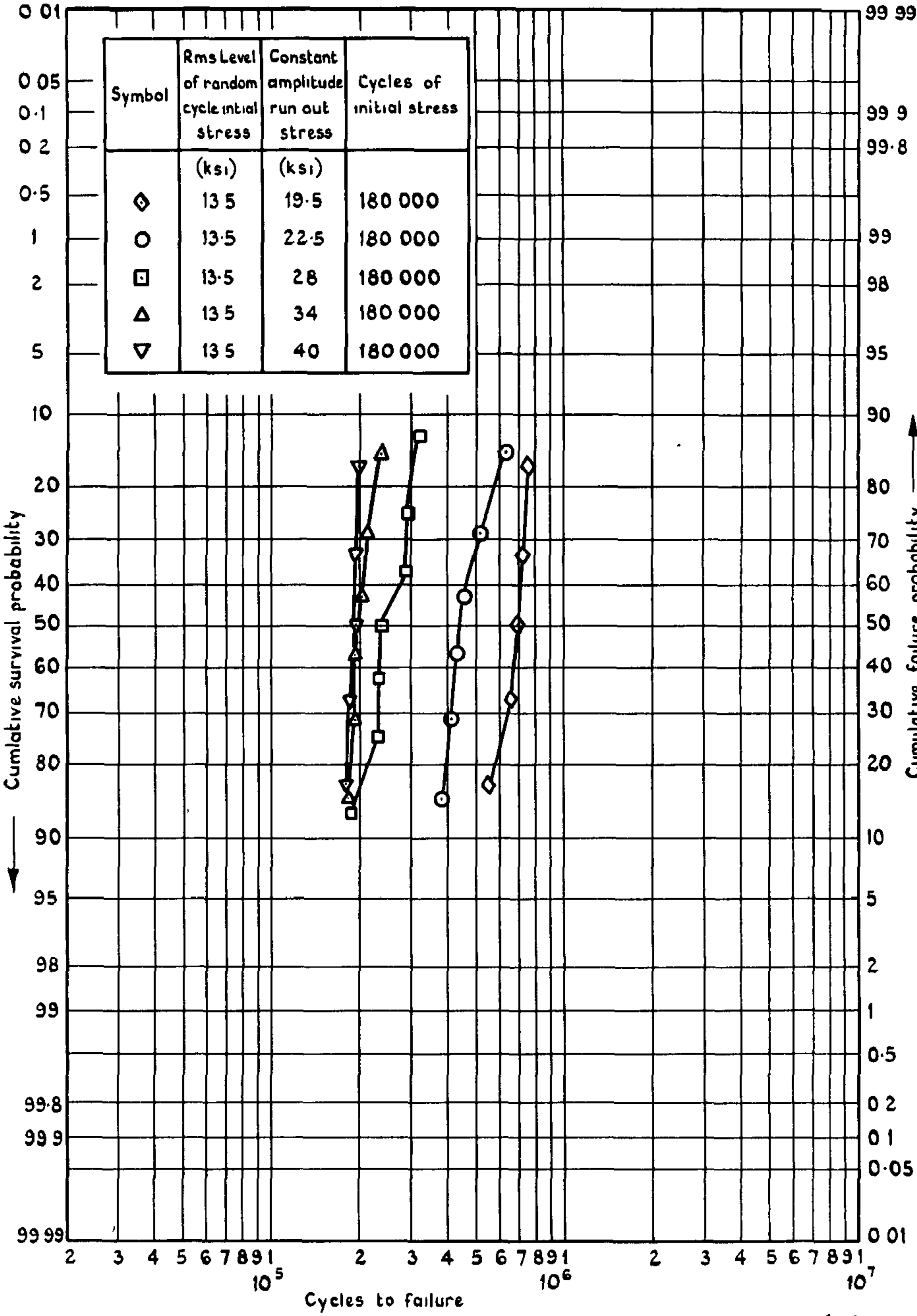


Fig 23 Unnotched, axial bar specimens of 2024 Ref (17).
Random cycle initial stressing, followed by constant amplitude run out stressing

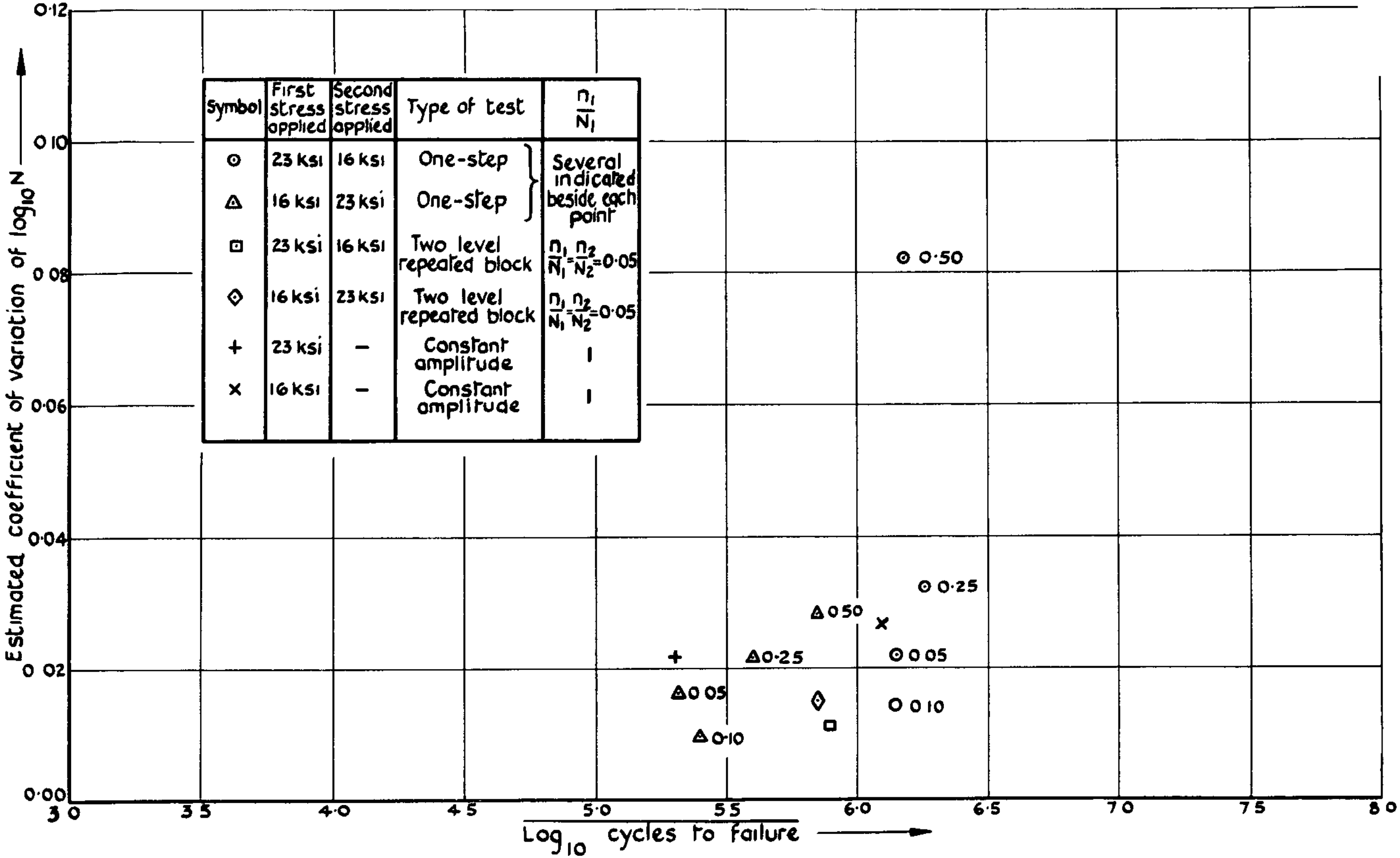


Fig. 24 Unnotched, clad 24 S-T sheet specimens, Ref (3).
One-step and two level repeated block test

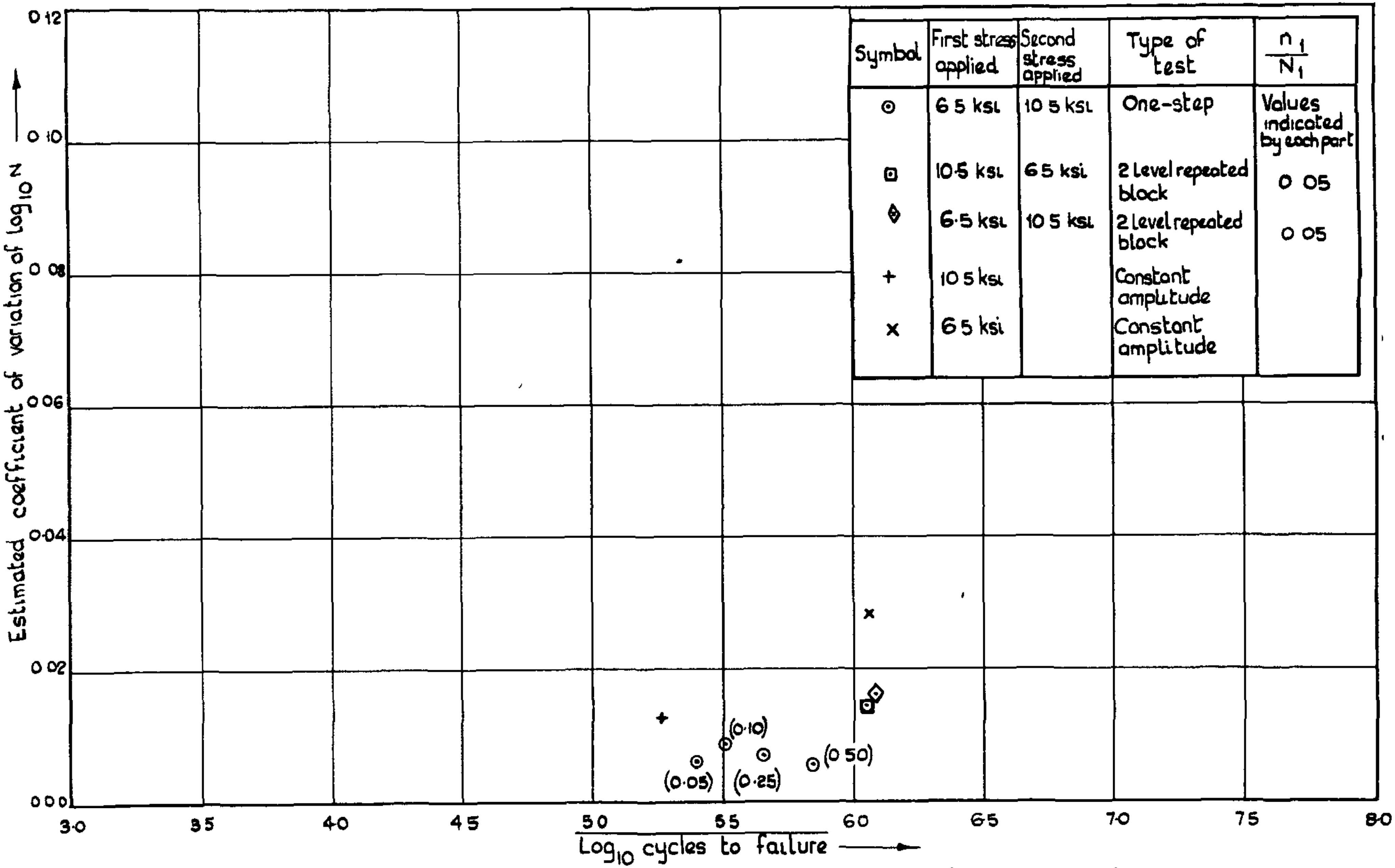


Fig. 25 Notched, clad 24 S-T sheet specimen Ref (3).
One step and two - level repeated block tests

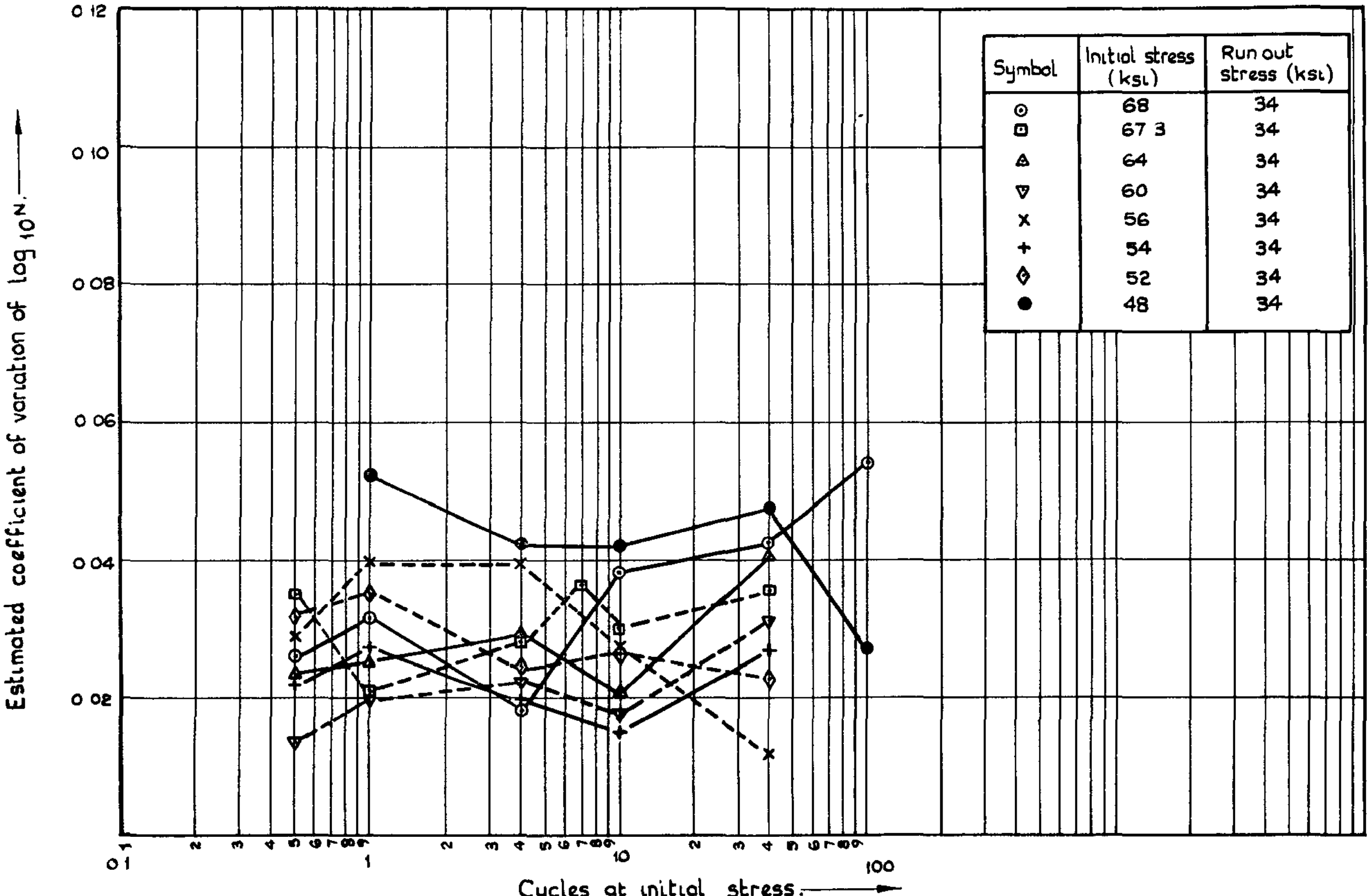


Fig. 26 Unnotched, axial specimens of 2024 material, Ref (2).
One-step tests with a manually applied initial loading

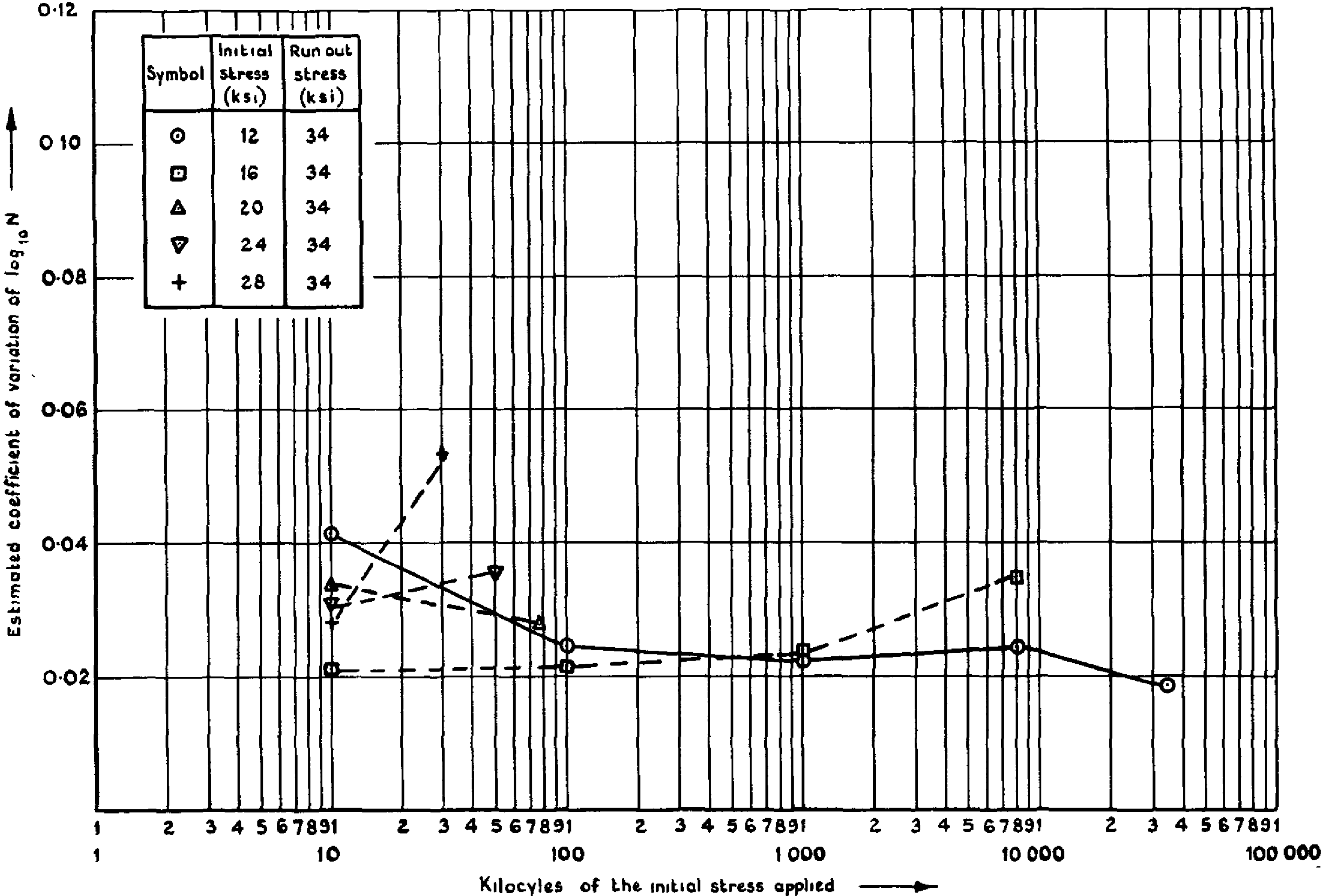


Fig. 27 Unnotched, axial specimens of 2024 material, Ref (2).
One step tests conducted on a vibrophore

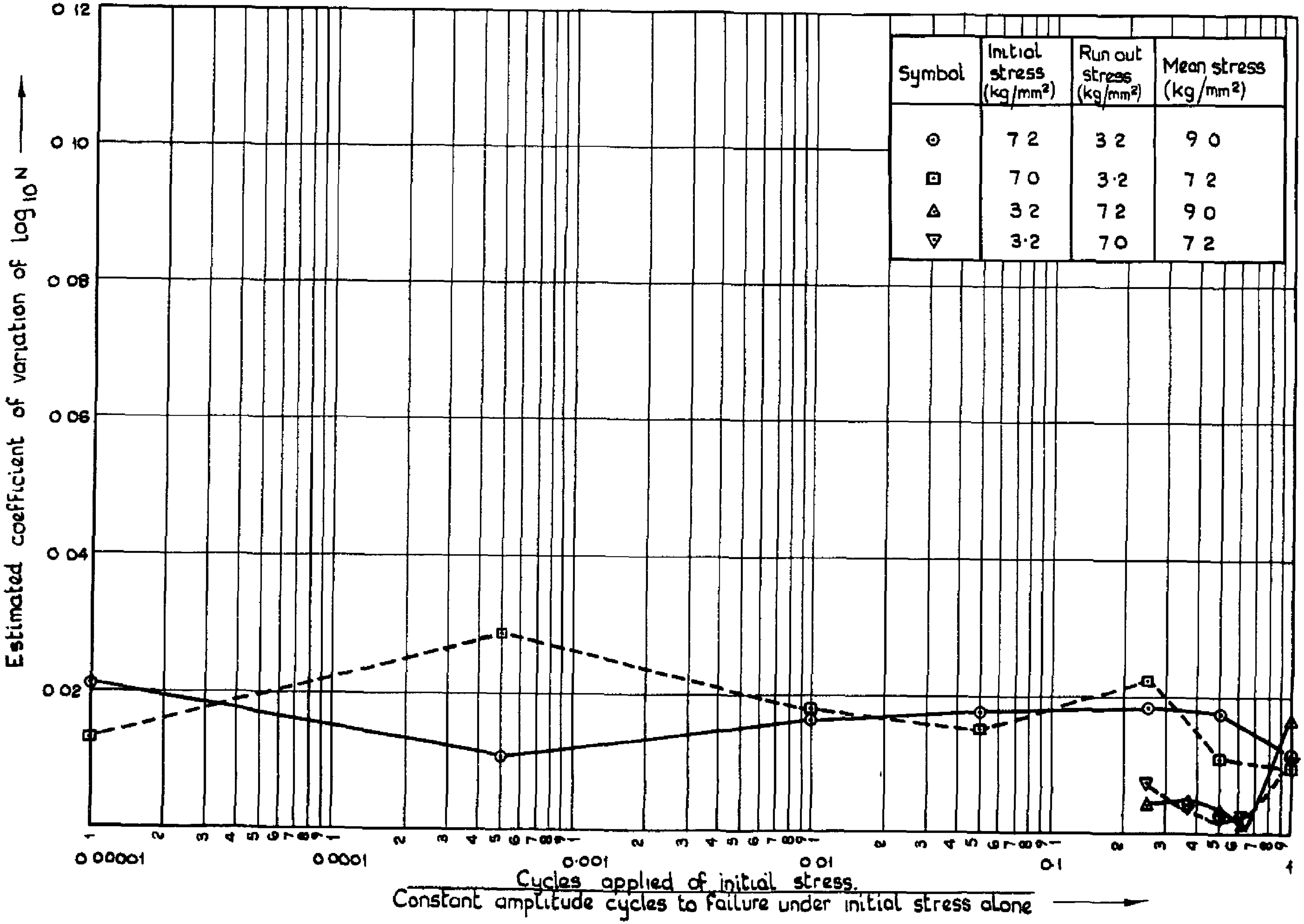


Fig. 28 Rivetted, alclad lap-joints of 24 S-T material, Ref (19). One-step loading

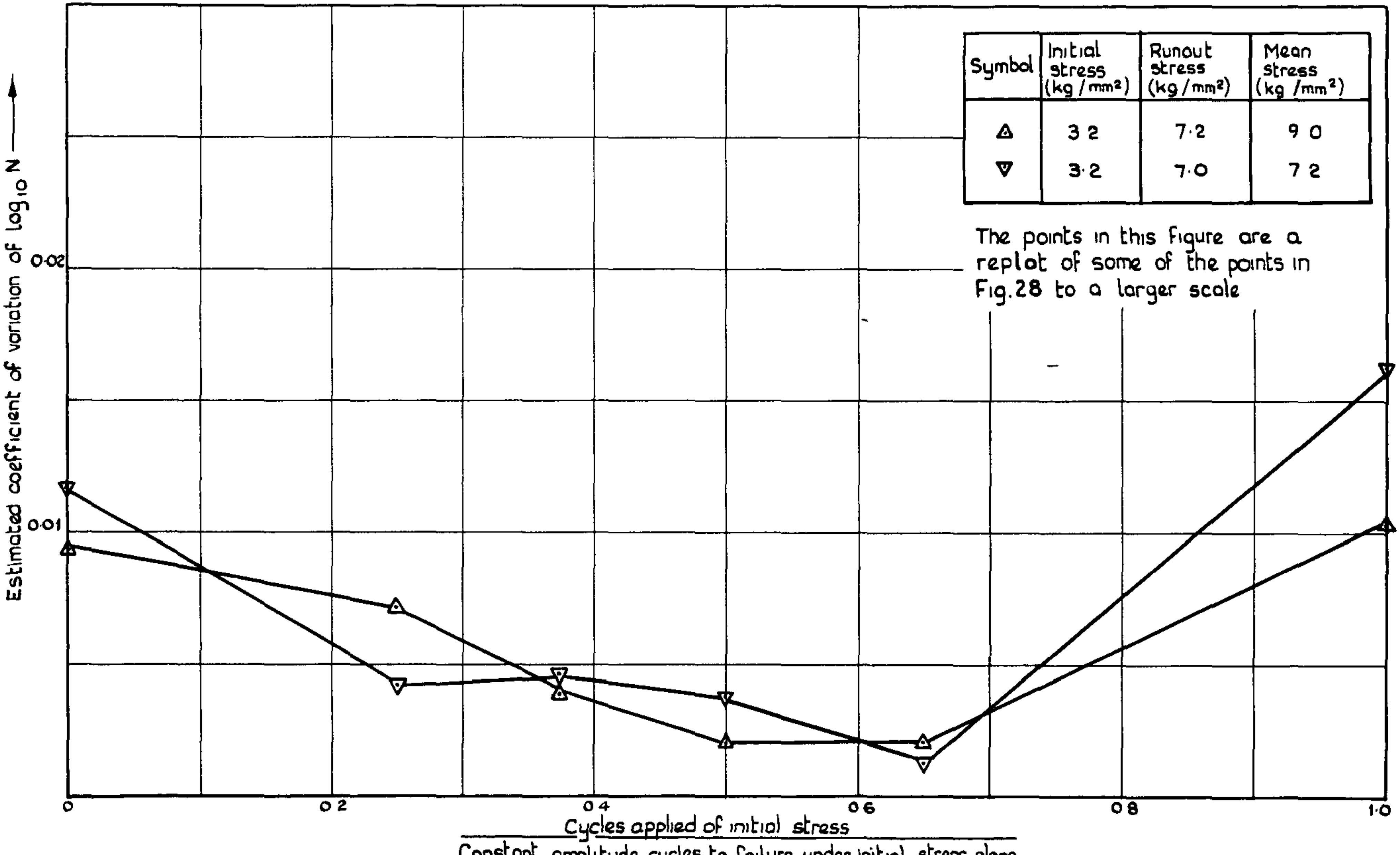


Fig. 29 Rivetted, lap-joints of alclad 24 S-T material, Ref (19). One step loading

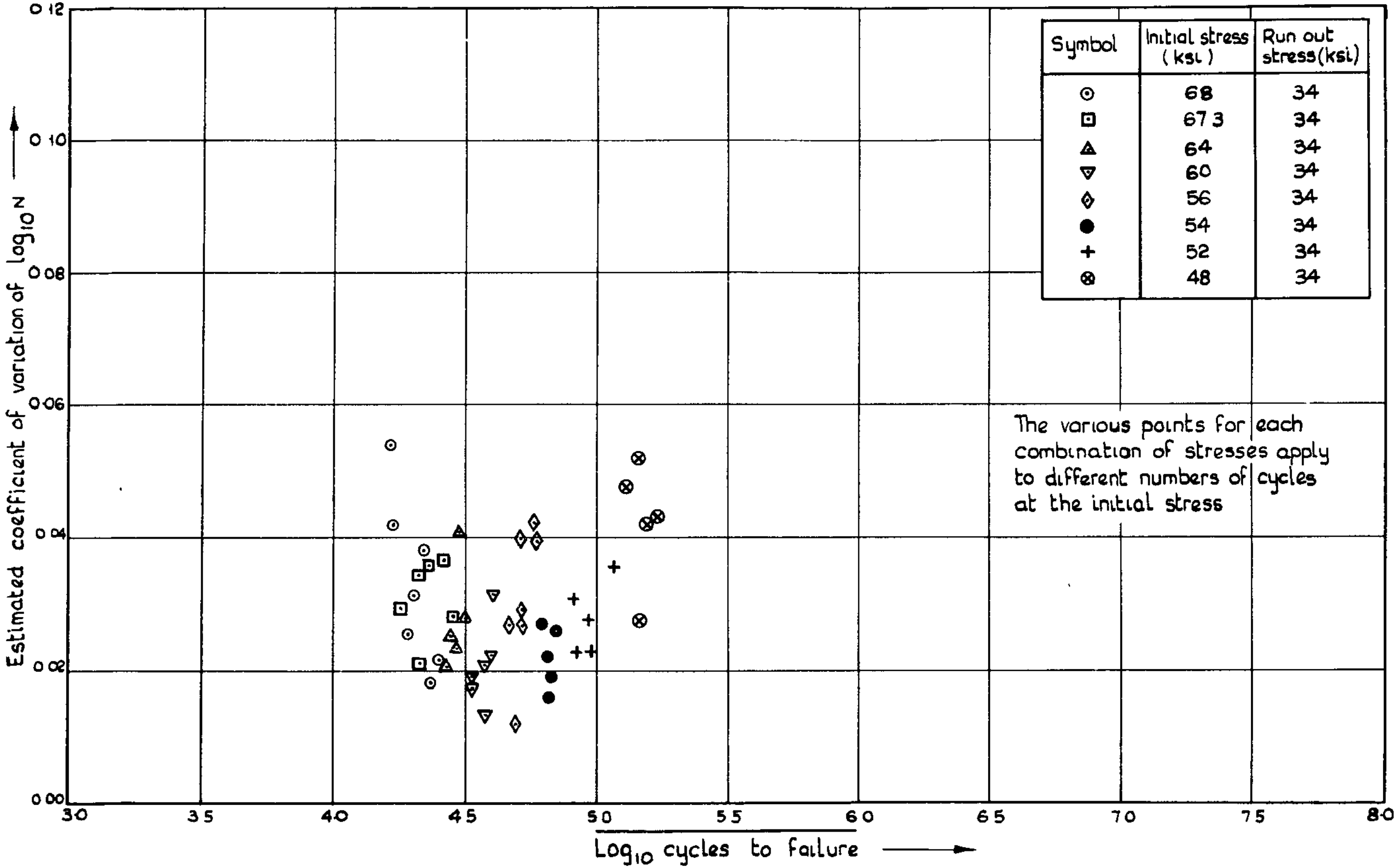


Fig. 30 Unnotched, axial specimens of 2024 material Ref (2).
One step tests with a manually applied initial loading

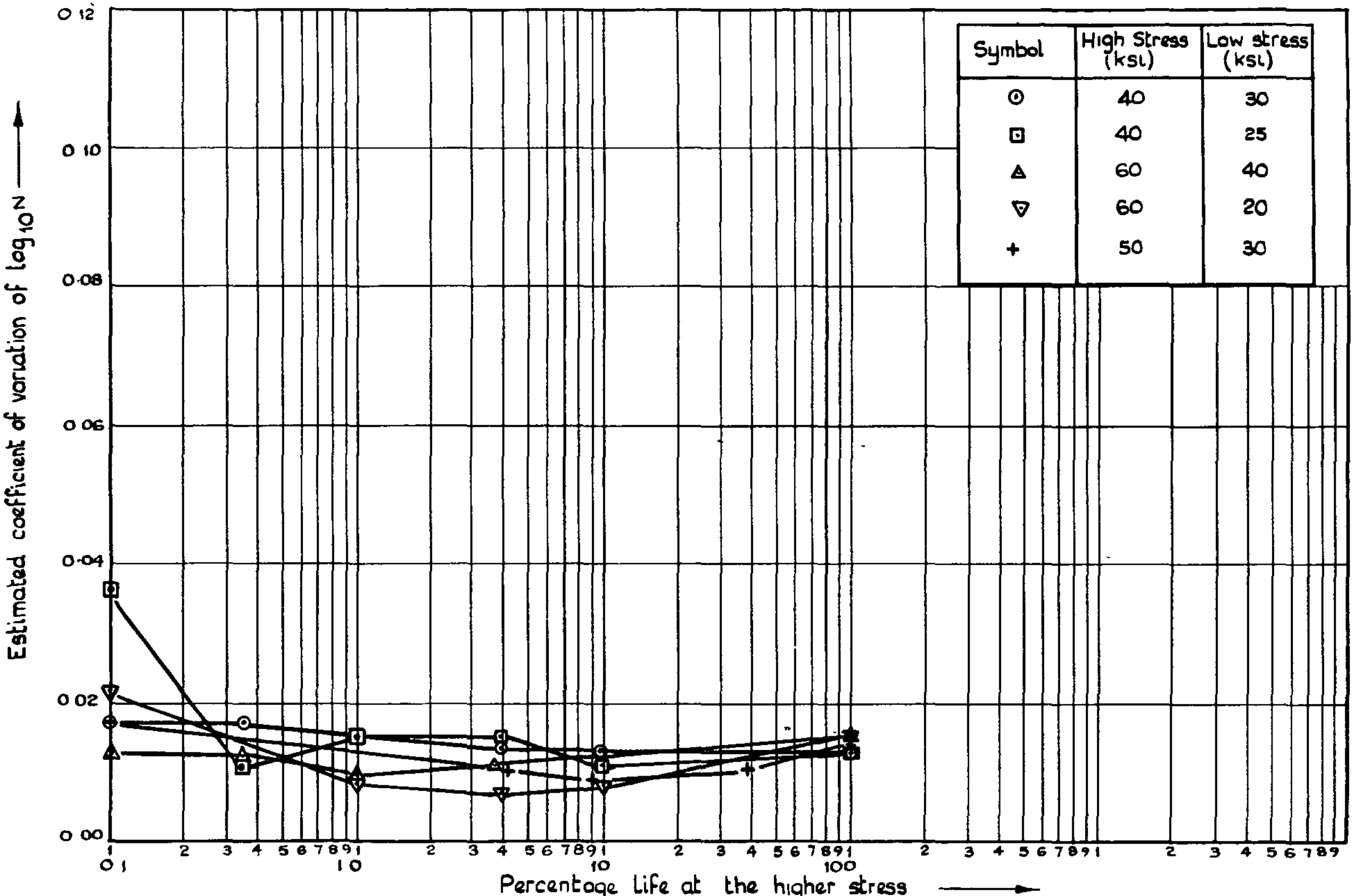


Fig. 31 Rotating beam, wire specimens of 2024-T4, Ref (20).
Two-level repeated block fatigue loading

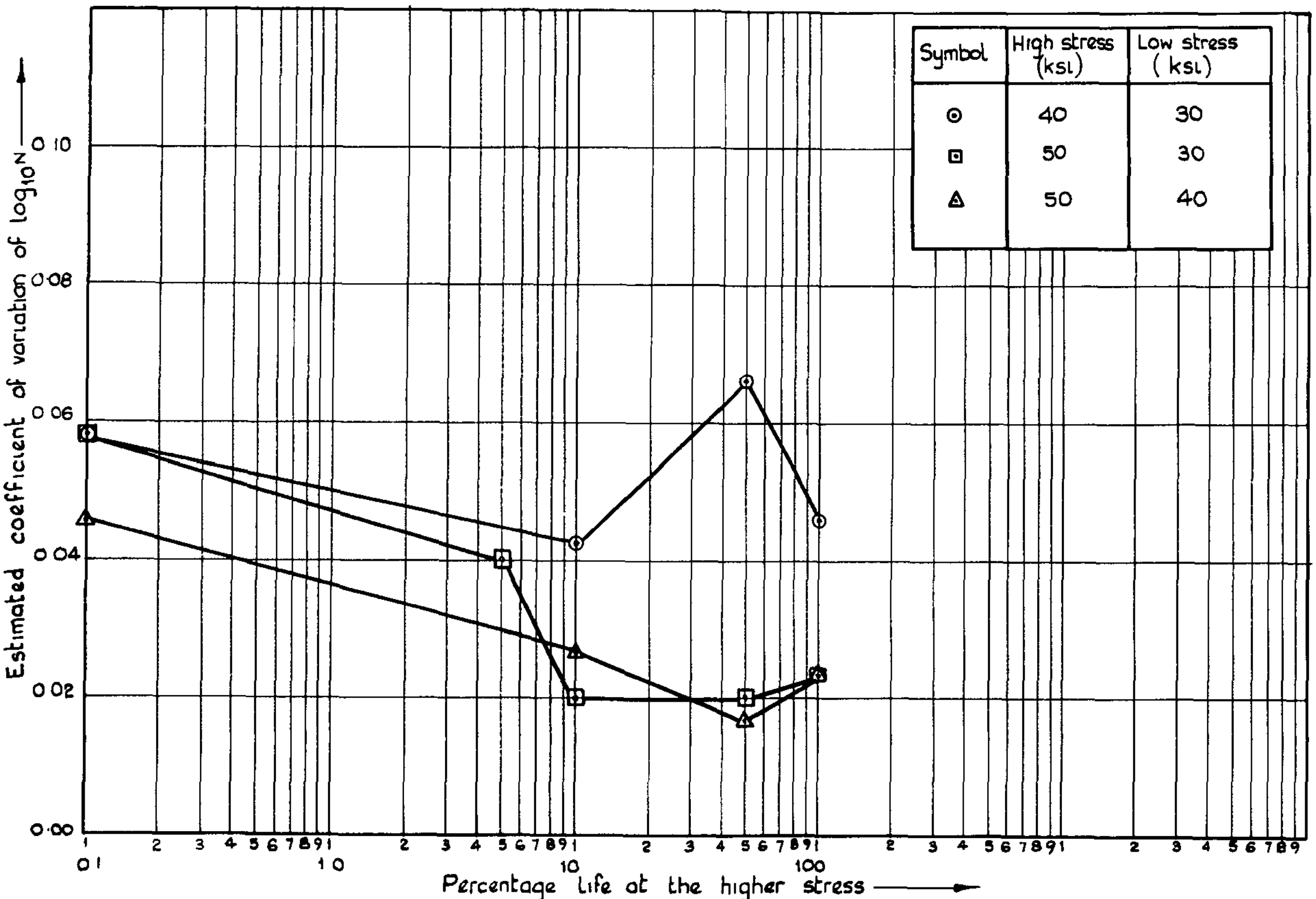


Fig. 32 Unnotched, rotating beam wire specimens of 7075-T6, Ref (5).
Two-level repeated block fatigue loading

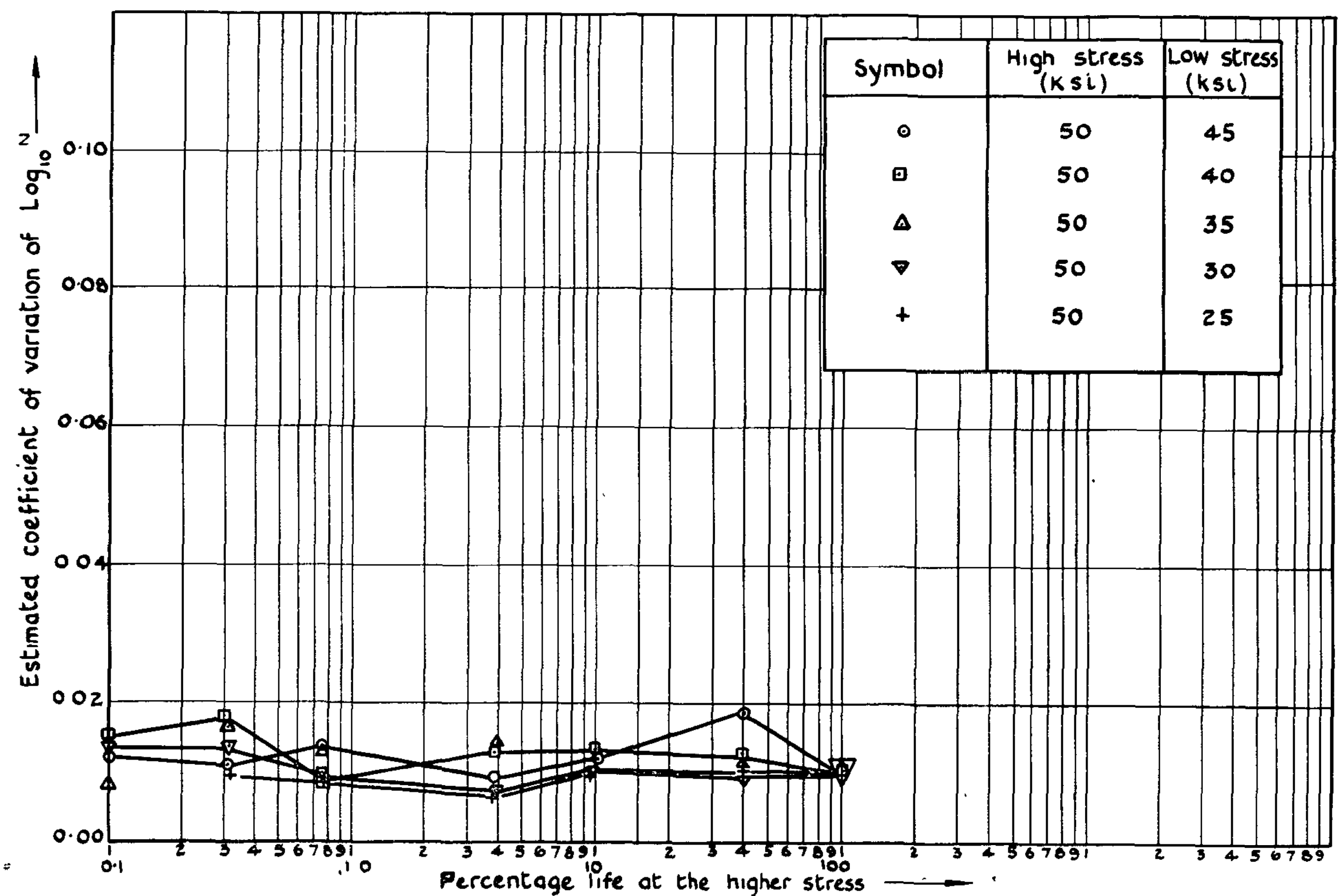


Fig. 33 Unnotched, rotating beam wire specimens of 2024-T4 material, Ref(21). Two level repeated block fatigue tests

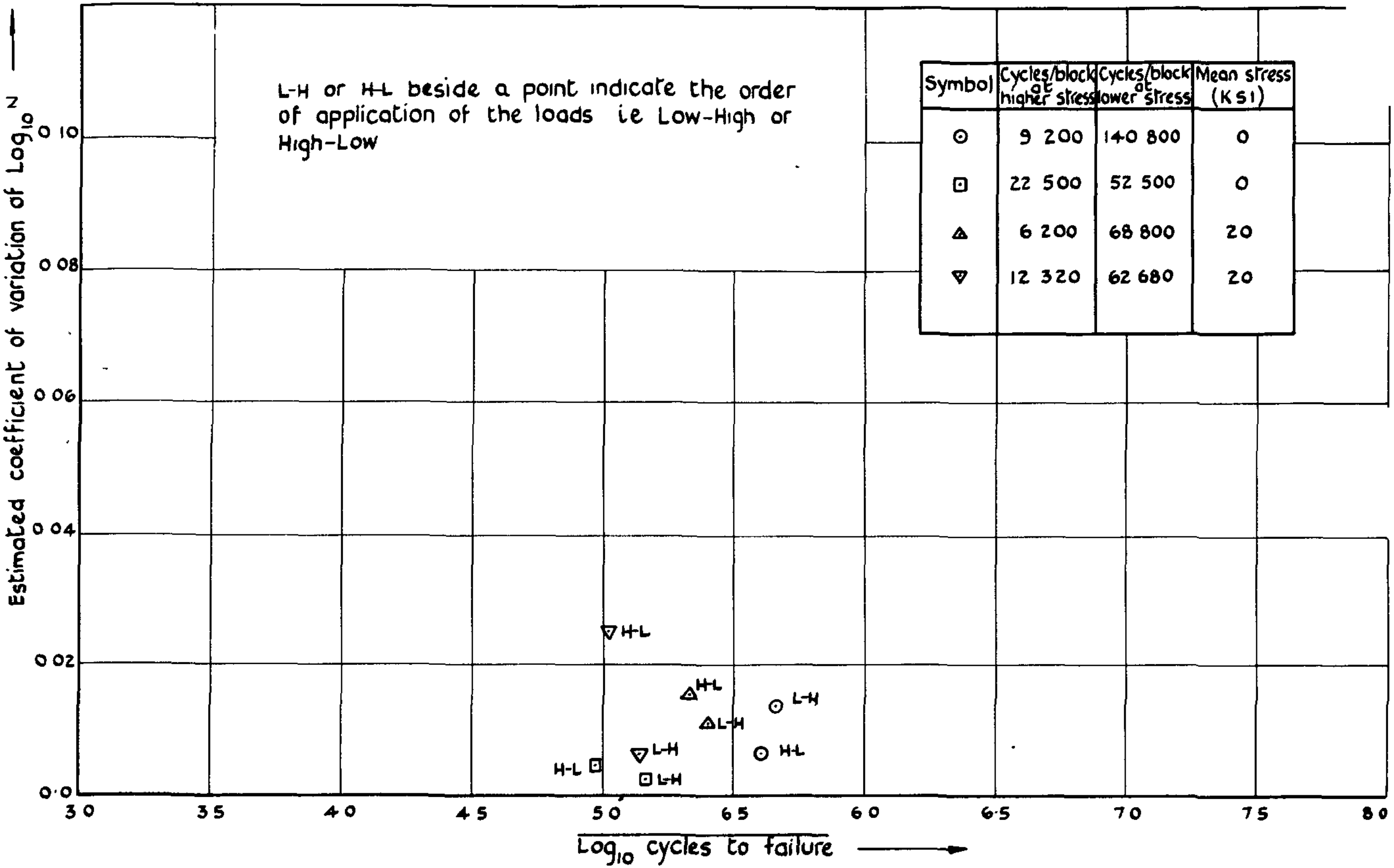


Fig. 34 Unnotched, axial sheet specimens (0·032 in thick) of 7075-T6 alloy, Ref (22). Two-level repeated block loading (stresses 30 and 16 ksi)

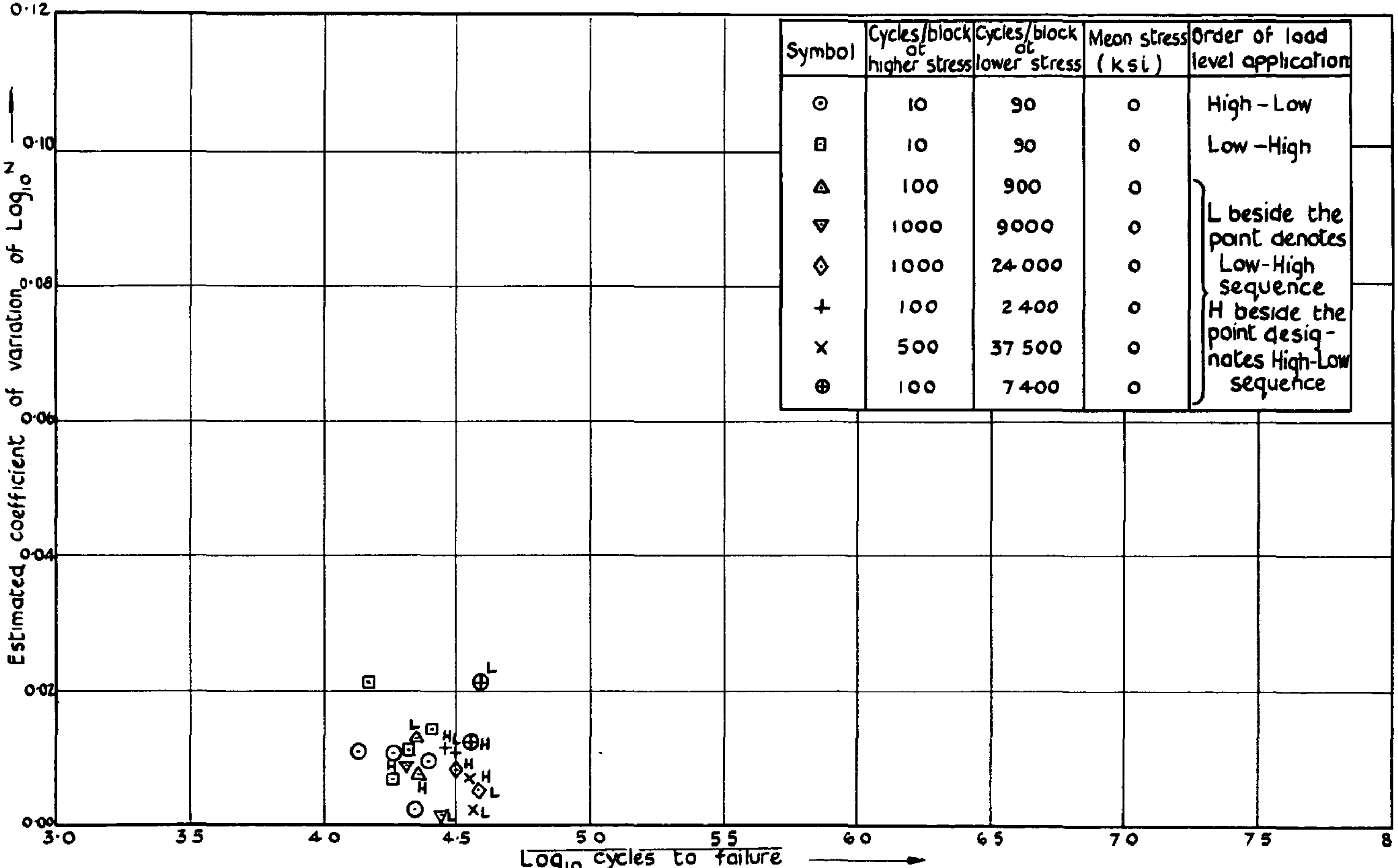


Fig. 35 Unnotched, axial sheet specimens (0·064 in thick) of 7075-T6 alloy, Ref(22). Two level repeated block loading (stresses 60 and 30 ksi)

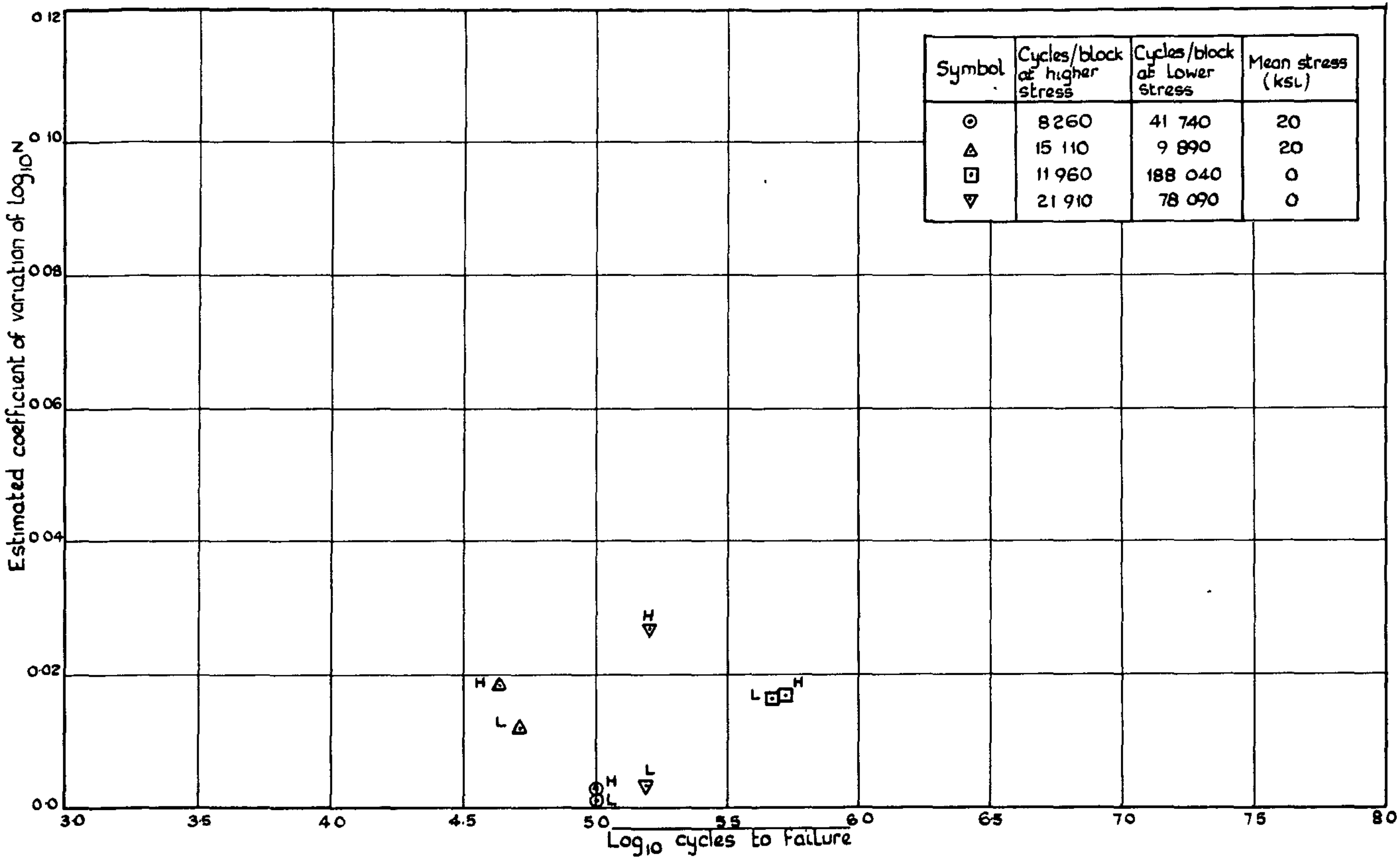
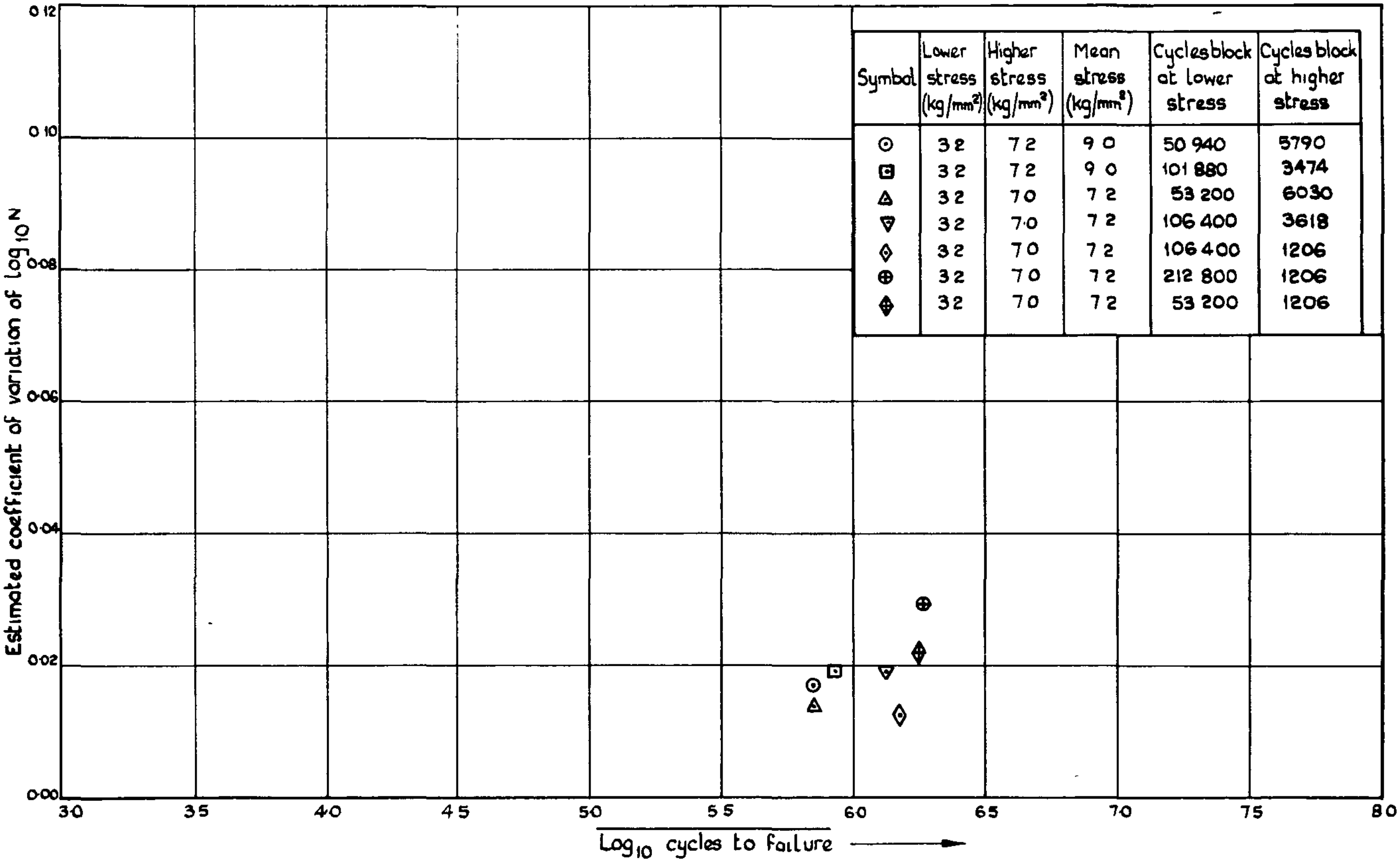


Fig. 36 Unnotched, axial sheet specimens (0·032 in thick) of 24S - T3 alloy. Ref (22).
Two-level repeated block loading (stresses 30 and 16 ksi)



Symbol	Lower stress (kg/mm²)	Higher stress (kg/mm²)	Mean stress (kg/mm²)	Cycles block at lower stress	Cycles block at higher stress
⊙	3 2	7 2	9 0	50 940	5790
⊡	3 2	7 2	9 0	101 880	3474
△	3 2	7 0	7 2	53 200	6030
▽	3 2	7·0	7 2	106 400	3618
◇	3 2	7 0	7 2	106 400	1206
⊕	3 2	7 0	7 2	212 800	1206
◈	3 2	7 0	7 2	53 200	1206

Fig.37 Axial, alclad rivetted lap-joints of 2024 S-T alloy, Ref(19).
Two level repeated block tests

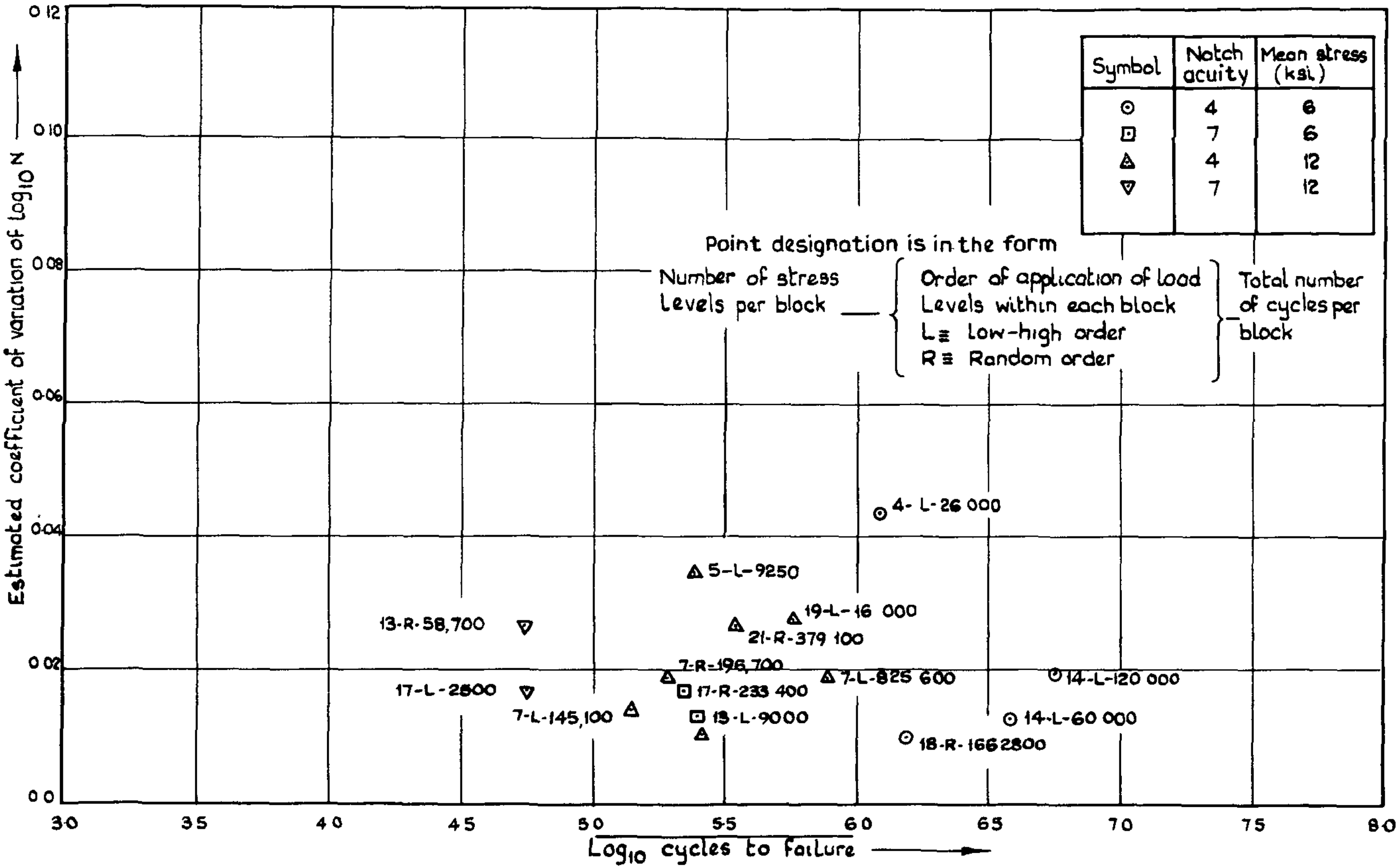
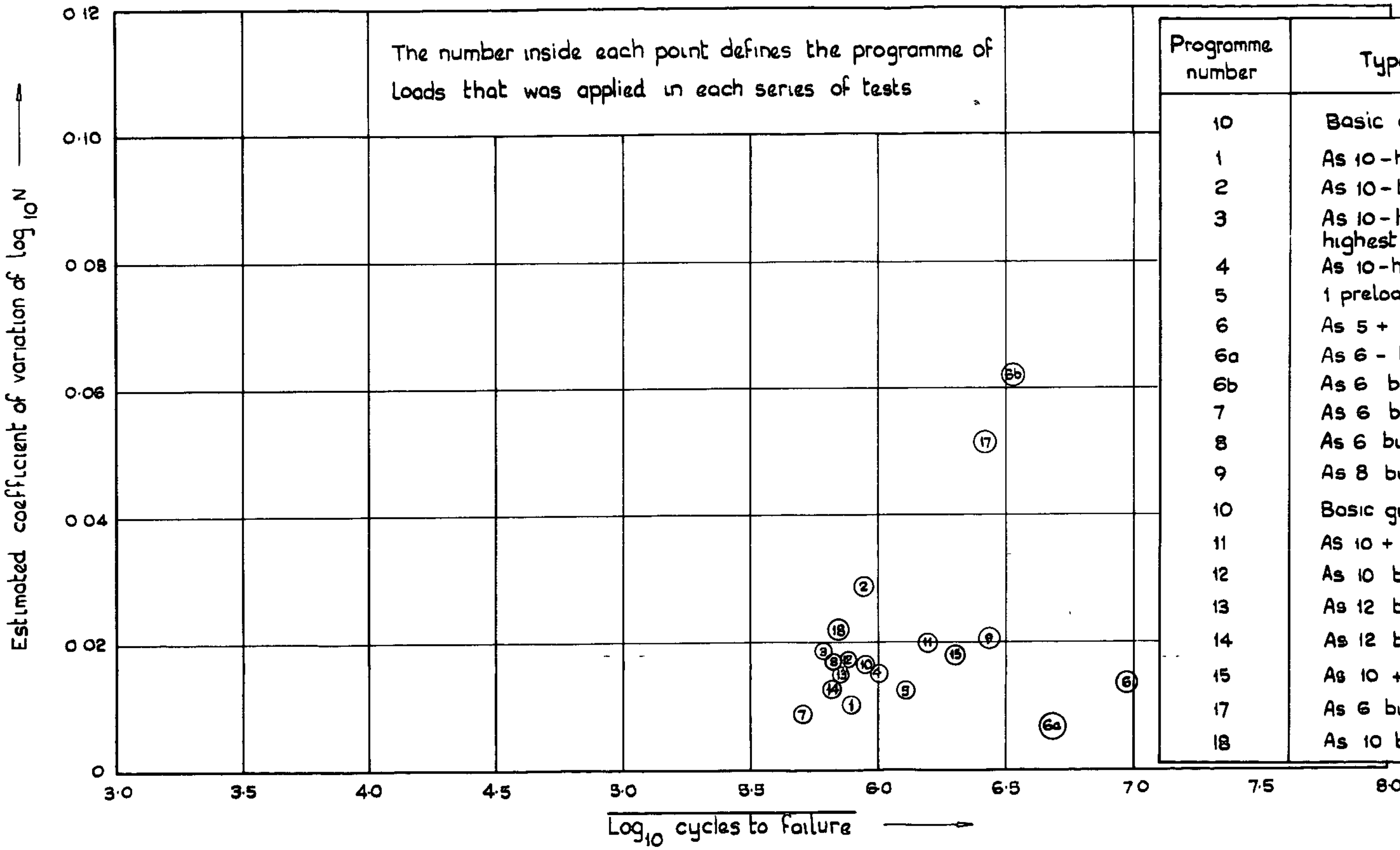


Fig. 38 Axial, notched coupon specimens of 7075–T6 alloy, Ref (30). Multi-level block variable amplitude testing to a gust spectrum



Programme number	Type of loading
10	Basic gust spectrum. 5 load levels low-high sequence.
1	As 10 - highest load level. Block size doubled
2	As 10 - two highest load levels
3	As 10 - highest load level + 2 × cycles at next highest load level.
4	As 10 - highest load level + 2 × cycles at lowest load level
5	1 preload + 10 - highest load level
6	As 5 + high positive half load after each block
6a	As 6 - high loads after 50th block
6b	As 6 but high loads after even blocks only
7	As 6 but negative instead of positive half cycles
8	As 6 but positive followed by negative half cycles
9	As 8 but order of high loads reversed
10	Basic gust spectrum 5 load levels low - high
11	As 10 + one lower load level
12	As 10 but block size doubled low - high
13	As 12 but but high - low
14	As 12 but low - high - low
15	As 10 + two lower load levels
17	As 6 but in high - low sequence
18	As 10 but block size doubled

Fig. 39 Axial, rivetted lap - joints of 7075 alloy, Ref (6).

Multi - level repeated block variable amplitude loading to a gust spectrum

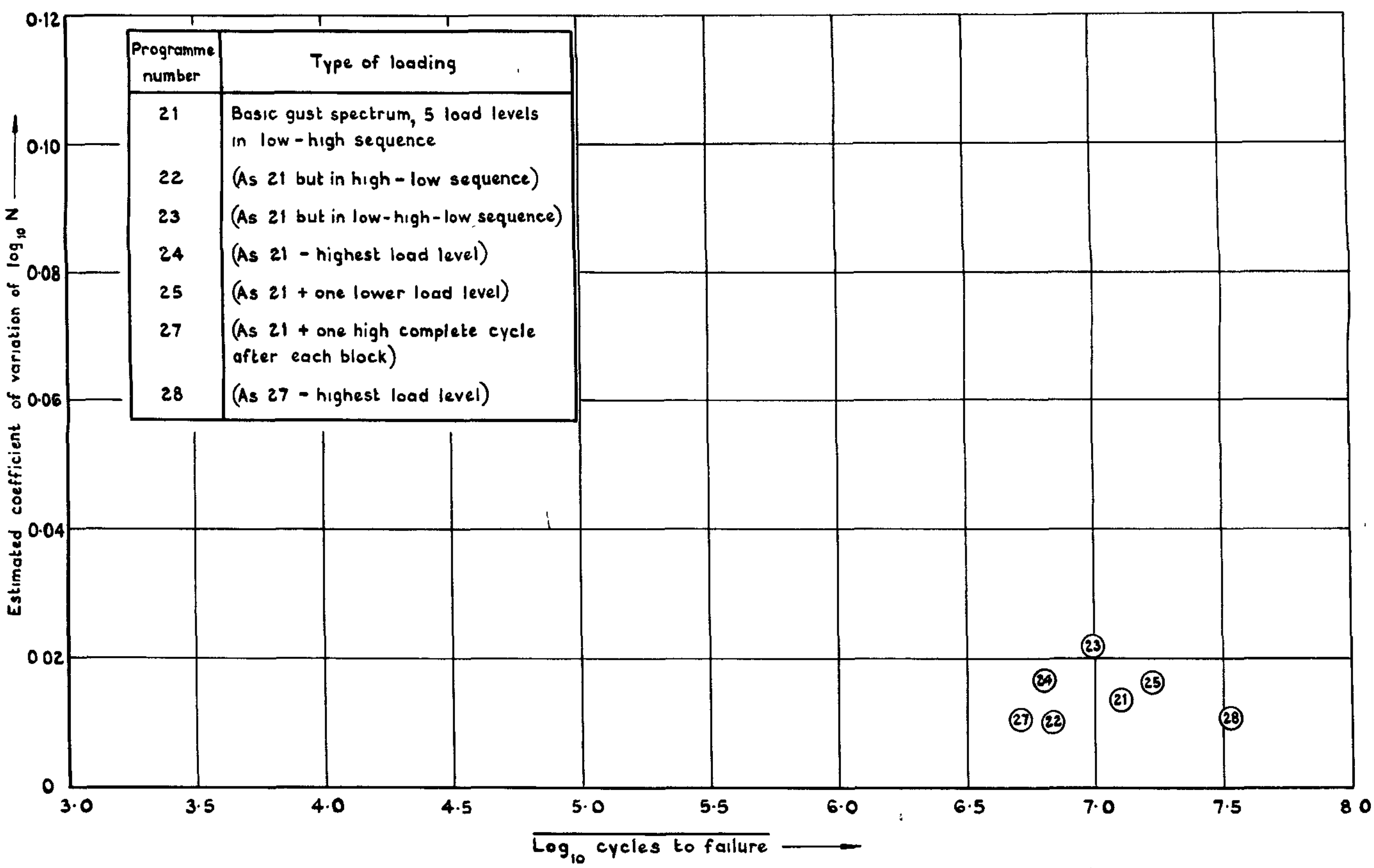


Fig. 40 Axial, rivetted lap-joints of 2024 alloy, Ref (6).
Multi-level repeated block variable amplitude loading to a gust spectrum

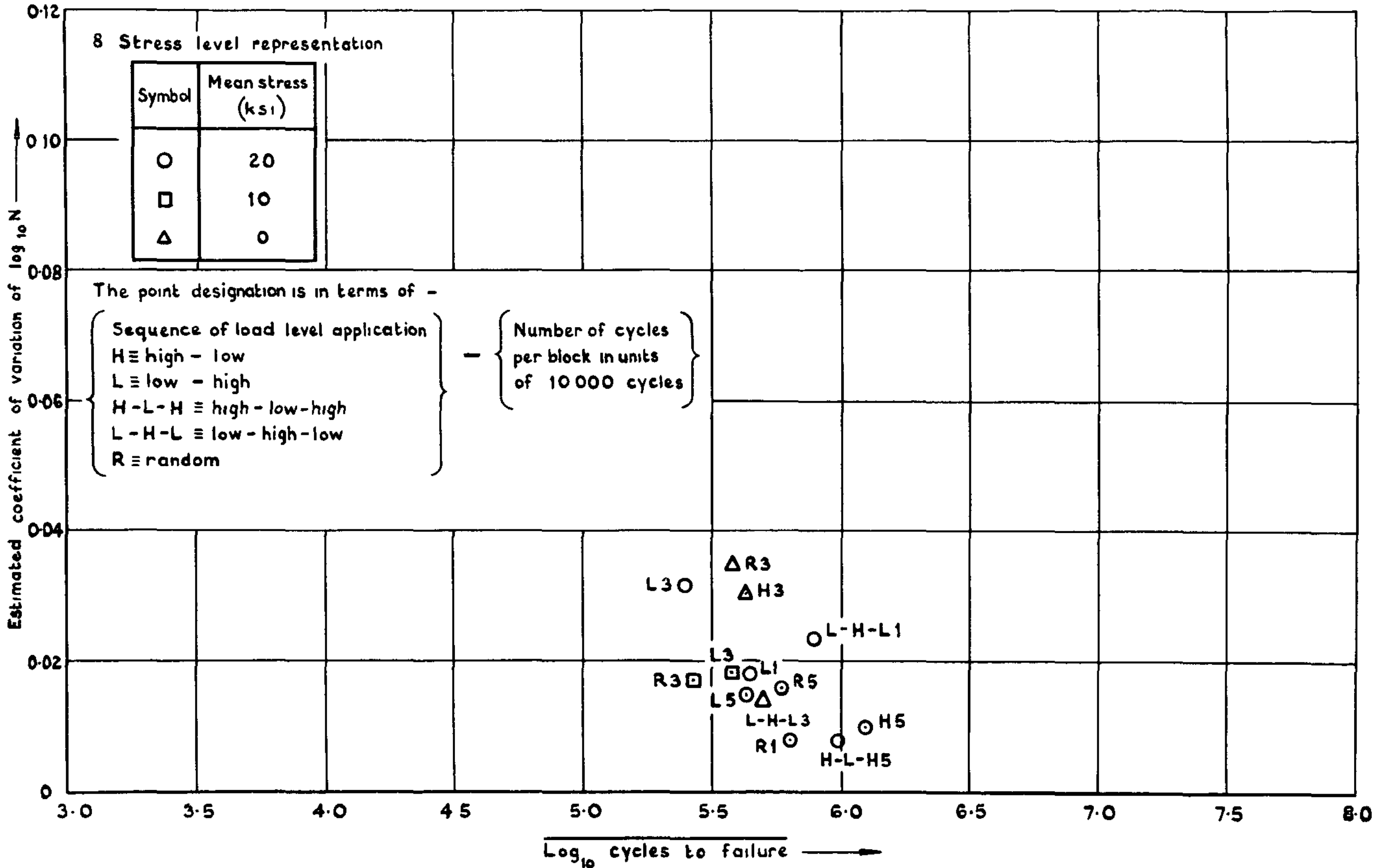


Fig. 41 Axial, edge-notched sheet $(K_t = 4{\cdot}0)$ of 7075-T6 alloy, Ref (24). Multi-level block variable amplitude loading to a gust spectrum

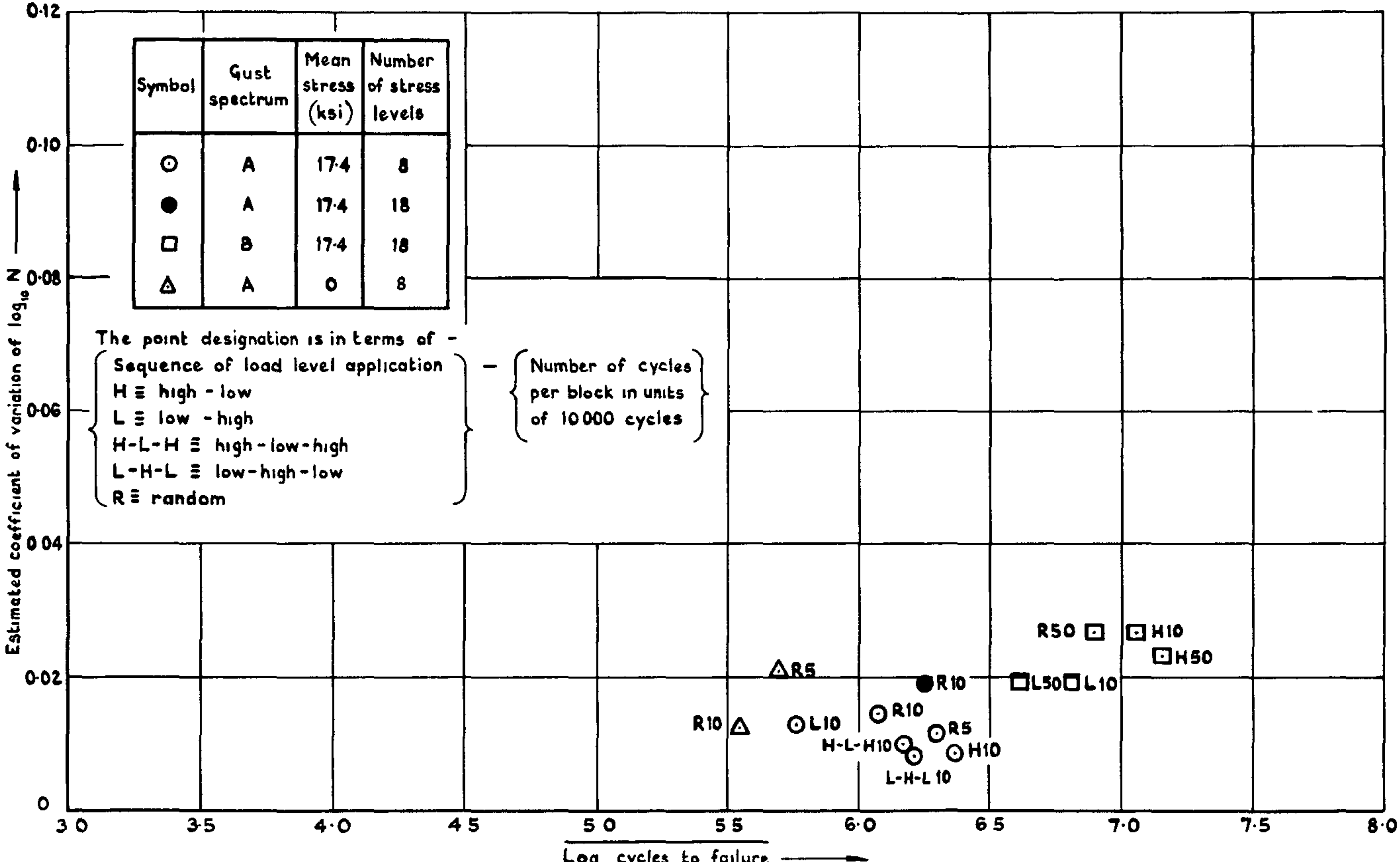


Fig 42 Axial, edge-notched sheet ($K_t = 4{\cdot}0$) of 2024-T3 alloy, Ref (24).
Multi-level block variable amplitude loading to a gust spectrum
Gust spectrum B has a greater proportion of the more damaging stress cycles than spectrum A

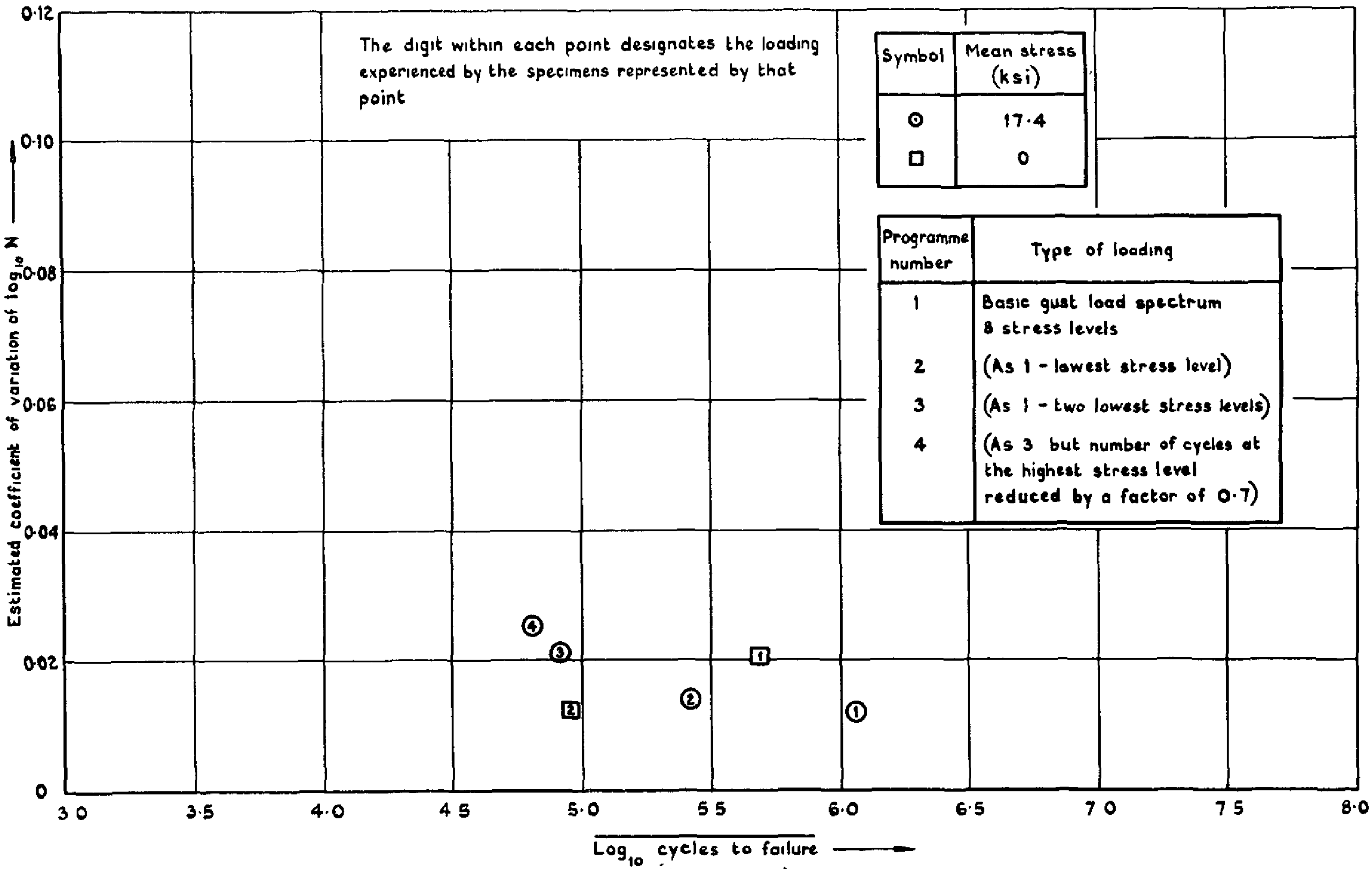


Fig. 43 Axial, edge-notched sheet ($K_t = 4 \cdot 0$) of 2024-T3 alloy, Ref (32). Multi-level repeated block variable amplitude loading to a gust spectrum

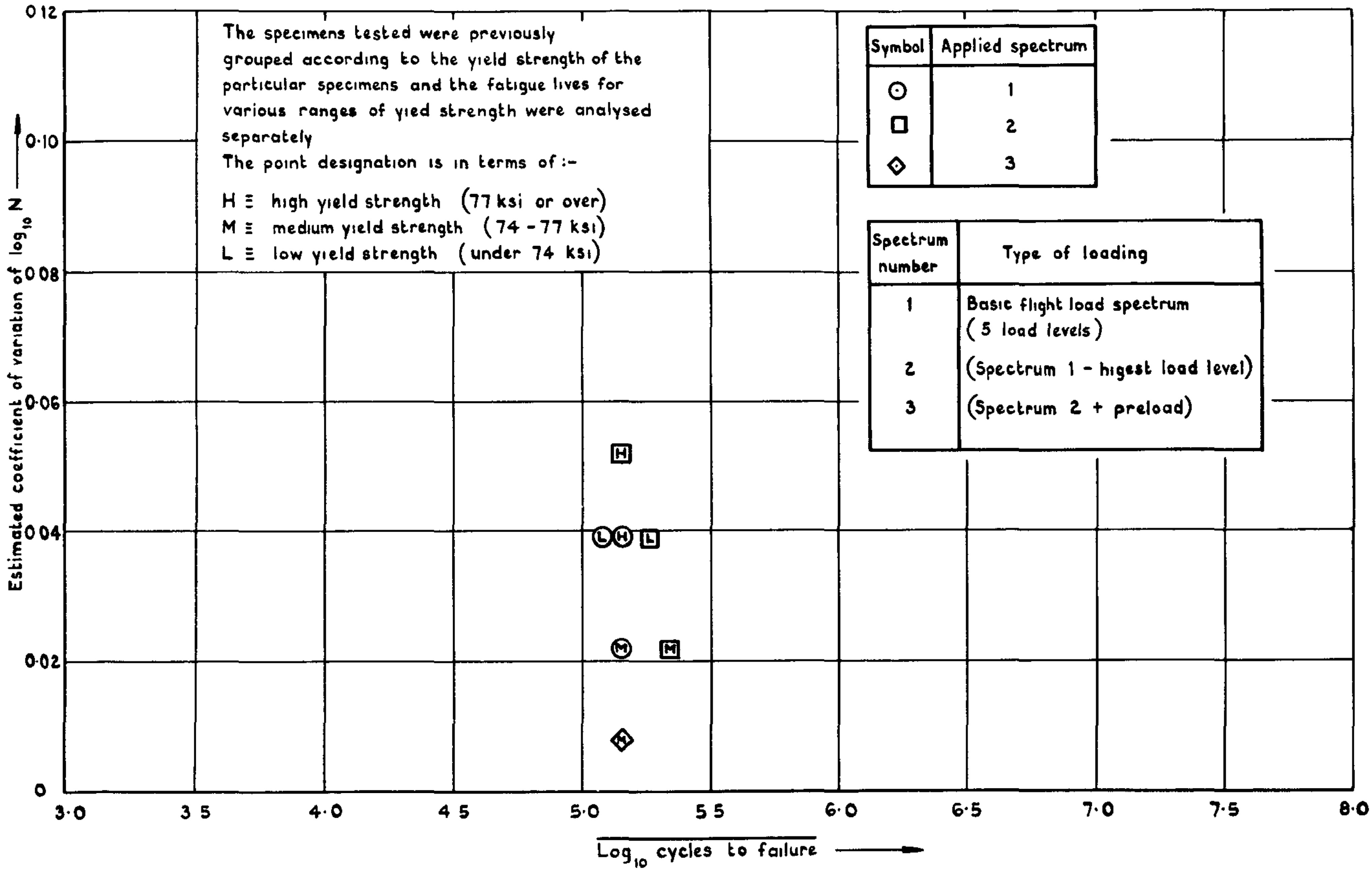


Fig. 44 Axial, double shear rivetted butt joints of 7075-T6, Ref (8).
Multi-level block variable amplitude loading to a service spectrum

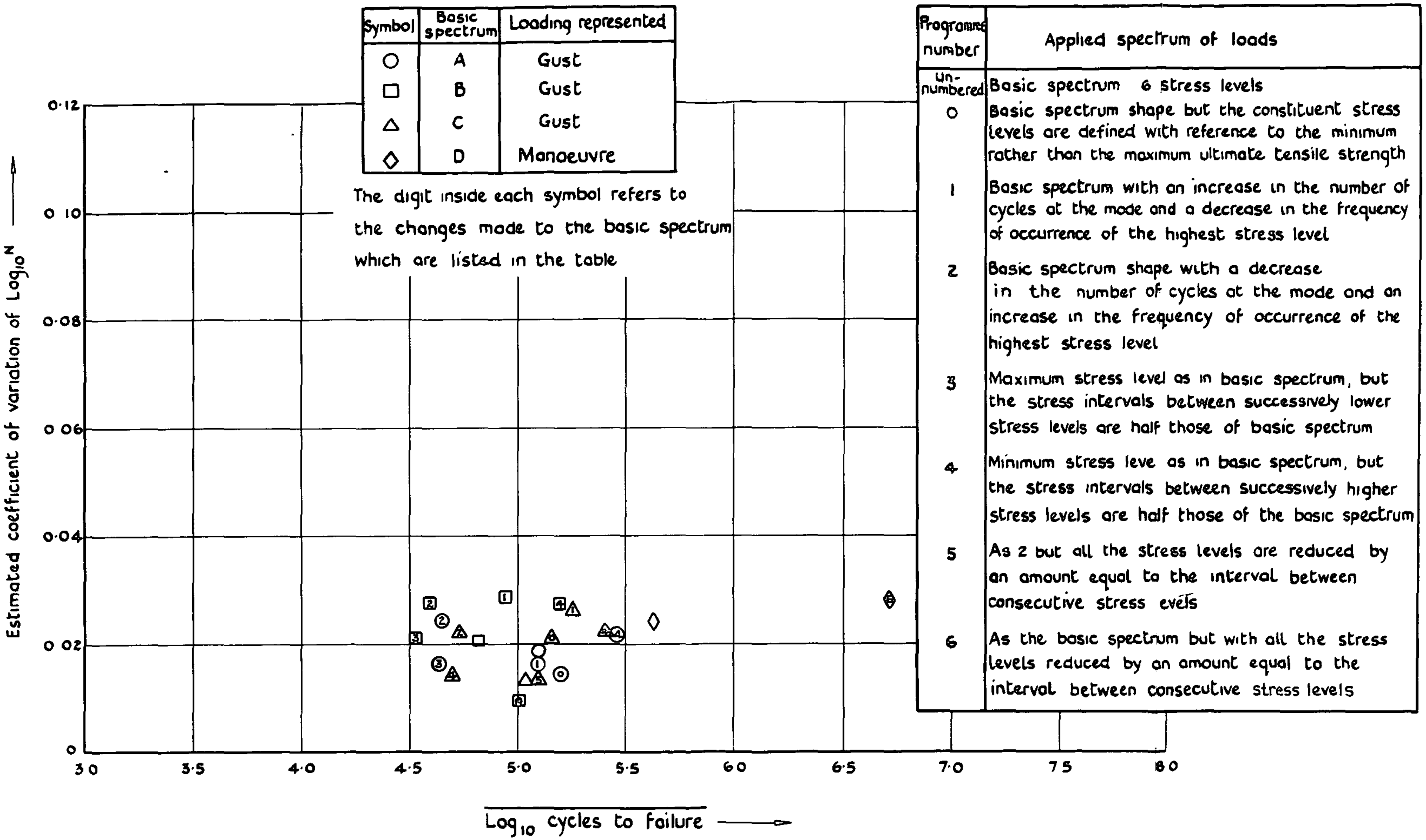
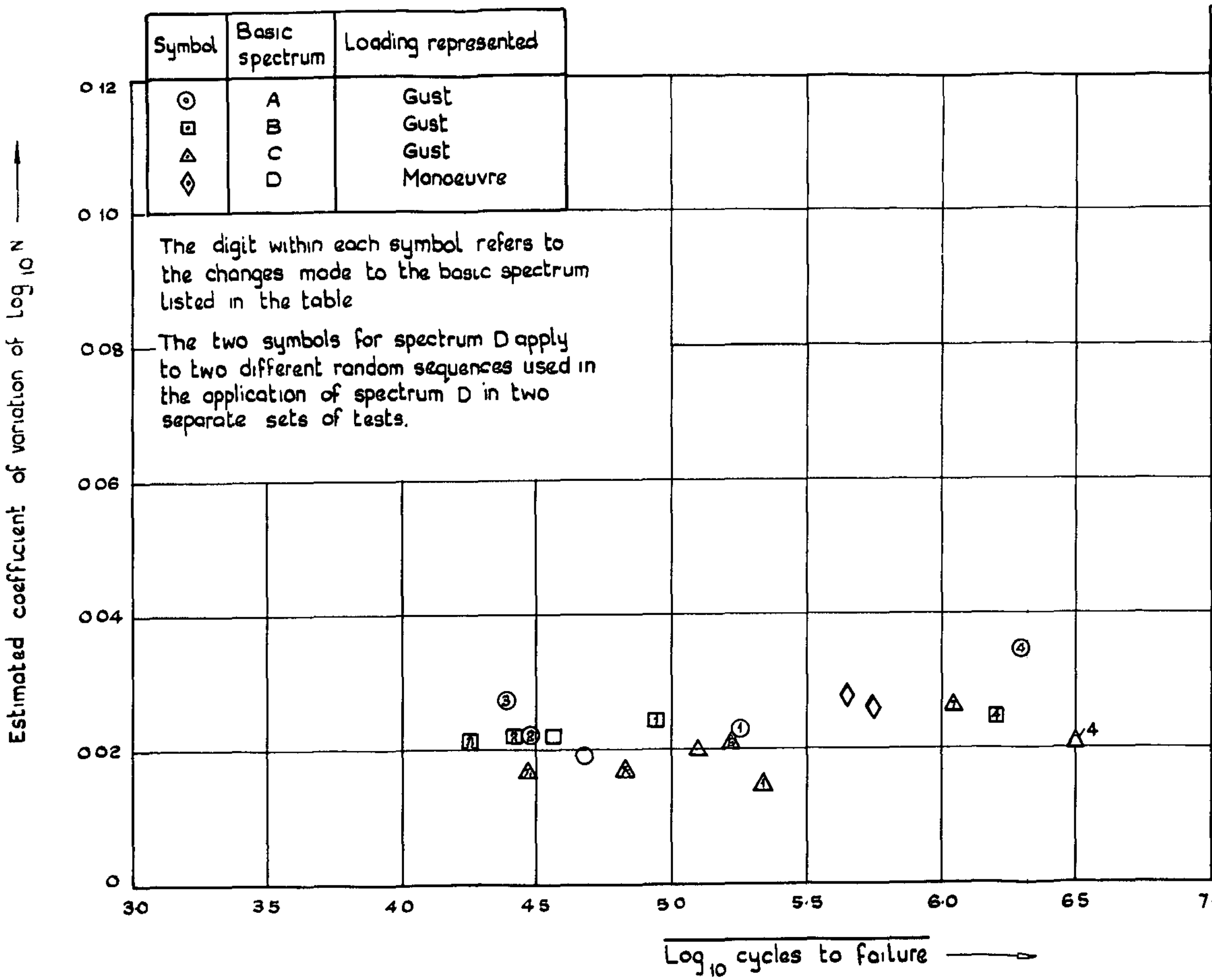


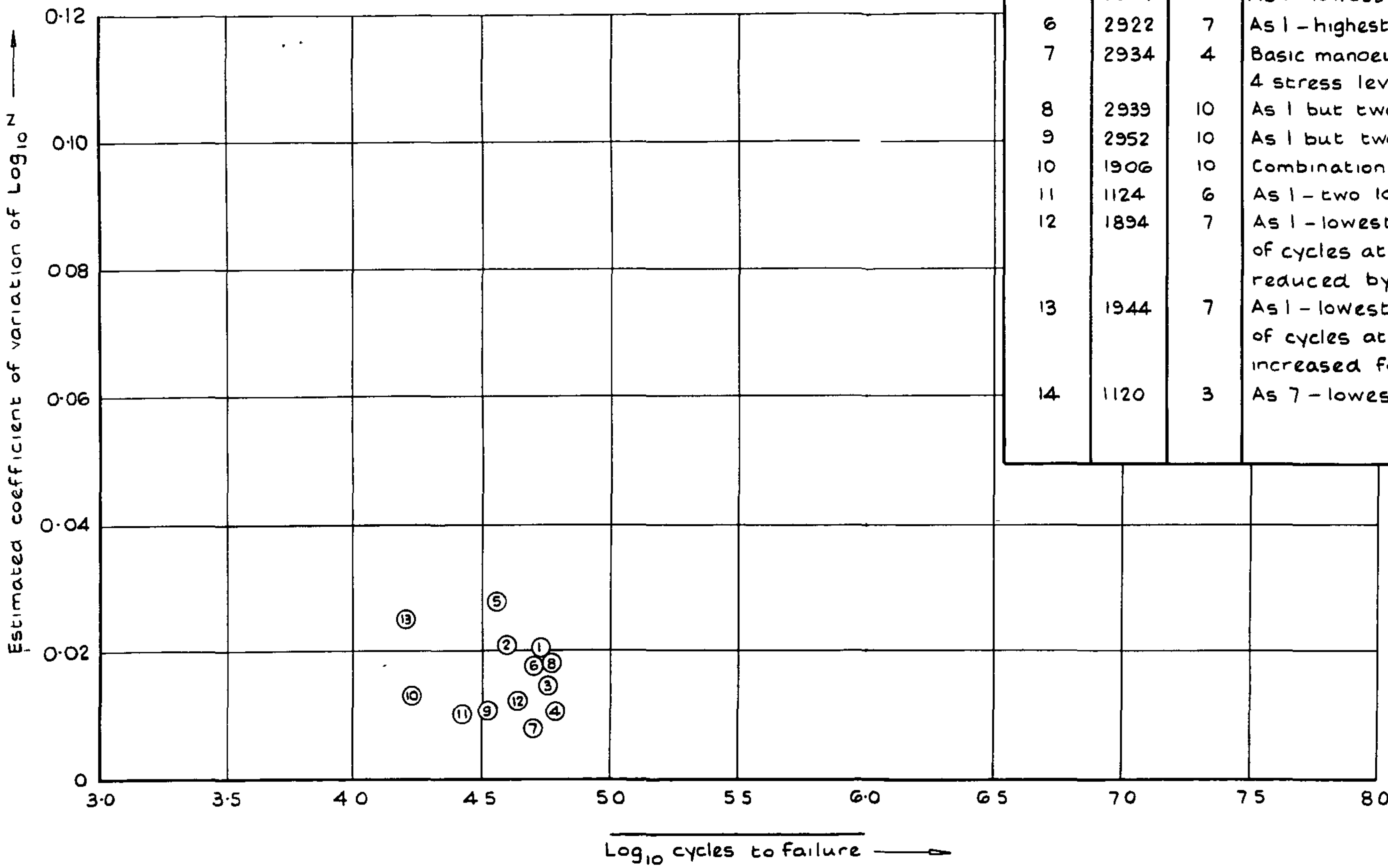
Fig.45 Unnotched, rotating beam specimens of 2024 alloy, Ref(11).

Multi-level random sequence block variable amplitude loading to gust and manoeuvre spectra



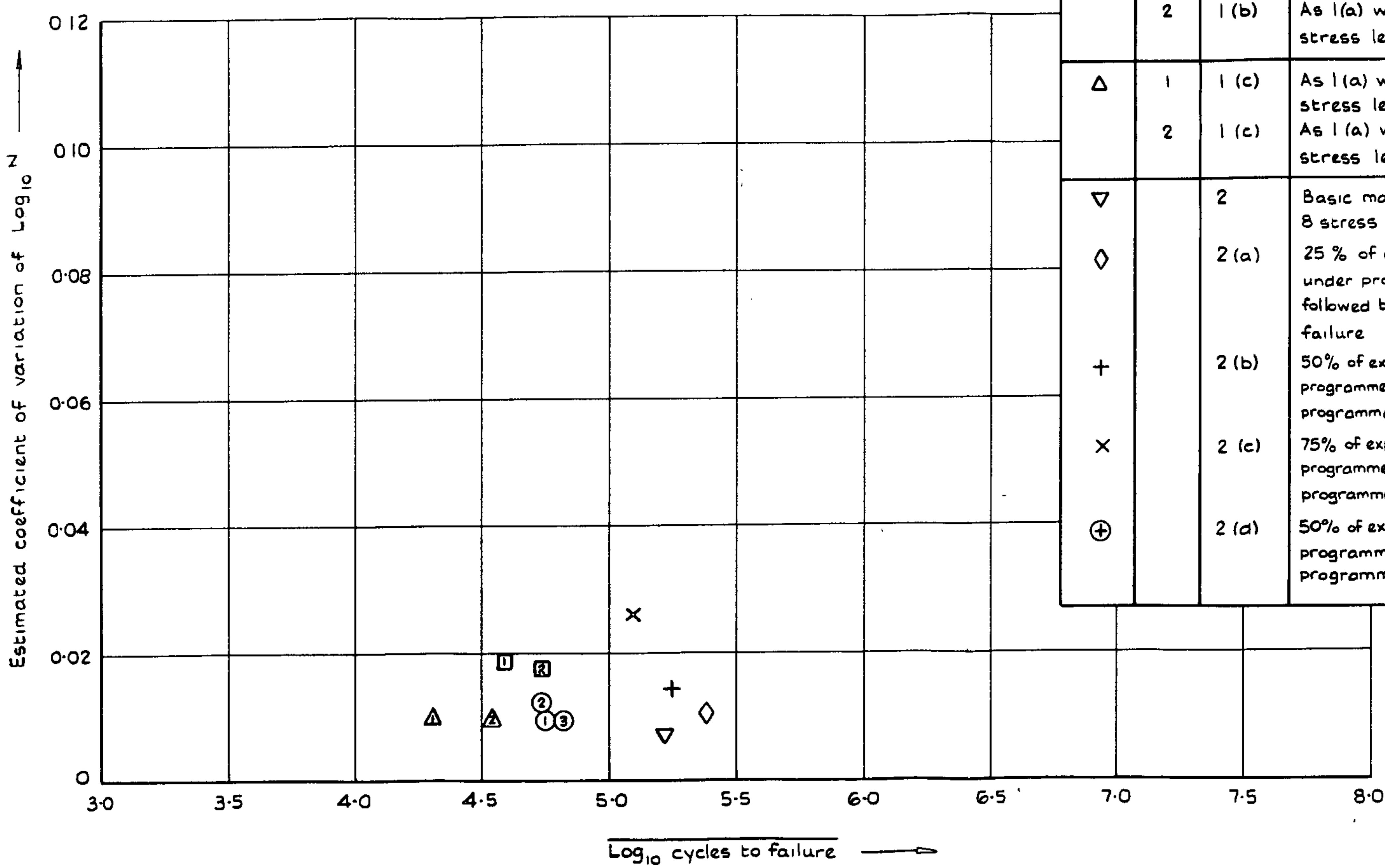
Programme Number	Applied spectrum of loads.
Unnumbered	Basic spectrum 6 stress levels
0	Basic spectrum shape but constituent stress levels are defined in terms of the minimum rather than the maximum ultimate tensile strength
1	Basic spectrum shape with an increase in the number of cycles at the mode and a decrease in the frequency of occurence of the highest stress level
2	Basic spectrum shape with a decrease in the number of cycles at the mode and an increase in the frequency of occurence at the highest stress level
3	Maximum stress level as in basic spectrum, but the stress intervals between successively lower stress levels are half of those of basic spectrum
4	Minimum stress level as in basic spectrum, but the stress intervals between successively higher stress levels are half of those of basic spectrum
5	As 2 but all the stress levels are reduced by an amount equal to the interval between consecutive stress levels
6	As the basic spectrum but with all the stress levels reduced by an amount equal to the interval between consecutive stress levels.
7	As 1 but all the stress levels are reduced by an amount equal to the interval between consecutive stress levels

Fig. 46 Unnotched rotating beam specimens of 7075 alloy, Ref (11).
Multi - level random sequence block variable amplitude loading to gust and manoeuvre spectra



Programme number	Cycle per block	Number of stress levels	Load spectrum
1	2934	8	Basic manoeuvre spectrum
2	293	8	As 1 but smaller block
3	879	8	As 1 but smaller block
4	14 670	8	As 1 but larger block
5	1904	7	As 1 – lowest stress level
6	2922	7	As 1 – highest stress level
7	2934	4	Basic manoeuvre spectrum but only 4 stress leve s in representation
8	2939	10	As 1 but two higher stress levels
9	2952	10	As 1 but two negative load levels
10	1906	10	Combination of 1, 8 and 9
11	1124	6	As 1 – two lowest stress levels
12	1894	7	As 1 – lowest stress level and number of cycles at highest stress level reduced by 0·9
13	1944	7	As 1 – lowest stress level and number of cycles at highest stress level increased four fold
14	1120	3	As 7 – lowest stress level

Fig. 47 Axial, edge notched sheet (K_t = 4·0) of 7075-T6, Ref (25) and (32).
Multi-level random sequence block variable amplitude fatique loading to a manoeuvre spectrum



Symbol	Digit	Programme	Definition of programme	Method of application of programme
○	1	1 (a)	Basic manoeuvre spectrum 8 stress levels (DLLF 7·3)	Block test on a semiautomatic machine
	2	1 (a)	Basic manoeuvre spectrum 8 stress levels (DLLF 73)	Random cycle test
	3	1 (a)	Basic manoeuvre spectrum 8 stress levels (DLLF 7 3)	Block test on an automatic machine
□	1	1 (b)	As 1(a) with 2 higher stress levels added	Random cycle test
	2	1 (b)	As 1(a) with 2 higher stress levels added	Block test
△	1	1 (c)	As 1 (a) with 2 negative stress levels added	Random cycle test
	2	1 (c)	As 1 (a) with 2 negative stress levels added	Block test
▽		2	Basic manoeuvre spectrum 8 stress levels (DLLF 6 0)	Block test
◇		2 (a)	25 % of expected life under programme 1 (a) followed by programme 2 to failure	Block test
+		2 (b)	50% of expected life under programme 1(a) followed by programme 2 to failure	Block test
×		2 (c)	75% of expected life under programme 1(a) followed by programme 2 to failure	Block test
⊕		2 (d)	50% of expected life under programme 2 followed by programme 1 (a) to failure	Block test

Fig. 48 Axial, edge notched sheet ($K_t = 4\ 0$) of 7075 material, Ref (26).
Multi-level variable amplitude fatique loading to a manoeuvre load spectrum

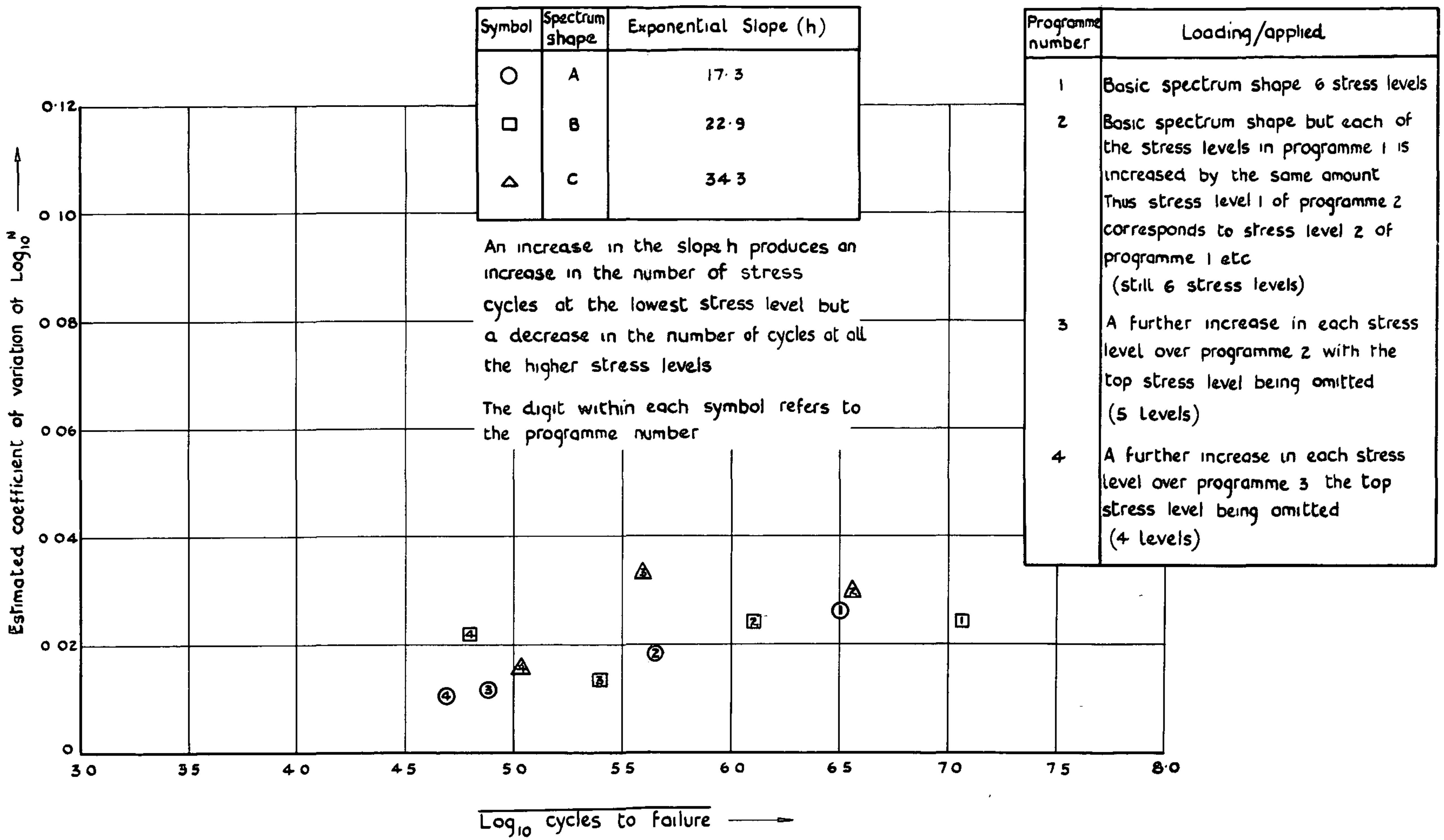


Fig. 49 Unnotched, rotating beam specimens of 2024 alloy, Ref (9).
Multi-level random sequence block variable amplitude loading to an exponented stress distribution

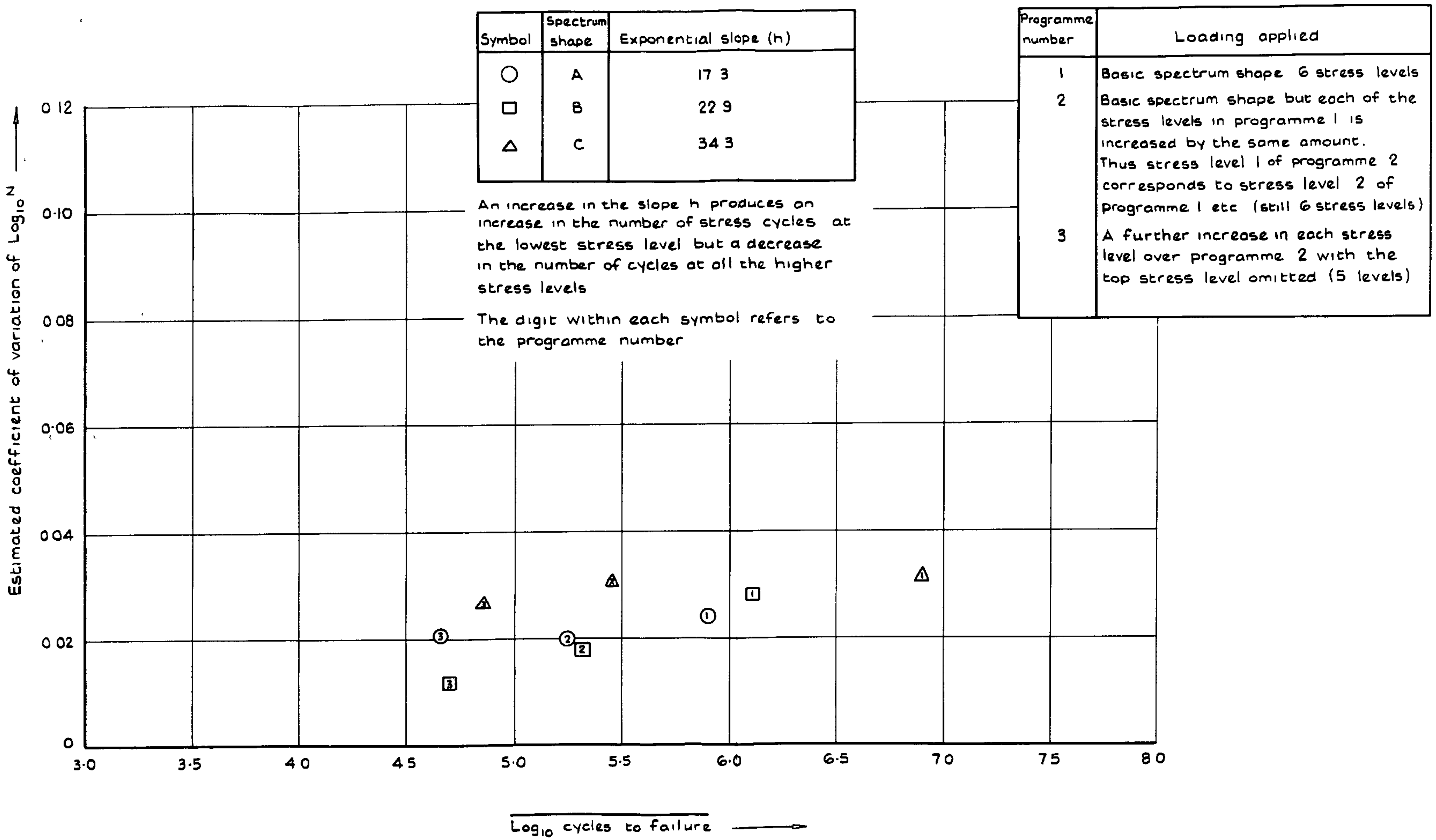


Fig. 50 Unnotched, rotating beam specimens of 2024 alloy, Ref (10).
Multi-level random sequence block variable amplitude loading to an exponented stress distribution

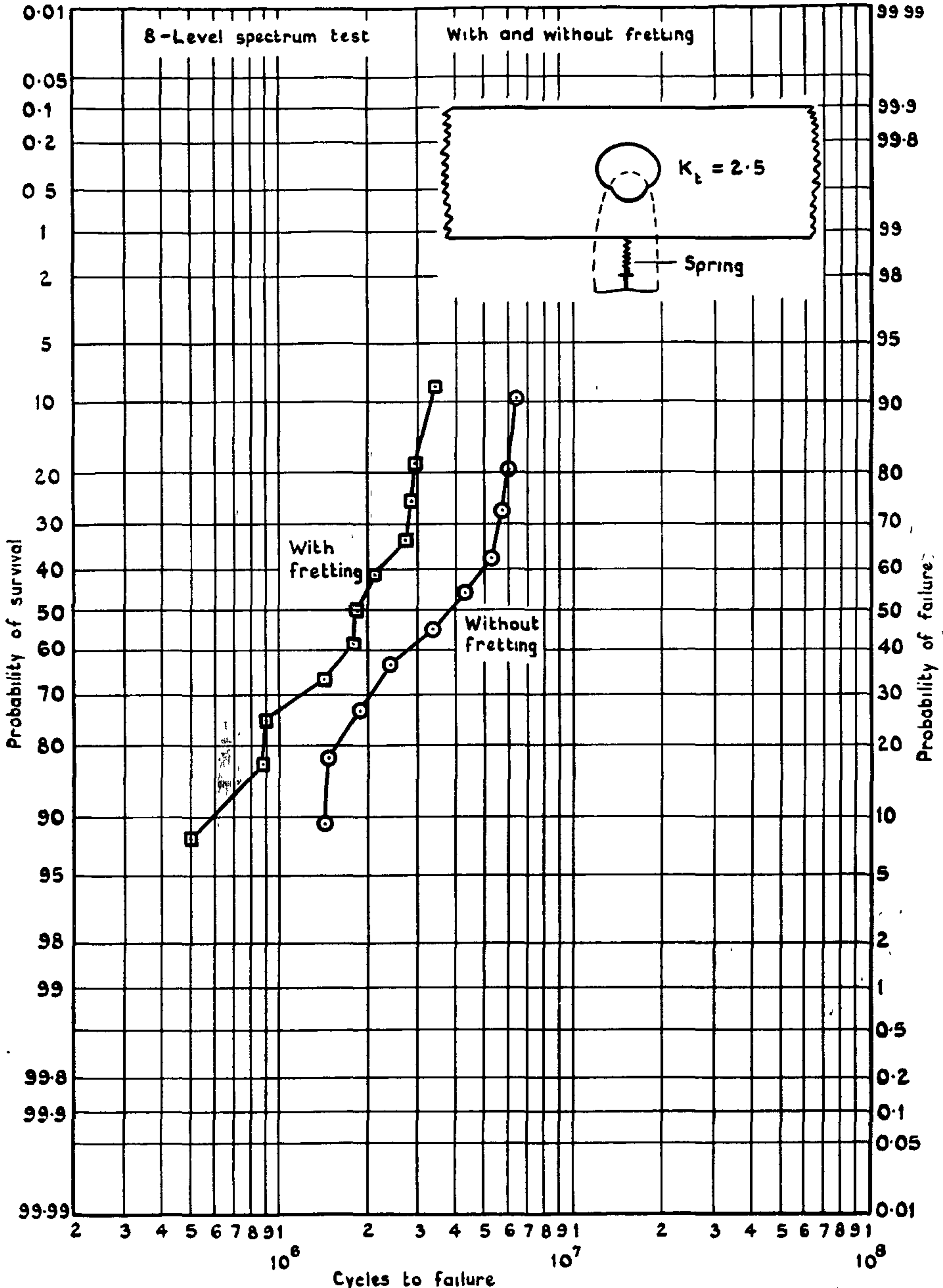


Fig 51 Axial, notched specimens of 2024 alloy, Ref(34). Multi-level repeated block variable amplitude test, to a combination of gust and manoeuvre loads

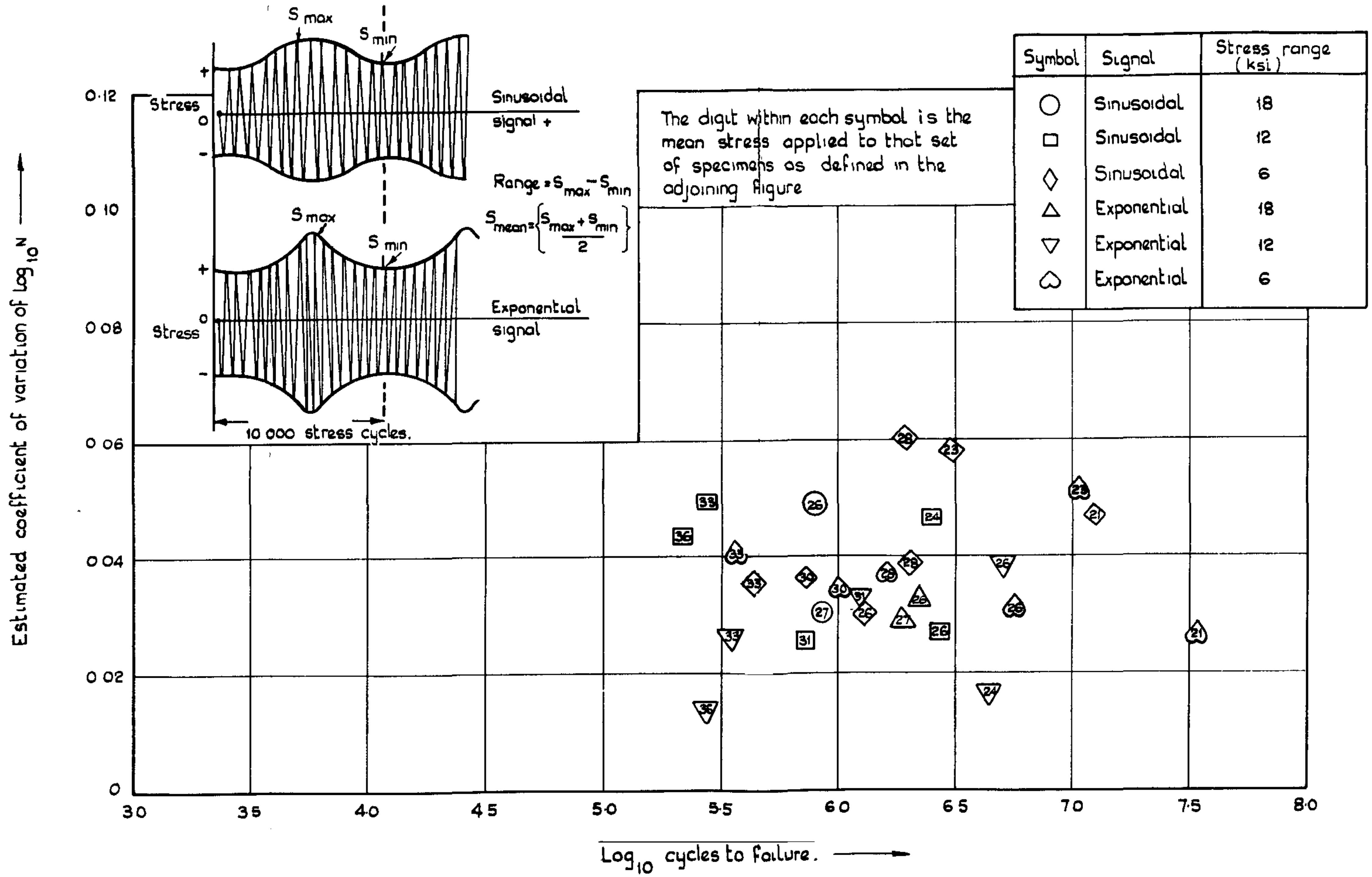


Fig. 52 Unnotched, rotating beam specimens of 2024 material, Ref (14)
Sinusoidal loading with amplitude continuously varying to a repeated signal shape

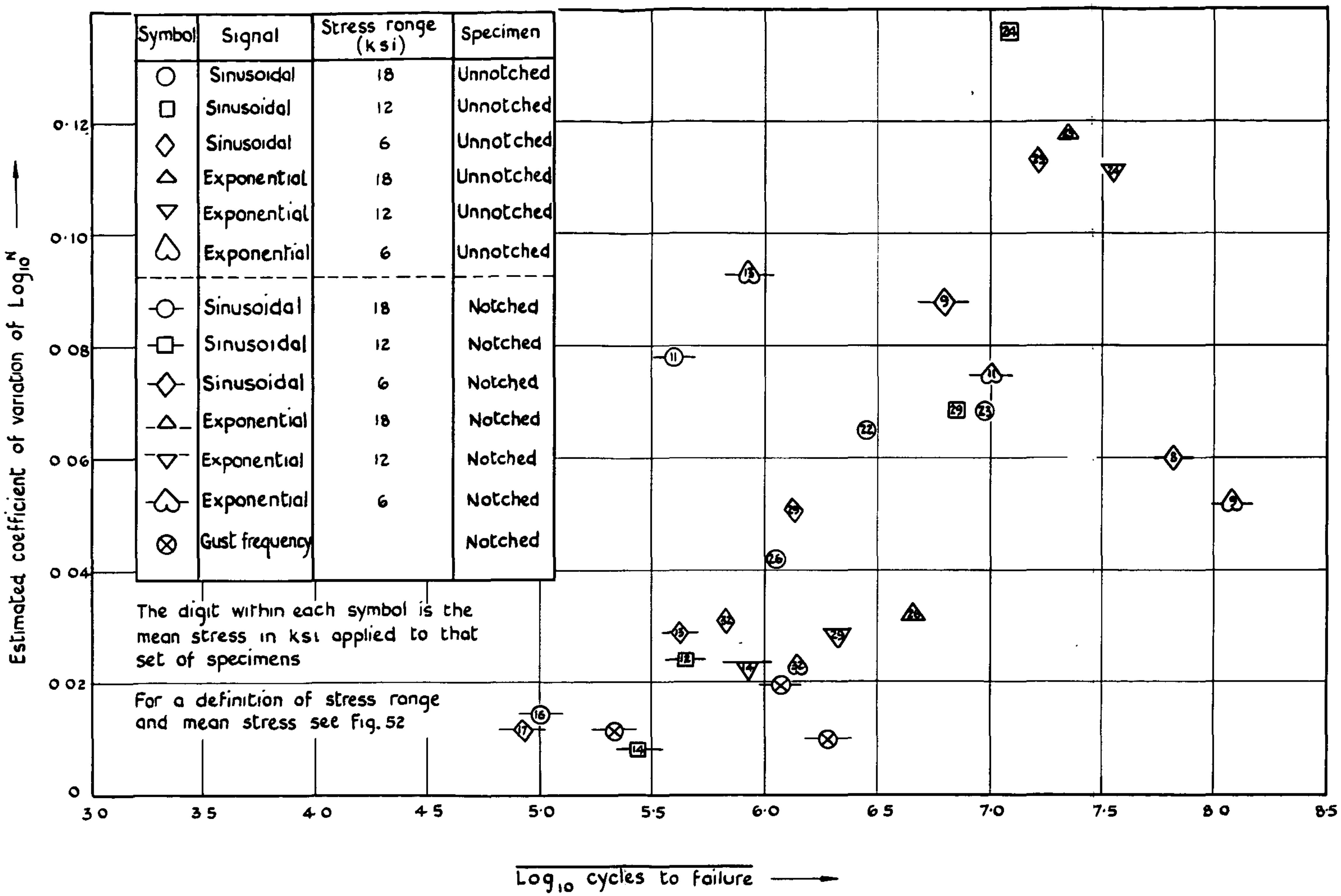


Fig. 53 Notched and unnotched, rotating beam specimens of 2024 material, Ref (15).
Sinusoidal loading with amplitude continuously varying to a repeated signal shape

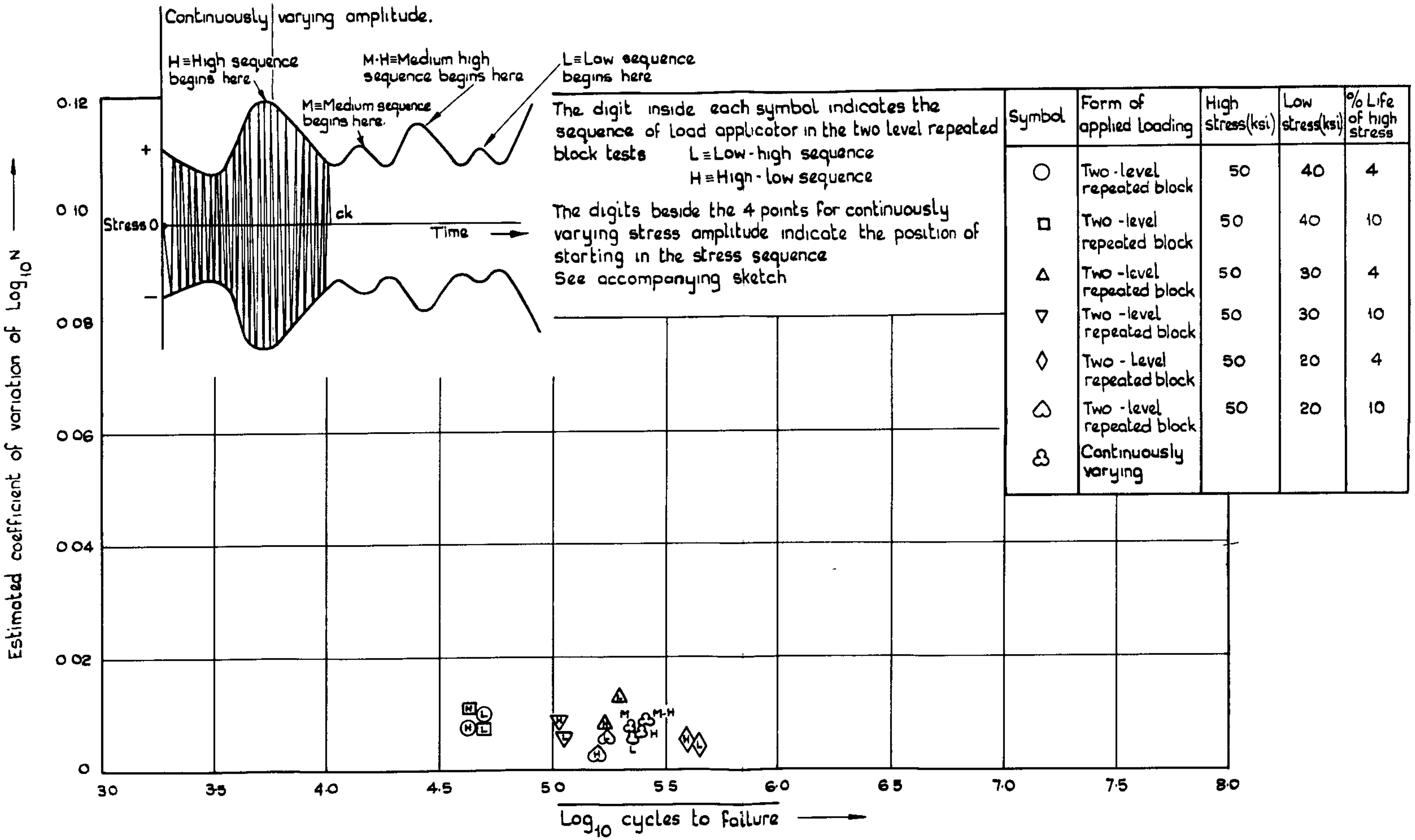
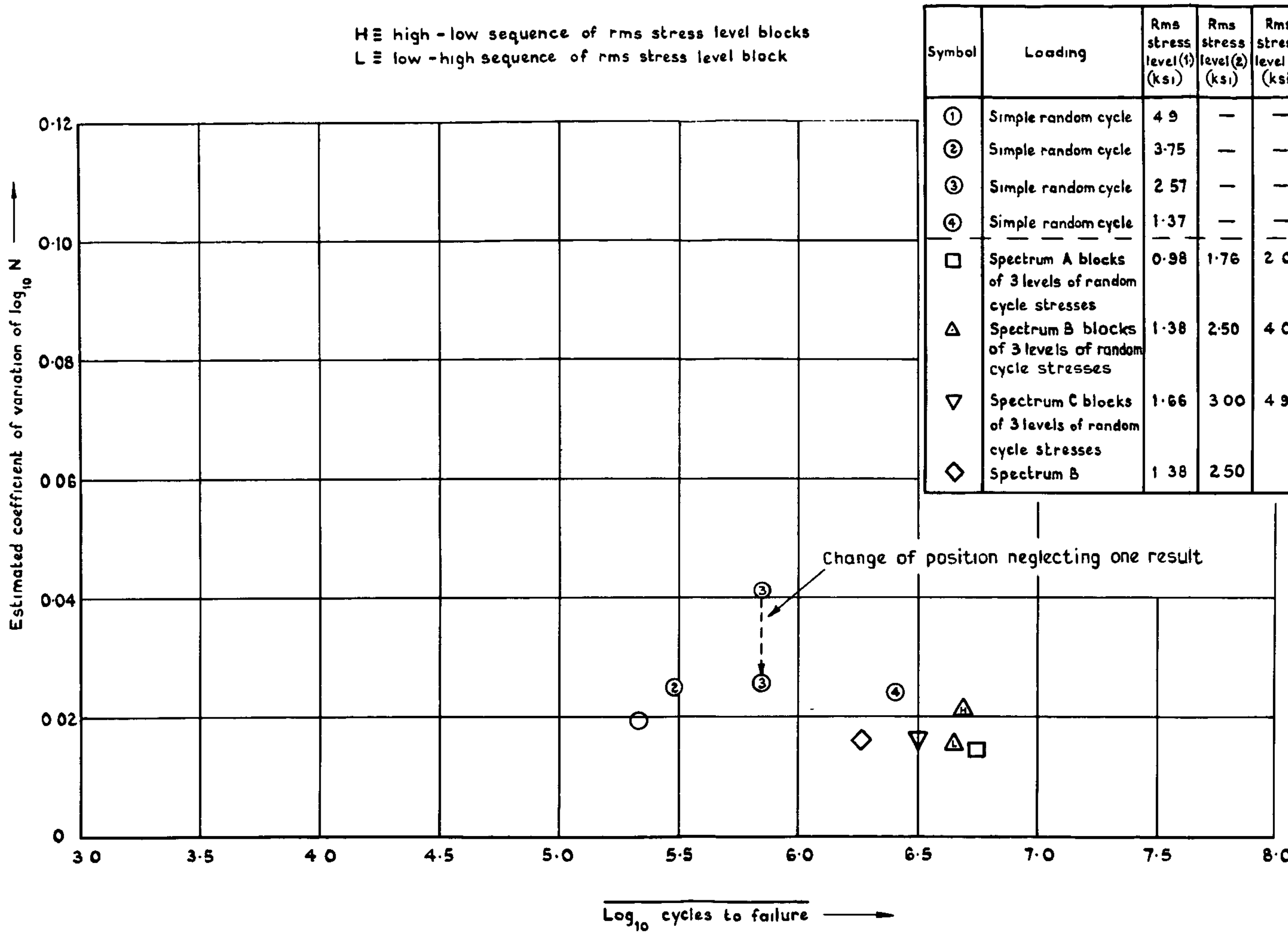


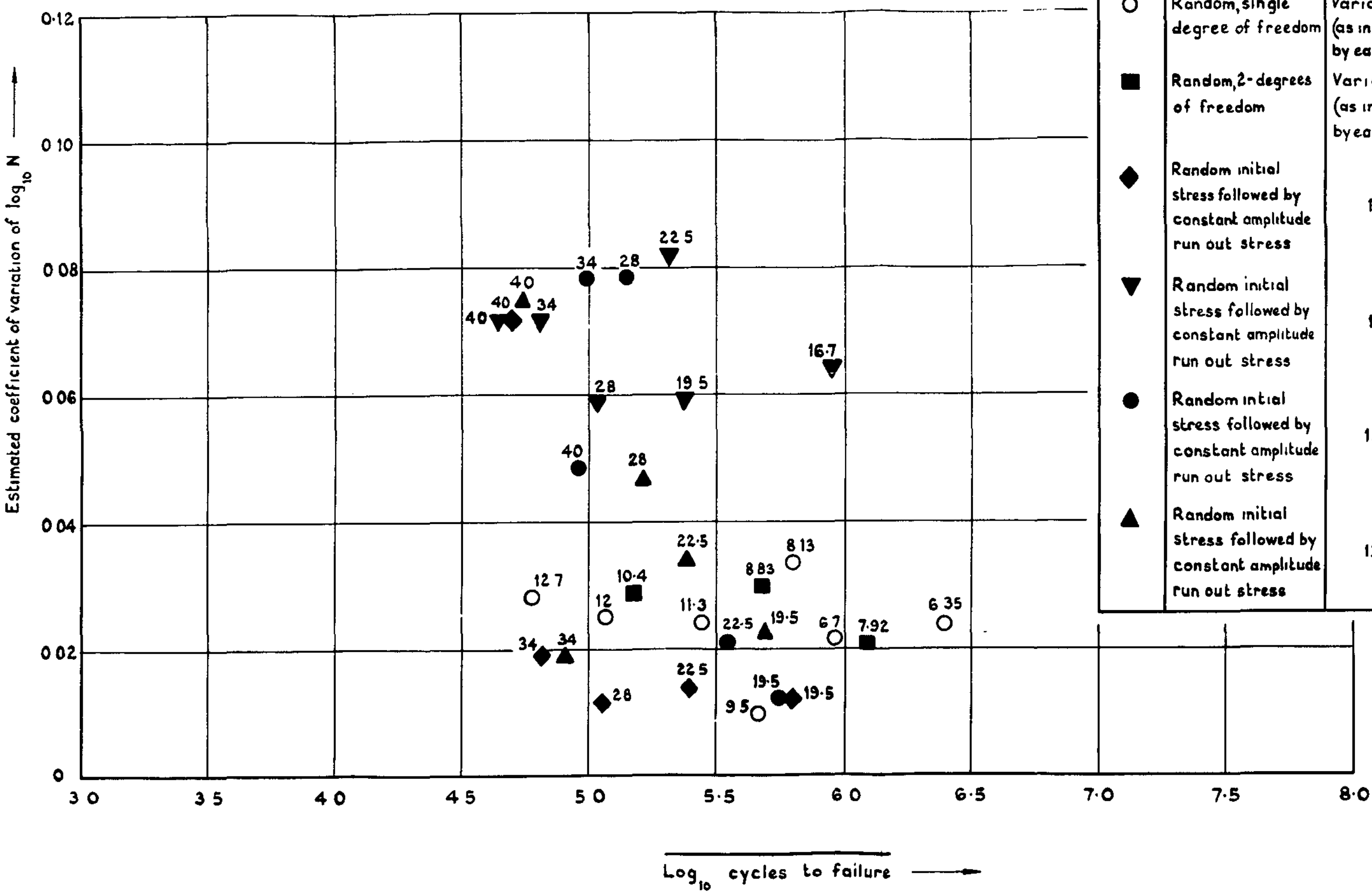
Fig. 54 Unnotched, rotating beam specimens of 7075-T6 alloy, Ref (23)

Two level repeated block and sinusoidal loading with amplitude continuously varying to a repeated signal shape



Symbol	Loading	Rms stress level (1) (ksi)	Rms stress level (2) (ksi)	Rms stress level (3) (ksi)	% of level (1)	% of level (2)	% of level (3)
①	Simple random cycle	4 9	—	—	100	—	—
②	Simple random cycle	3·75	—	—	100	—	—
③	Simple random cycle	2 57	—	—	100	—	—
④	Simple random cycle	1·37	—	—	100	—	—
□	Spectrum A blocks of 3 levels of random cycle stresses	0·98	1·76	2 0	77	21	2
△	Spectrum B blocks of 3 levels of random cycle stresses	1·38	2·50	4 05	77	21	2
▽	Spectrum C blocks of 3 levels of random cycle stresses	1·66	3 00	4 90	77	21	2
◇	Spectrum B	1 38	2·50		78	22	

Fig. 55 Simple lug specimens of 2L 65 material, Ref (35).
Random cycle and 3 level random cycle tests



Symbol	Type of loading	Rms level of random loading (ksi)	Cycles of random loading applied	Constant amplitude run out stress level (ksi)
○	Random, single degree of freedom	Various (as indicated by each point)	To failure	—
■	Random, 2-degrees of freedom	Various (as indicated by each point)	To failure	—
◆	Random initial stress followed by constant amplitude run out stress	16	45 000	Various (as indicated by each point)
▼	Random initial stress followed by constant amplitude run out stress	16	90 000	Various (as indicated by each point)
●	Random intial stress followed by constant amplitude run out stress	13·5	180 000	Various (as indicated by each point)
▲	Random initial stress followed by constant amplitude run out stress	13·5	90 000	Various (as indicated by each point)

Fig 56 Unnotched, axial bar specimens of 2024 material, Ref (17).
Various forms of loading involving random cycle stressing

Symbol	Peak history	Loading represented
○	A	White noise DLL = 4·0
□	A	White noise DLL = 5·0
△	B	Atmospheric turbulence
▽	C	Unimodal load spectrum
◇	D	Bimodal load spectrum

Programme number	Method of load counting and programming
	A basic peak history is the starting point for all these methods
1	The peaks were programmed as they occurred in the basic peak history ie in random cycle sequence
2	Only the maximum peaks of the peak history between zero crossings were applied ie in random cycle sequence
3	The peaks defined by means and amplitudes counting method were applied ie in random cycle sequence
4	The peaks of programme (2) were converted to a cumulative stress frequency distribution and then programmed in random sequence blocks of 8 stress levels
5	As (4) but applied to programme (3) and not (2)
6	As (5) but applied to only the amplitudes from the means and amplitudes counting method
7	As (4) but using the level crossings counting method rather than the maximum peaks method
8	As (7) but using the level crossing eliminating small fluctuations counting method
9	As (5) but the mean stresses in each block could take one of 7 values rather than being fixed
10	As (9) but using the means and amplitudes eliminating small fluctuations counting method

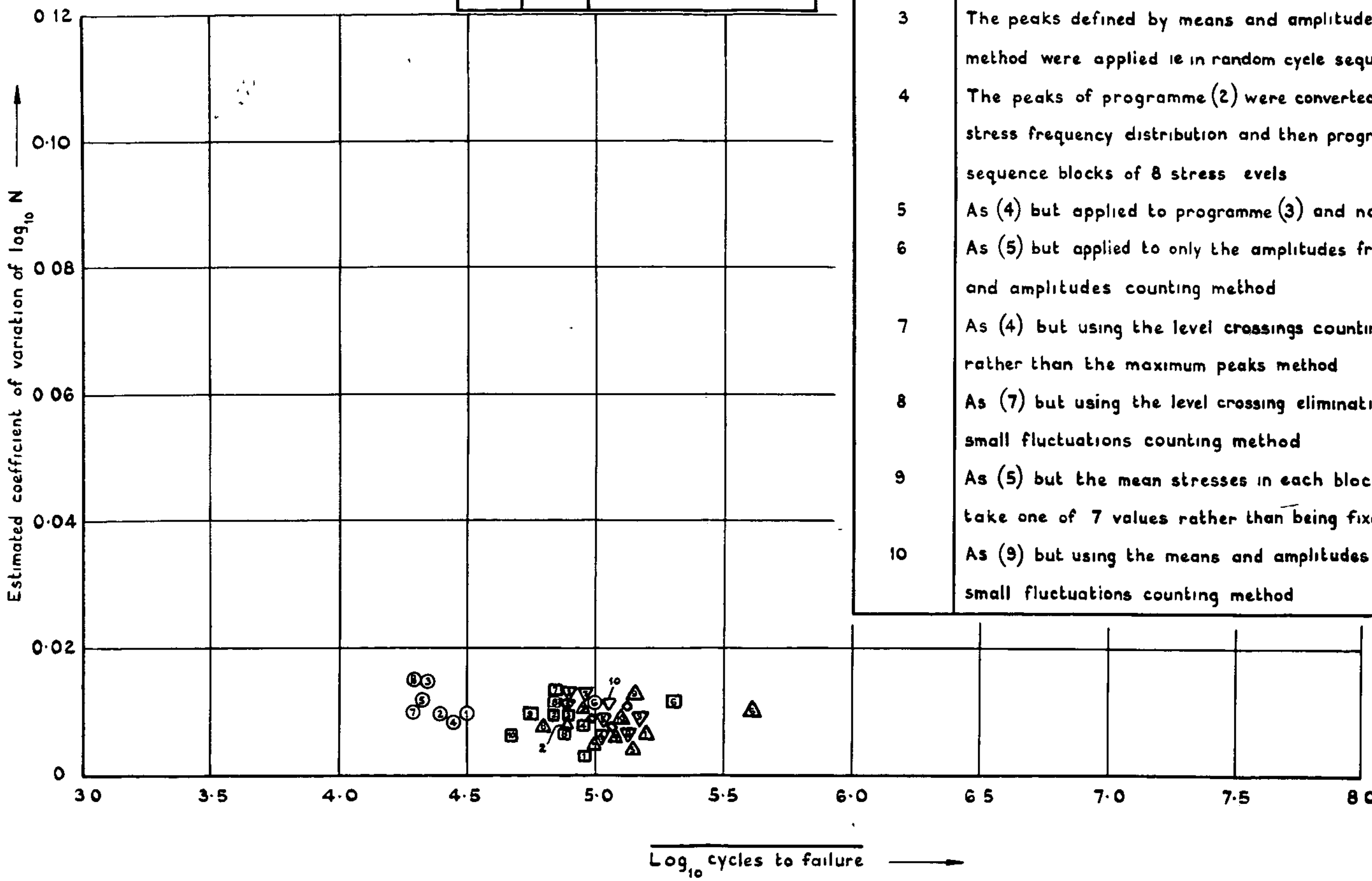
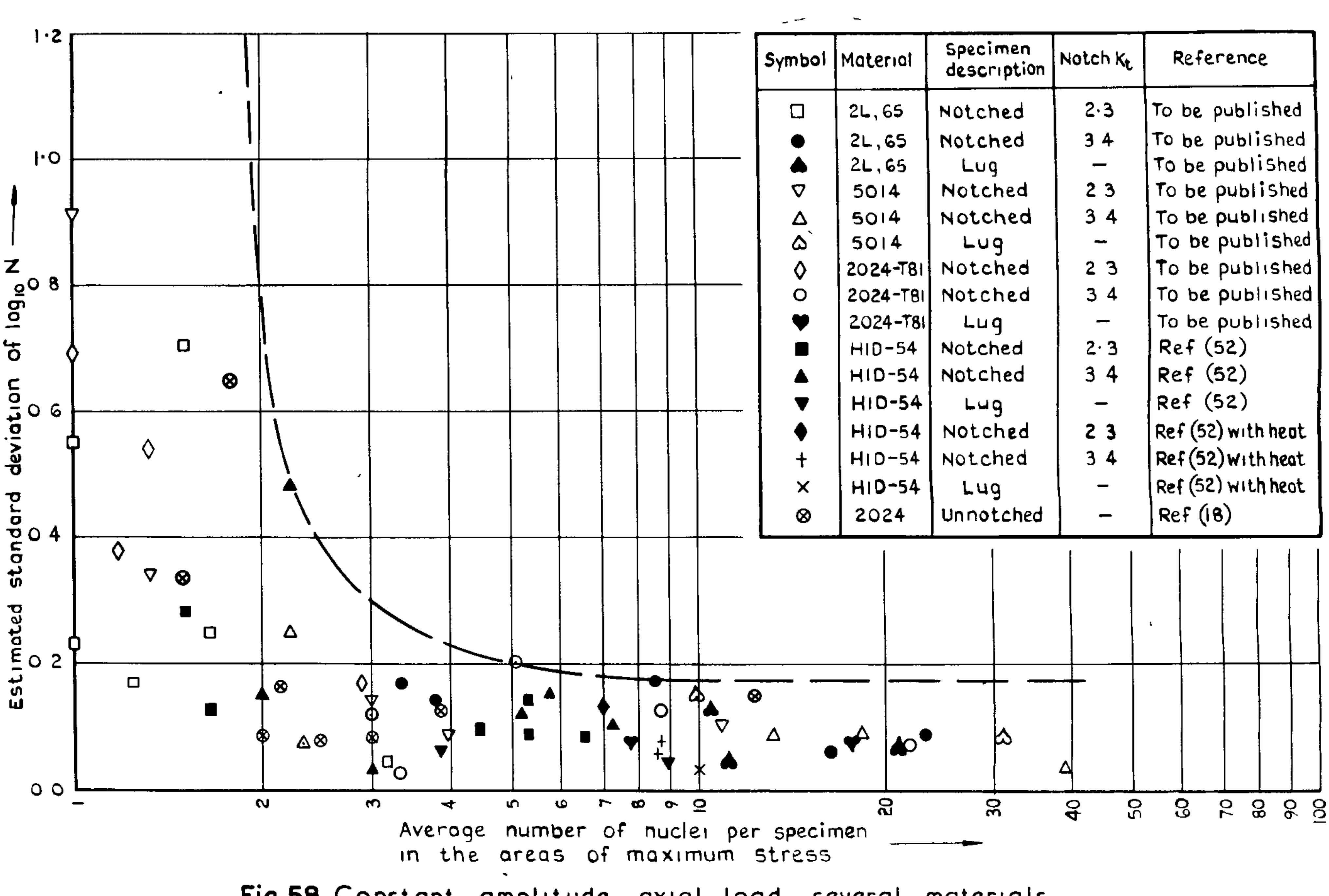


Fig. 57 Axial edge-notched sheet (K_t=4·0) of 2024-T3, Ref (27)
Random and programmed variable amplitude fatigue tests



Symbol	Material	Specimen description	Notch K_t	Reference
□	2L, 65	Notched	2·3	To be published
●	2L, 65	Notched	3 4	To be published
♠	2L, 65	Lug	—	To be published
▽	5014	Notched	2 3	To be published
△	5014	Notched	3 4	To be published
♤	5014	Lug	—	To be published
◊	2024-T81	Notched	2 3	To be published
○	2024-T81	Notched	3 4	To be published
♥	2024-T81	Lug	—	To be published
■	H10-54	Notched	2·3	Ref (52)
▲	H10-54	Notched	3 4	Ref (52)
▼	H10-54	Lug	—	Ref (52)
♦	H10-54	Notched	2 3	Ref (52) with heat
+	H10-54	Notched	3 4	Ref (52) with heat
×	H10-54	Lug	—	Ref (52) with heat
⊗	2024	Unnotched	—	Ref (18)

Fig. 58 Constant amplitude, axial load, several materials
Correlation of scatter with the number of crack nuclei. **Reproduced from Ref (1)**

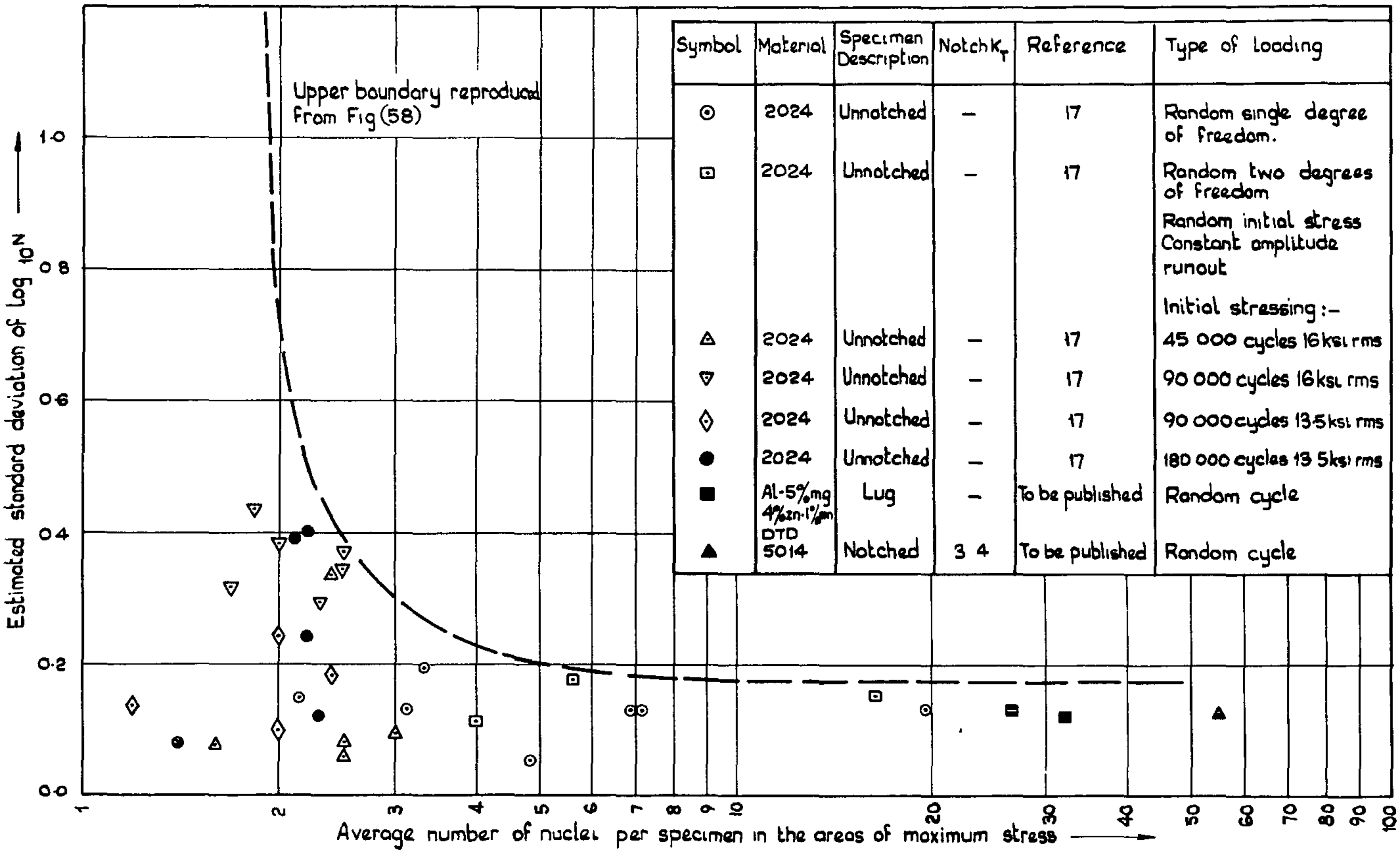


Fig. 59 Variable amplitude axial load several materials. Correlation of scatter with the number of crack nuclei

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A.R.C. C.P. No.1123
May 1969

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Stagg, A.M.

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