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The use of Cross-Correlation and Power Spectral Techniques for the Identification of the Hunter Mk.12 Dynamic Response

by

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THE USE OF CROSS-CORRELATION AND POWER SPECTRAL TECHNIQUES FOR THE IDENTIFICATION OF THE HUNTER MK.12 DYNAMIC RESPONSE

by

D. E. Fry

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SUMMARY

This Report describes a method of identifying the short period longitudinal transfer function, and impulse response of the Hunter Mk.12 from data recorded in flight. The method uses cross-correlation and power spectral techniques. The input was a pilot induced psuedo-random binary sequence on the elevator via the control column, and the output the pitch rate response of the aircraft as measured by a rate gyro. Digital computer programmes were used to calculate the relevant auto and cross-correlation functions, and the power spectra. The results, Bode plots and time responses are compared with theoretical results and give good agreement.

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* Replaces R.A.E. Technical Report 69156 - A.R.C. 31760.

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1 INTRODUCTION

The need to identify the parameters of a system transfer function frequently arises in the design of control systems, in this case a manoeuvre-demand, electrically signalled, pilot input control system[1,7] for the Hunter Mk.12. Although the aerodynamic derivatives of the aircraft have been reasonably well established, (from wind tunnel tests and estimates based on previous knowledge), it was thought necessary to confirm these values in flight prior to fitting the control system. A further and more long term object of the experiment was to assess the merits of the method used, and compare with other types of identification.

There are numerous methods of identification of which a few are model matching, correlation techniques, Shinbrot[9] and maximum likelihood[10]. Each tend to produce answers in a slightly different form, e.g. 'Shinbrot' identifies the coefficients of the basic equations of motion where as the 'maximum likelihood' method identifies the coefficients of the Z-transform of the system. The cross correlation method is described in this note, although the other three have been investigated and will be reported in other notes. A comparison of the above four methods will be made at a later date, although this Report does compare some results obtained via cross-correlation and the model-matching method[2].

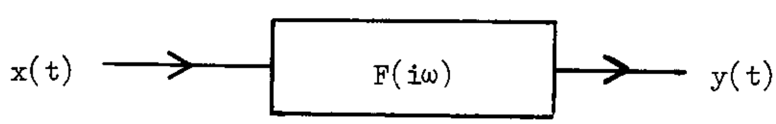
Frequency response methods of determining system dynamic characteristics from sinusoidal inputs are lengthy, unwieldy and expensive, especially from the aircraft point of view and the number of flying hours involved. Using the method of cross-correlation, the frequency response is essentially calculated over all frequencies (in practice a limited bandwidth of frequencies) in one run. The essence of the method is that a random signal, $x(t)$ is applied as excitation to the system, and the cross-correlation function between this signal and the resulting output $y(t)$ is calculated. The cross power spectrum is then obtained by taking the Fourier transform of the cross-correlation function, and hence the frequency response by dividing the cross-spectrum by the input spectrum. This technique has the advantages of one, the experiment may be performed while the system is functioning in its normal mode, and two, measurements are immune from extraneous noise, provided that this noise is not correlated with the input noise $x(t)$. To obtain an accurate cross-correlation function would ideally take an infinite length of time; however, inputs taking the form of periodic chain code noise give good estimates of the correlation functions, by integrating over one complete period of the noise. One such chain code is the so-called P.R.B.S., psuedo-random binary sequence, the logic and properties of which are briefly described in Appendix B. The particular code used in this experiment was of 127 bits long with a clock frequency of 2 c/s, i.e. one complete sequence of the random noise was 63.5 sec. A visual display of this P.R.B.S. was presented to the pilot, and he was asked to move the control column in phase with a light system as dictated by the input noise signal. Laboratory tests indicated that the frequency of 2 c/s was about the fastest a pilot could follow for a minute. Since the clock frequency governs the bandwidth of the input noise, no identification is possible of modes with frequencies greater than 2 c/s. The frequency range of the basic aircraft longitudinal short period motion is approximately from 1-0.25 c/s.

The elevator movement and rate of pitch were recorded in flight on magnetic tape in frequency-modulated form. Later the results were converted to analogue, passed through some data logging equipment, and sampled at 10 times a second onto paper tape. The data was then processed through a digital computer to finally obtain the frequency response. The data-logging equipment had some quite severe limitations which effectively put noise into the system. The use of Avionics Department's new hybrid computer system would eliminate most of these sampling and rate of sampling errors.

Comparisons are made with theoretical transfer functions, and also with results obtained by a model matching technique.

2 DESCRIPTION OF METHOD

One method of determining empirically the dynamic characteristics or frequency response of a system is by cross-correlating the input with the output.



It can be shown (Appendix A) that if the input is 'white' noise, the cross-correlation function of output/input is proportional to the system $(F(i\omega))$ impulse response. The Fourier transform of the correlation function is described as the power spectrum. Two important formulae used in this Report, the first of which is derived in Appendix A are:-

$$F(i\omega) = S_{xy}(i\omega)/S_{xx}(\omega) \tag{1}$$

$$|F(i\omega)|^2 = S_{yy}(\omega)/S_{xx}(\omega) \tag{2}$$

where $F(i\omega)$ = transfer function of system
$S_{xy}(i\omega)$ = cross power spectrum of input/output
$S_{yy}(\omega)$ = power spectrum of output
$S_{xx}(\omega)$ = power spectrum of input.

It should be noted that $S_{xy}(i\omega)$ is complex and not symmetrical. It can be shown (Appendix A) that:

$$S_{xy}(i\omega) = \int_0^\infty \phi_{xy}(\tau) \cos(\omega\tau)\, d\tau + \int_0^\infty \phi_{yx}(\tau) \cos(\omega\tau)\, d\tau$$

$$- i\left[\int_0^\infty \phi_{xy}(\tau) \sin(\omega\tau)\, d\tau - \int_0^\infty \phi_{yx}(\tau) \sin(\omega\tau)\, d\tau\right] \quad . \tag{3}$$

This formula has been used to calculate the cross-spectra $S_{xy}(i\omega)$. The power spectra of both input and output $S_{xx}(\omega)$ and $S_{yy}(\omega)$ are real and symmetrical and are defined as:

$$S_{xx}(\omega) = 2\int_0^\infty \phi_{xx}(\tau) \cos(\omega\tau)\, d\tau \tag{4}$$

$$S_{yy}(\omega) = 2\int_0^\infty \phi_{yy}(\tau) \cos(\omega\tau)\, d\tau \tag{5}$$

where $\phi_{xx}(\tau)$ = autocorrelation function of input
$\phi_{yy}(\tau)$ = autocorrelation function of output
$\phi_{xy}(\tau)$ = cross-correlation function of input/output
$\phi_{yx}(\tau)$ = cross-correlation function of output/input.

(defined in Appendix A)

The input $x(t)$ in this series of experiments was a P.R.B.S. (psuedo-random binary sequence) which approximates to 'white' noise over a limited bandwidth and is exactly repeatable (see Appendix B). The pilot was asked to move the elevator in phase with the P.R.B.S. input. The elevator movement and resulting rate of pitch were recorded in F/M form on a magnetic tape recorder. These tapes were subsequently fed through a data processer, the final answers being produced in the form of paper tape in sampled data form, at a time interval of 0.1 second.

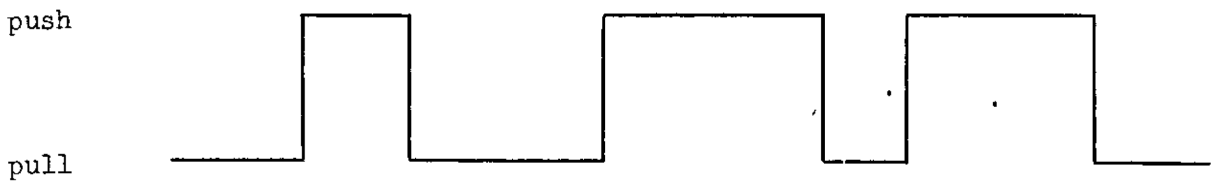
Digital computer programmes were written to calculate the auto and cross-correlation functions and hence by Fourier transform the power spectra. Clearly, any digital discrete calculation involving Fourier transforms can only be an approximation, since it involves integrating from plus to minus infinity. Several weighting or 'window' functions are available, which give improved estimates of the power spectra. These tend to overcome the errors due to 'aliasing' or folding and due to the truncation of the correlation function. A short investigation into various 'window' functions was carried out (Appendix A), the so-called 'Hamming' filter being eventually used.

3 FLIGHT TRIALS

The aircraft, a Hunter Mk.12 was flown at four different flight conditions, i.e.

Flight case	Mach No.	Height	No. of runs
1	0.3	6000 ft 1800 m	3
2	0.6	6000 ft 1800 m	1
3	0.9	5000 ft 1500 m	2
4	0.9	40000 ft 12000 m	3

At each flight condition, the pilot was asked to fly straight and level in a trimmed condition, and then perform the P.R.B.S. manoeuvre, holding speed constant. This manoeuvre required that he moved the stick in phase with a director system of lights controlled by the P.R.B.S., and attempting to keep the amplitude constant. The output of the P.R.B.S. generator is a series of pulses on and off, e.g. on/push, off/pull, the length of time of 'push' and 'pull' being indicated by the number of lights on at any particular instant.



A photograph of the light system displayed to the pilot is shown in Fig.1.

Tests were carried out on a simulator prior to the flight trials to determine the maximum frequency of the clock rate of the P.R.B.S. and for how long the pilot could hold his concentration on this manoeuvre. These turned out to be approximately 2 c/s and one minute. These figures governed the characteristics of the P.R.B.S., i.e. clock frequency 2 c/s, number of bits 127 ($2^7 - 1$), i.e. duration of complete cycle 63.5 seconds. Since the clock frequency governs the bandwidth of the input noise spectra, and it was known that the aircraft's natural frequency was in the 0.25-1.0 c/s range, it was thought the bandwidth of the input was only just sufficiently wide enough to justify the experiment. Clearly, for the identification of modes of higher frequency, a faster clock rate would be necessary, and the input would have to by-pass the pilot and be fed directly to the elevator.

The 127 bit sequence of the P.R.B.S. contained at least one 6 bit 'push or pull', and it was so arranged that the long duration pushes or pulls came at the end of the sequence. This was an attempt to keep speed errors as small as possible.

The amplitude of the stick input was decided by the pilot after trial runs, the second pilot attempting to keep it constant over the run. The extent of his success can be seen in Fig.17, which shows an analogue trace of the elevator movement.

This particular aircraft has a 'flying' tail and, although, the pilots were asked to fix the tail, both elevator and tailplane movement were measured separately, i.e.

$$\eta' = \eta + k\eta_T$$

η' = effective control movement

η = elevator

η_T = tailplane

k = constant, depending on Mach number and altitude.

The respective values of k for the four flight conditions are:- 1.76, 2.22, 3.40, and 2.32.

Some stick jerks (and the corresponding pitch rate responses) were also recorded in flight to obtain 'impulse' responses.

4 ANALYSIS OF RESULTS

Fig.2 shows the autocorrelation function and power spectrum of the input stick movement, i.e. the psuedo-random binary sequence. Both agree reasonably well with the theoretical results as described in Appendix B and as shown in the diagram. The autocorrelation function is pulse-like, and the power spectra exhibits the required characteristics. Good agreement is shown between the two extremes of the flight envelope, indicating the accuracy with which the pilots performed their task of moving the stick in phase with the P.R.B.S. light display.

Theoretical transfer functions substantiated by model-matching techniques[2], were obtained for each of the four flight conditions. They all had the form of:-

$$q/\eta = a(s + b)/(s^2 + cs + d)$$

Flight case	Mach No.	Altitude	a	b	c	d
1	0.3	6000 ft 1800 m	4.46	0.56	1.42	2.79
2	0.6	6000 ft 1800 m	15.8	1.23	3.84	8.51
3	0.9	5000 ft 1500 m	18.0	1.41	5.45	24.4
4	0.9	40000 ft 12000 m	10.3	0.61	1.98	3.93

Flight case 1 M = 0.3, 6000 ft

Three separate runs were achieved at this flight condition. Fig.3 shows a typical cross-correlation function, i.e. an approximation to the aircraft's impulse response. Comparison of the actual aircraft's response to a stick jerk, with the cross-correlation function is made for flight case 4, M = 0.9, 40000 ft (12000 m). Appendix A shows how the system transfer function in terms of amplitude and phase can be calculated from the cross power spectrum and the input power spectrum.

$$F(i\omega) = S_{xy}(i\omega)/S_{xx}(\omega) .$$

In addition the amplitude can also be derived from

$$|F(i\omega)|^2 = S_{yy}(\omega)/S_{xx}(\omega) .$$

Fig.4 shows the amplitude and phase of the aircraft's short period transfer function for the three separate runs at this flight condition.

Good agreement between runs is shown for the amplitude, and exceptional agreement for the phase calculations. The amplitude plot gives a peak at 0.3 c/s, i.e. aircraft's short period of approximately 3.3 second. The phase plot indicates that the system transfer function has a first order zero, indicated by the phase lead between 0 and 0.25 c/s and second order pole. This agrees with the theoretical transfer function which has the form:-

$$a(s + b)/(s^2 + cs + d) .$$

Accuracy of the results rapidly tails off after about 1.3 cps, due to the fact that the input spectrum is small above this value (see Fig.2). The cut-off frequency of the input spectrum is determined by the clock frequency of the P.R.B.S. (i.e. 2 c/s). Fig.5 shows the amplitude versus frequency, as obtained from the second method, and compared with theoretical values. The theoretical transfer function agreed well with that obtained from a model-matching technique[2]. In general, the values of $|F(i\omega)|$ obtained from the second method are slightly more consistent within themselves and agree better with the theoretical values than those derived from the cross-spectra. This merely means that the calculation of $S_{yy}(\omega)$ is more accurate than the complex calculation of $S_{xy}(i\omega)$ (equation (A-12)).

Fig.6 shows an alternative method of presentation of the transfer function in the form of a Nyquist diagram.

<u>Flight case 2</u> M = 0.6, 6000 ft (1800 m)

Fig.7 shows the aircraft's impulse response for this flight condition as derived from $\phi_{xy}(\tau)$, the cross-correlation function. Actually the digital programme calculated the individual values of the cross-correlation coefficients; the difference in scaling (between correlation function and coefficients) being indicated in Appendix A. The diagram shows that the aircraft is lightly damped with a natural period of about 1.5 seconds.

Fig.8 shows the amplitude and phase of the aircraft's transfer function as derived from $S_{xy}(i\omega)/S_{xx}(\omega)$. Only one run was obtained at this flight condition. The comparison between the flight test amplitude/frequency results and the theoretical values is shown in Fig.9. In all of these (amplitude/frequency) diagrams, the natural frequency of the system is not at the peak, due to the effect of the phase advance of the zero in the transfer function.

Flight case 3 M = 0.9, sea level

Fig.10 gives the impulse response $(\phi_{xy}(\tau))$ showing a fairly well damped mode with a period of about 1.4 second.

Figs.11 and 12 demonstrate the transfer function characteristics as derived from the two methods and a comparison with the theoretical results. Reasonably good agreement is shown between runs.

Flight case 4 M = 0.9, 40000 ft (12000 m)

Fig.13 shows the impulse response derived via the cross-correlation function, and an analogue response of an actual airborne stick jerk. This response was shifted in time and scaled, but the agreement shown is very good.

Fig.14 shows the amplitude and phase of the aircraft transfer function for the three separate runs. It can be seen that there is a large discrepancy between run 1 and runs 2 and 3. The theoretical value of the amplitude also agreed fairly well with the first run (see Fig.15). The impulse response of run 2 was plotted (Fig.16) and it also showed a large difference from run 1 (Fig.13). It was not until the analogue recordings of the input, pitch rate and airspeed were produced that the reason for the discrepancy was found. These analogue recordings are shown in Fig.17. The first trace is the P.R.B.S. input, the second, a pitch rate response of a theoretical model (from an analogue computer model of the theoretical transfer function), the third trace is the pitch rate response of run 1, and the fourth trace the airspeed variations during the run. The final two traces are the pitch rate response and airspeed for run 2. (The input for run 2 is not shown, but it was practically identical to that of run 1.). It can be seen that the pitch rate response of run 1 agrees very well with the pitch rate response of the model, with the airspeed remaining fairly constant. The pitch rate response of run 2 does not agree with the model response, especially at the end of the run. The frequency appears higher and the damping less. The airspeed has gradually increased over the whole run by some 20 kts (10.3 m/sec). It is known that some aerodynamic derivatives change rapidly at this Mach number, especially m_w, the main contribution to the frequency, so that the results obtained from the power spectra are not strictly valid due to the speed change, i.e. it is essential to the method that the coefficients of the transfer function do not change radically throughout the P.R.B.S. run.

5 CONCLUSIONS

Different identification techniques give their final results in different form. The method used in this Report produces impulse responses, and frequency response in Bode-plot form.

The principle of using psuedo-random binary sequences as random noise inputs seems sound, although the described system had several limitations; (1) the bandwidth of the input noise could have been preferably wider, i.e. clock frequency higher, (2) the data sampling equipment was noisy and limited to a maximum rate of 10 per second.

The results obtained agree well with those obtained from a 'model-matching' technique and also with theoretical values of transfer functions.

Appendix A

SUMMARY OF MATHEMATICS OF METHOD

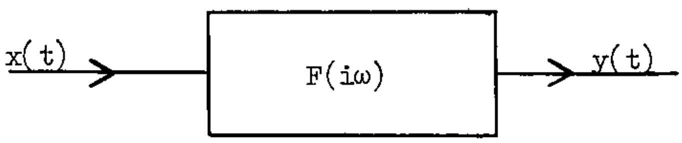
Definitions

$$\phi_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, x(t - \tau)\, dt, \quad \text{autocorrelation function} \qquad (A\text{-}1)$$

$$\phi_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t)\, x(t - \tau)\, dt, \quad \text{cross-correlation function} \qquad (A\text{-}2)$$

$$S_{xx}(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_{xx}(\tau)\, e^{-i\omega\tau}\, d\tau, \quad \text{power spectrum} \qquad (A\text{-}3)$$

$$S_{xy}(i\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau, \quad \text{cross power spectrum} \qquad (A\text{-}4)$$



If $w(t)$ is the impulse response of system $F(i\omega)$ then by superposition integral;

$$y(t) = \int_{-\infty}^{\infty} w(\tau)\, x(t - \tau)\, d\tau \qquad (A\text{-}5)$$

substituting (A-5) into (A-2) gives

$$\phi_{xy}(\tau_1) = \int_{-\infty}^{\infty} w(\tau)\, d\tau \left[\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t - \tau)\, x(t - \tau_1)\, dt \right]$$

$$= \int_{-\infty}^{\infty} w(\tau)\, \phi_{xx}(\tau - \tau_1)\, d\tau \quad . \qquad (A\text{-}6)$$

Now if the input is 'white' noise, i.e. $\phi_{xx}(\tau)$ is a delta function then,

$$\frac{\phi_{xy}(\tau_1) \; = \; c\, w(\tau_1)}{(c \; = \; \text{constant})} \tag{A-7}$$

i.e. the cross-correlation function $\phi_{xy}(\tau)$ is proportional to the impulse response $w(\tau)$.

Formulae used in Report

$$F(i\omega) = S_{xy}(i\omega)/S_{xx}(\omega) \tag{A-8}$$

$$|F(i\omega)|^2 = S_{yy}(i\omega)/S_{xx}(\omega) \tag{A-9}$$

$$\beta = |S_{xy}(i\omega)|^2/S_{yy}(\omega) . \; S_{xx}(\omega) \; . \tag{A-10}$$

Equation (A-10) defines the coherency function, which should approach one from below for coherent results.

Derivation of $F(i\omega) = S_{xy}(i\omega)/S_{xx}(i\omega)$

By definition:-

$$\phi_{xy}(\tau) = \overline{x(t)\; y(t + \tau)} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\; y(t + \tau)\; dt$$

$$= \int_0^\infty w(\tau_1)\; x(t)\; x(t + \tau - \tau_1)\; d\tau_1 \qquad \text{using super-position}$$

$$= \int_0^\infty w(\tau_1)\; \phi_{xx}(\tau - \tau_1)\; d\tau_1$$

Now,

$$S_{xy}(i\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$= \frac{1}{\pi} \int_{0}^{\infty} w(\tau_1)\, e^{-i\omega\tau_1}\, d\tau_1 \int_{-\infty}^{\infty} \phi_{xx}(\tau - \tau_1)\, e^{-i\omega(\tau-\tau_1)} d\tau$$

$$= \frac{1}{\pi} \int_{0}^{\infty} w(\tau_1)\, e^{-i\omega\tau_1}\, d\tau_1 \int_{-\infty}^{\infty} \phi_{xx}(t)\, e^{-i\omega t}\, dt$$

$$= F(i\omega)\, S_{xx}(i\omega) \qquad \text{(A-11)}$$

therefore,

$$F(i\omega) = S_{xy}(i\omega)/S_{xx}(\omega) \quad .$$

Equation (A-9) is derived by Lanning and Batten[4] and numerous other typical texts.

<u>Numerical calculation of cross-power spectrum</u>

$$S_{xy}(i\omega) = \int_{-\infty}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau \quad : \text{ignoring scaling factor } \frac{1}{\pi}$$

$$= \int_{0}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, dt + \int_{-\infty}^{0} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau$$

$$= \int_{0}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau - \int_{\infty}^{0} \phi_{xy}(-\tau)\, e^{i\omega\tau}\, d\tau$$

$$= \int_{0}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau + \int_{0}^{\infty} \phi_{xy}(-\tau)\, e^{i\omega\tau}\, d\tau$$

$$= \int_{0}^{\infty} \phi_{xy}(\tau)\, e^{-i\omega\tau}\, d\tau + \int_{0}^{\infty} \phi_{yx}(\tau)\, e^{i\omega\tau}\, d\tau$$

since

$$\phi_{xy}(-\tau) = \phi_{yx}(\tau) \quad .$$

Resolving into sine and cosine components:

$$S_{xy}(i\omega) = \left[\int_0^\infty \phi_{xy}(\tau)\cos(\omega\tau)\,d\tau + \int_0^\infty \phi_{yx}(\tau)\cos(\omega\tau)\,d\tau\right] - i\left[\int_0^\infty \phi_{xy}(\tau)\sin(\omega\tau)\,d\tau - \int_0^\infty \phi_{yx}(\tau)\sin(\omega\tau)\,d\tau\right] . \quad \text{(A-12)}$$

Digital computer programmes have been written to calculate the above formula.

Digital computer calculation of power spectra

In order to obtain better estimates of power spectra derived digitally from the correlation function, it is usual to apply a smoothing or 'window' filter[6] to the correlation function prior to Fourier transformation,

i.e.

$$\phi'_{xy}(\tau) = G(\tau)\,\phi_{xy}(\tau)$$

where $G(\tau)$ is the 'window' filter.

Four different filters were tried, to obtain estimates of the power spectra; they were:

(1) Impulse $= 1, \quad |\tau| < T$
$\quad = 0, \quad |\tau| > 0$

(2) Bartlett $1 - |\tau|/T, \quad |\tau| < T$

(3) Hanning $0.5 + 0.54 \cos(\pi\tau/T)$

(4) Hamming $0.54 + 0.46 \cos(\pi\tau/T)$

where $T = N\,\Delta t$

$\tau = n\,\Delta t$

N = total number of delays or lags.

The Hamming filter was eventually used for the calculation of the results in this Report.

Correlation coefficients

The previous theory applies to the correlation function, but correlation coefficients were calculated in the digital programme.

Now,

$$S_{xy}(i\omega) = \sigma_x \sigma_y \int_{-\infty}^{\infty} \frac{\phi_{xy}(\tau)}{\sigma_x \sigma_y} e^{-i\omega\tau} d\tau$$

but,

$$\phi'_{xy}(\tau) = \phi_{xy}(\tau)/\sigma_x \sigma_y$$

was actually calculated in digital programme, therefore

$$S_{xy}(i\omega) = \sigma_x \sigma_y S'_{xy}(i\omega) \quad .$$

Similarly,

$$S_{xx}(\omega) = \sigma_x^2 \; S'_{xx}(\omega)$$

therefore,

$$S_{xy}(i\omega)/S_{xx}(\omega) = \sigma_y/\sigma_x \quad S'_{xy}(i\omega)/S'_{xx}(\omega)$$

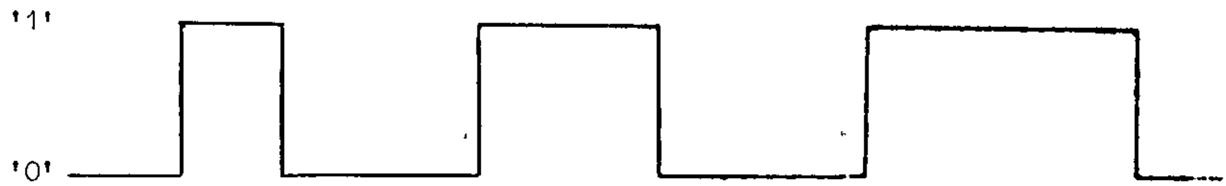
i.e. conversion factor between calculated and theoretical values is σ_y/σ_x
where σ_y^2 = variance of output
σ_x^2 = variance of input.

Flight case	σ_x	σ_y	σ_y/σ_x
1	3.5	6.3	1.8
2	2.3	6.9	3.0
3	1.7	4.6	2.7
4	2.3	5.5	2.4

Appendix B

LOGIC AND PROPERTIES OF MAXIMUM LENGTH BINARY SEQUENCES

Maximum length binary sequences[6,8] (P.R.B.S.) provide a simple way to obtain periodic noise. The sequence is generated by an n-stage shift register which has the output of two or more of its stages, fed back through a modulo-two gate to the input of its first stage. There are $(2^n - 1)$ digits in any sequence, all zeros being non-valid.



Example of part of P.R.B.S., i.e. 010011001110 where Δt is time interval or inverse of clock frequency.

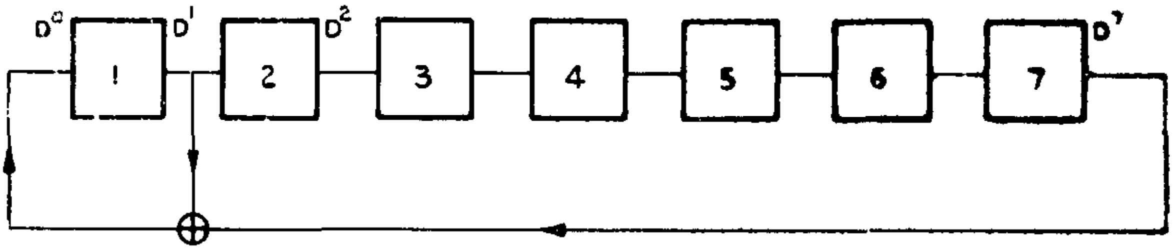


Diagram of 7-shift register P.R.B.S. generator

where $\oplus$ represents modulo-two addition, see following table.

B \ A	0	1
0	0	1
1	1	0

The characteristic equation of the above generator can be written thus:

$$D^7 \oplus D^1 \oplus D^0 = 0 \qquad \text{(B-1)}$$

or

$$D^7 \oplus D^1 = D^0 \qquad \text{(B-2)}$$

where $D \equiv$ unit delay operator

$D^7 \equiv$ output of shift-register 7.

Period of maximum length sequence $= 2^7 - 1 = 127$ units of Δt.

Some properties of P.R.B.Ss are:-

(1) Easily produced, reliable and repeatable.

(2) Mean zero, known variance.

(3) Bandwidth proportional to clock frequency.

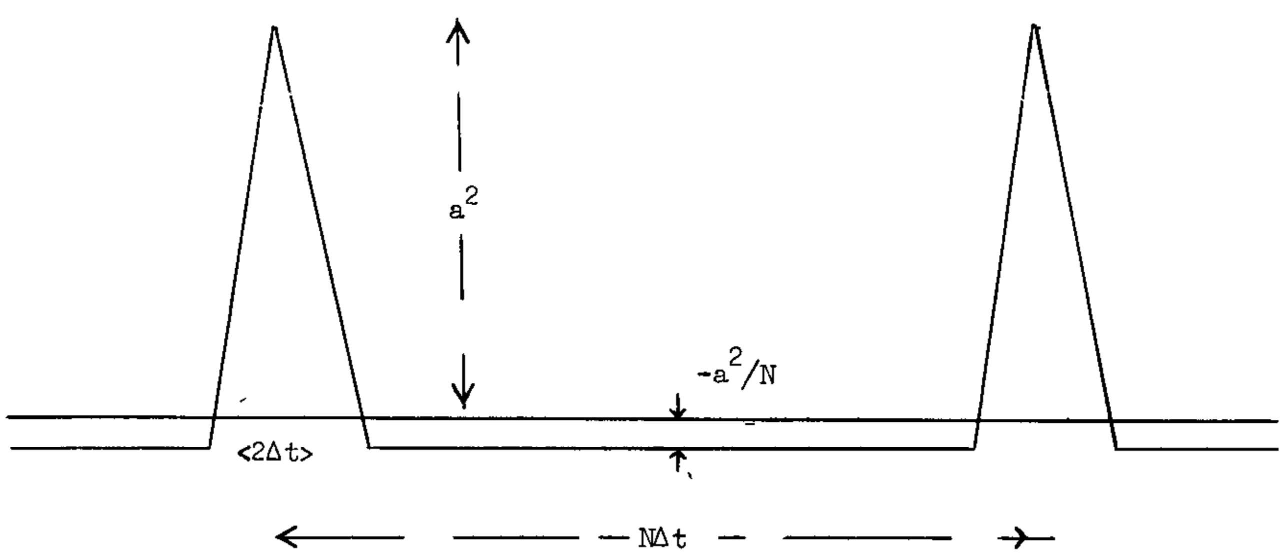
(4) Exhibits the 'shift-and-add' property, i.e. delayed versions of the same sequence readily obtainable.

(5) Pulse-like autocorrelation function.

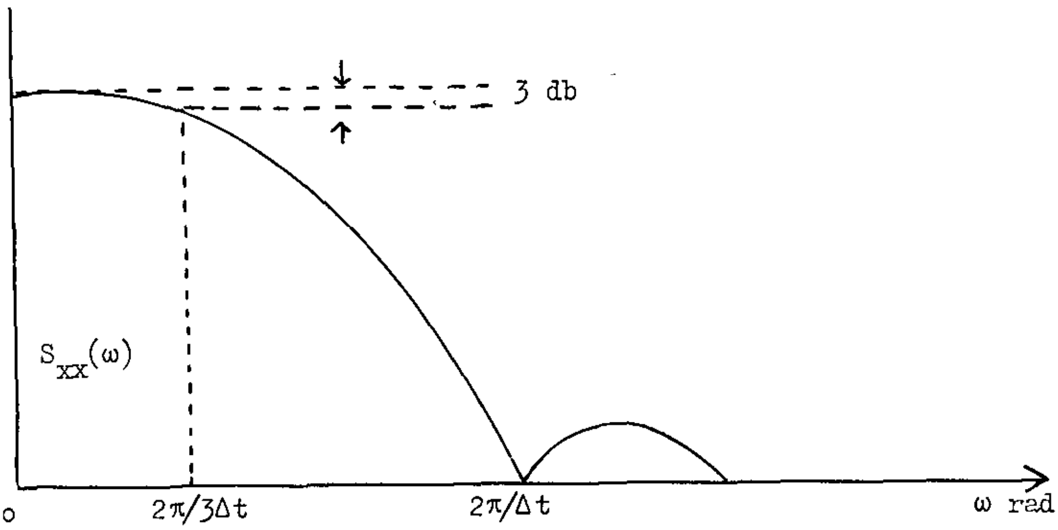
(6) Flat power spectrum within limits.

(7) Binary output permits logical or analogue multiplication by simple switching.

Some of these properties are amplified below.



Autocorrelation function of P.R.B.S.



Power spectrum of P.R.B.S. (actually a multiple of line spectra spaced at $1/(2^n - 1).\Delta t$ intervals

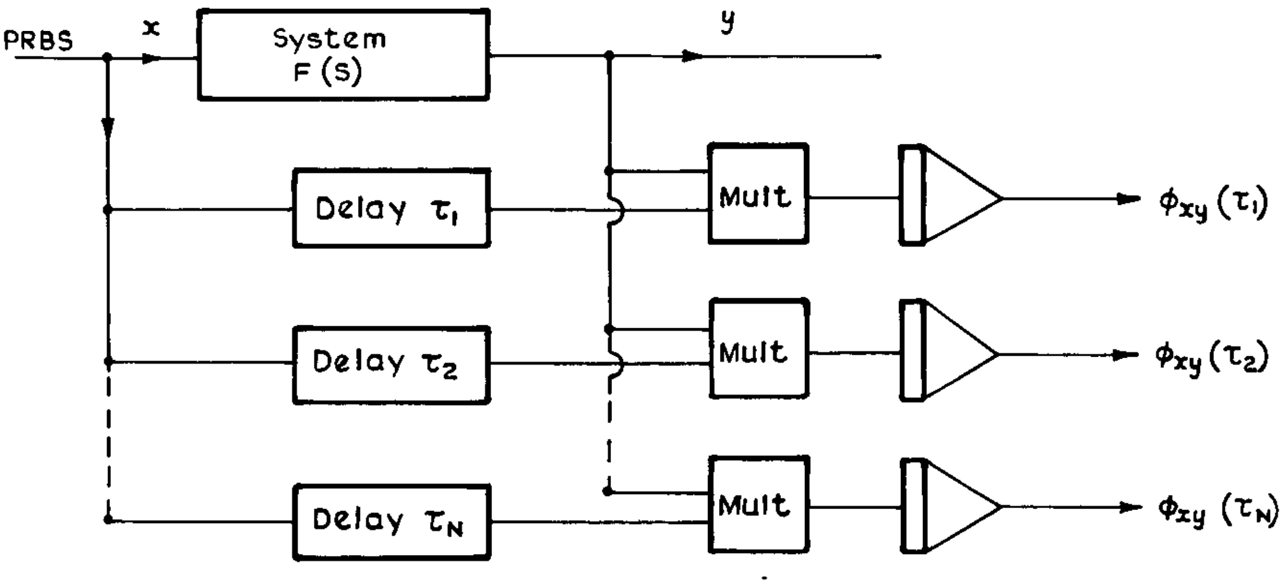
$$S_{xx}(\omega) = \left(\frac{N+1}{N}\right) a^2 \, \Delta t \sum_{\tau} \left[\frac{\sin(\tau\pi/N)}{(\tau\pi/N)}\right]^2$$

$$a^2 = \phi_{xx}(o)$$
$$-a^2/N = \phi_{xx}(i) \simeq o; \quad i \neq o .$$

Shift-and-add property

Term by term modulo-two addition of a maximum-length shift register sequence $\{a_k\}$ and a delayed sequence $\{a_{i+k}\}$ yields another delayed sequence $\{a_{i+m}\}$; otherwise identical. This property can be very useful when calculating cross-correlation functions on a analogue computer, i.e.

$$\phi_{xy}(\tau) = \int_o^T x(t)\, y(t - \tau)\, dt \equiv \int_o^T x(t + \tau)\, y(t)\, dt \quad .$$



Now from (B-1), (B-2),

$$D^7 = D^1 \oplus D^0$$

therefore,

$$D^8 = D^2 \oplus D^1$$

$$D^9 = D^3 \oplus D^2$$

$$\vdots \qquad \vdots$$

$$D^{13} = D^7 \oplus D^6 = D^1 \oplus D^0 \oplus D^6$$

$$D^{14} = D^8 \oplus D^7 = D^2 \oplus D^1 \oplus D^7 = D^2 \oplus D^0$$

therefore;

$$D^{15} = D^3 \oplus D^1$$

$$D^{16} = D^4 \oplus D^2 \quad \text{etc.}$$

i.e. any delayed version of the sequence can be obtained by modulo-two addition of the available shift-register outputs. Davies gives a method for generating any particular delayed sequence by long division, e.g. the delay by 16 intervals,

$$D^{16} = ?$$

$$
\begin{array}{r|l}
 & D^{9} \oplus D^{3} \oplus D^{2} \\
\hline
D^{7} \oplus D^{1} \oplus D^{o} & D^{16} \\
 & \underline{D^{16} \oplus D^{10} \oplus D^{9}} \\
 & \qquad D^{10} \oplus D^{9} \\
 & \qquad \underline{D^{10} \oplus \qquad D^{4} \oplus D^{3}} \\
 & \qquad\qquad D^{9} \oplus D^{4} \oplus D^{3} \\
 & \qquad\qquad \underline{D^{9} \qquad \oplus D^{3} \oplus D^{2}} \\
 & \qquad\qquad\qquad D^{4} \oplus D^{2}
\end{array}
$$

Remainder gives expression for delay 16, i.e.

$$D^{16} = D^{4} \oplus D^{2}$$

(see previous derivation).

= modulo-two sum of output of 4th and 2nd register.

SYMBOLS

a, b, c, d	coefficients of transfer function
β	coherency function
D	unit delay operator
D^7	delay 7
M	Mach number
η	elevator angle
η_T	tailplane angle
σ_x^2	variance of x
$\phi_{xx}(\tau)$	autocorrelation function of x
$\phi_{xy}(\tau)$	cross-correlation function of x and y
q	aircraft's pitch rate
$S_{xx}(\omega)$	power spectrum of input
$S_{xy}(i\omega)$	power spectrum of input/output
w(t)	impulse response

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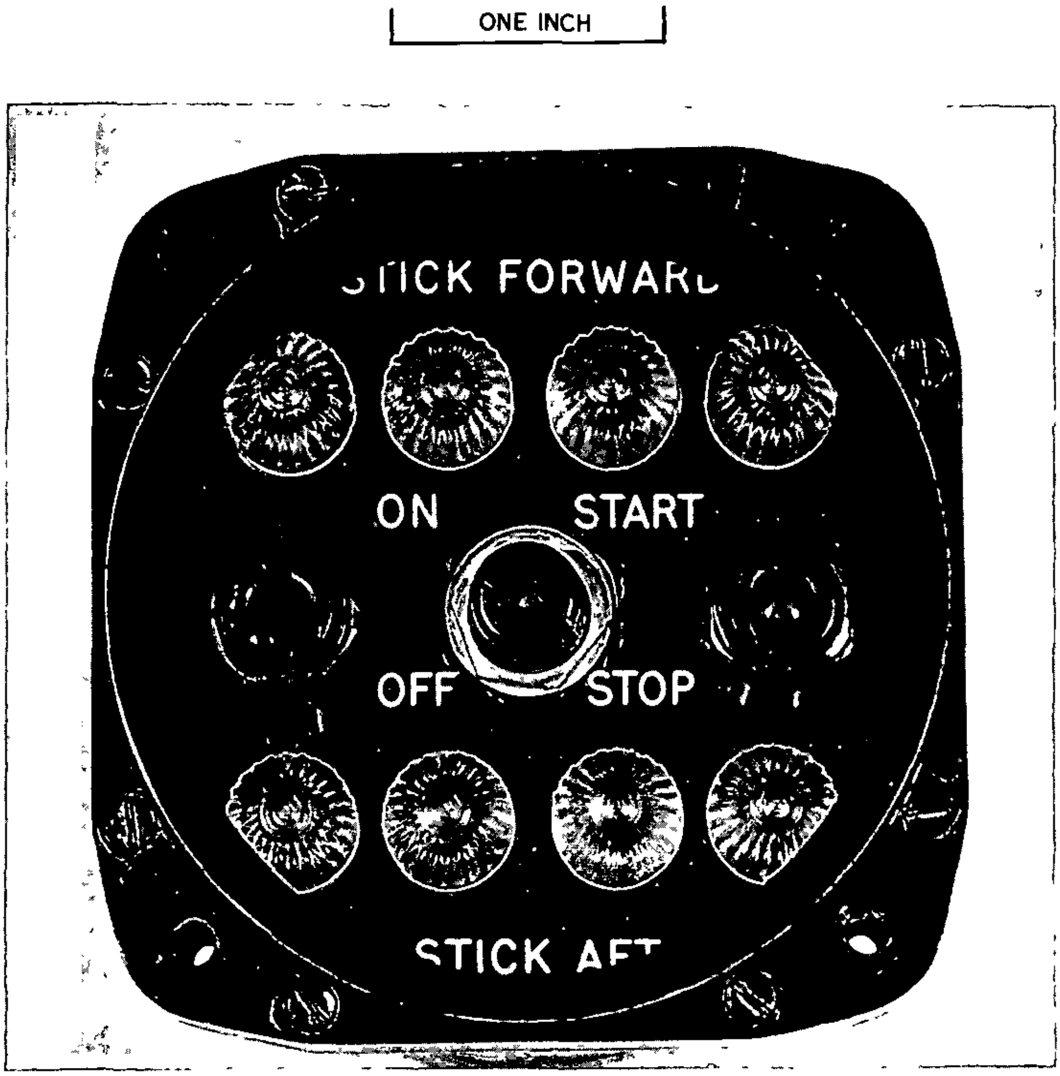


Fig.1. Pilot's psuedo-random binary sequence display

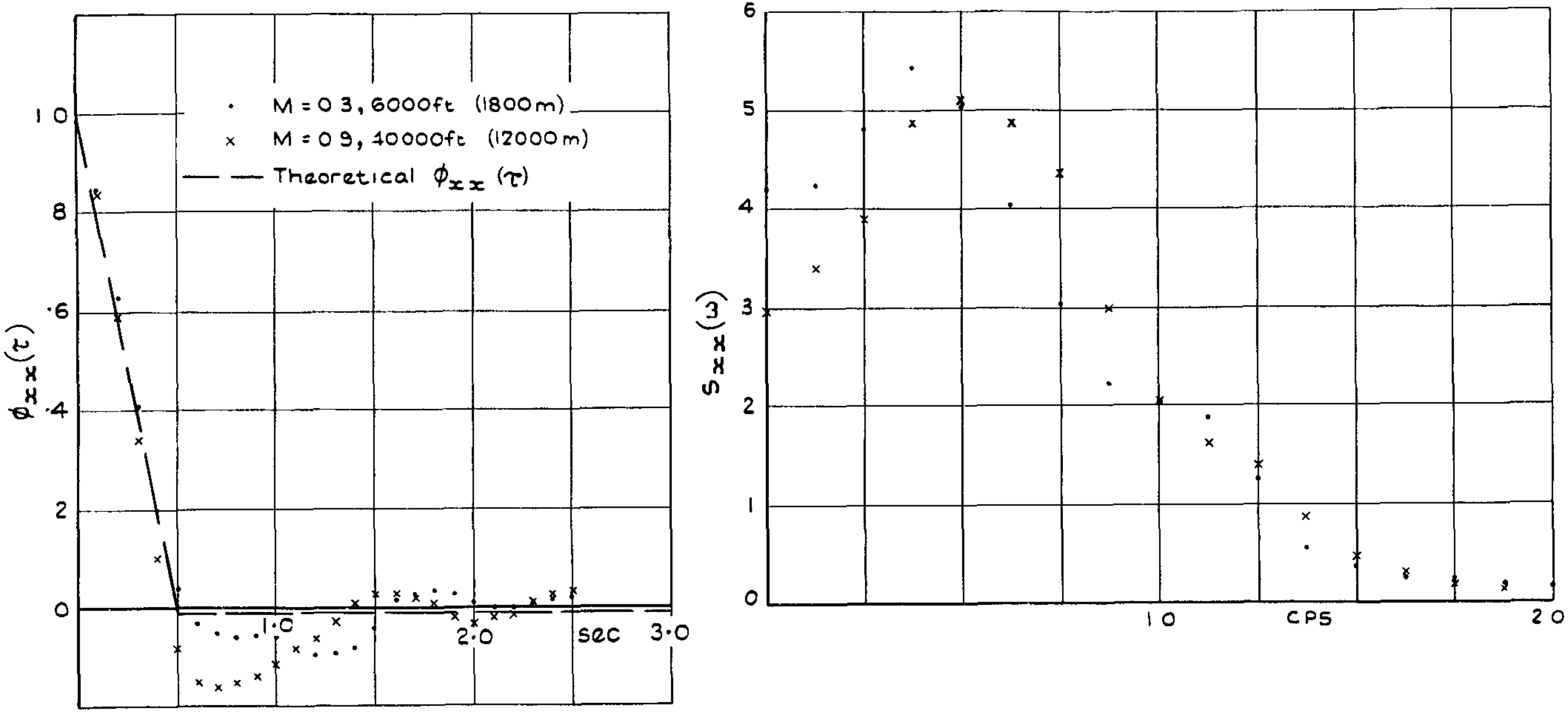


Fig. 2 Autocorrelation and power spectrum of input PRBS
Flight case 1, M = 0·3, 6000ft (1800m)
Flight case 4, M = 0 9, 40000ft (12000m)

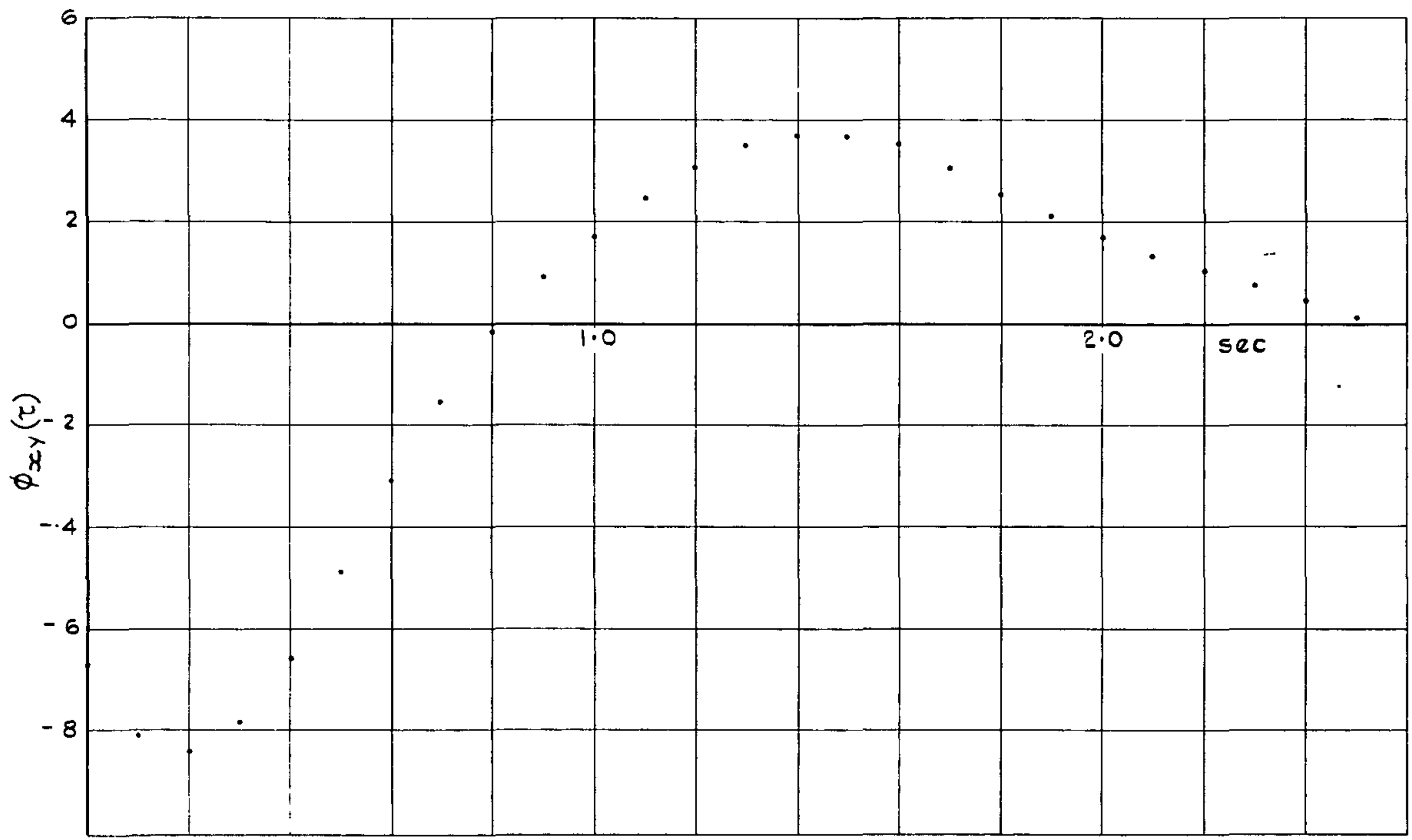


Fig. 3 Impulse response, q, flight case 1
M = 0·3, 6000ft (1800m)

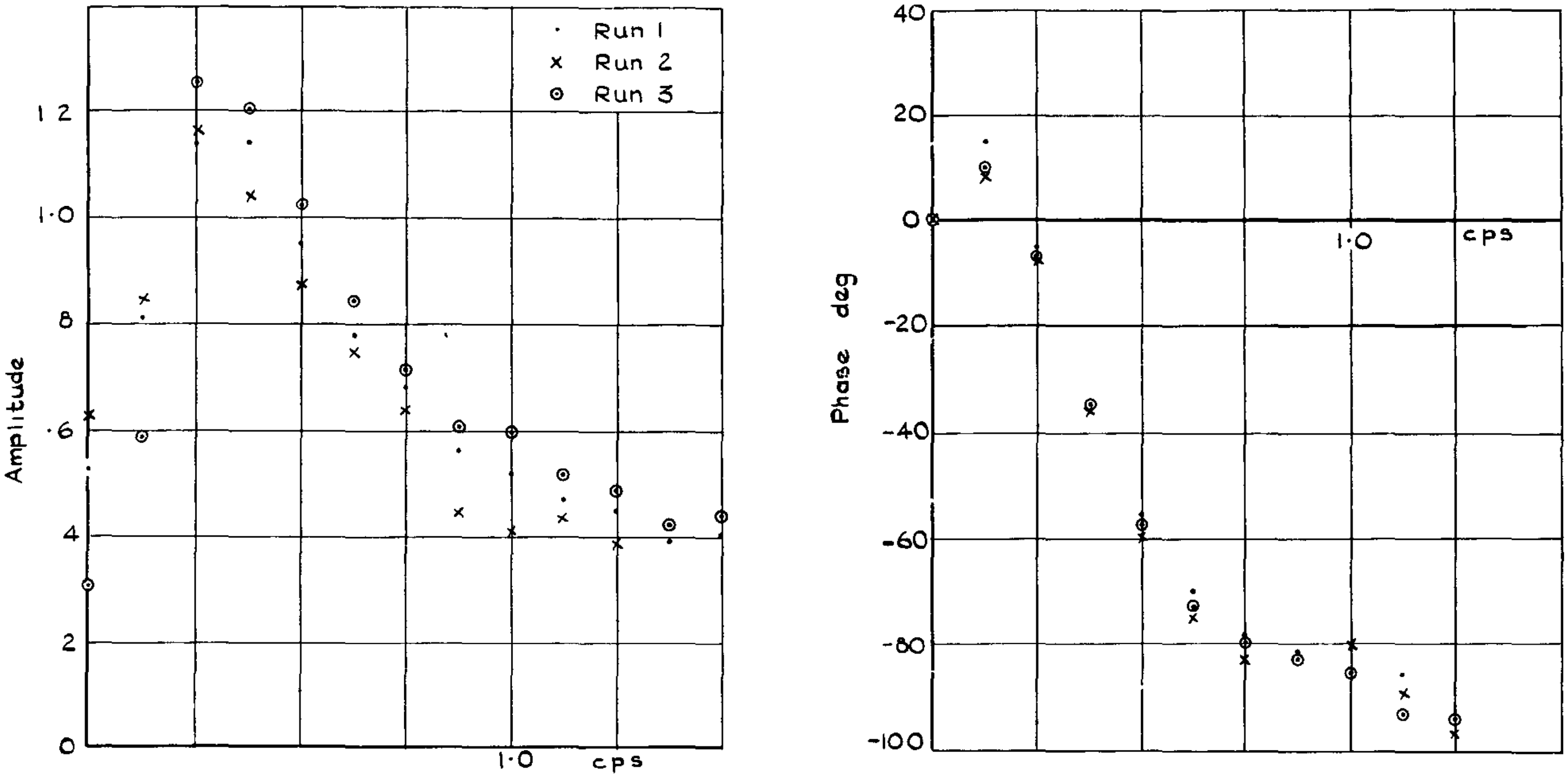


Fig 4 Amplitude and phase of transfer function, q/η derived from $S_{xy}(i\omega)/S_{xx}(\omega)$

Flight case 1, M = 0·3, 6000 ft (1800 m)

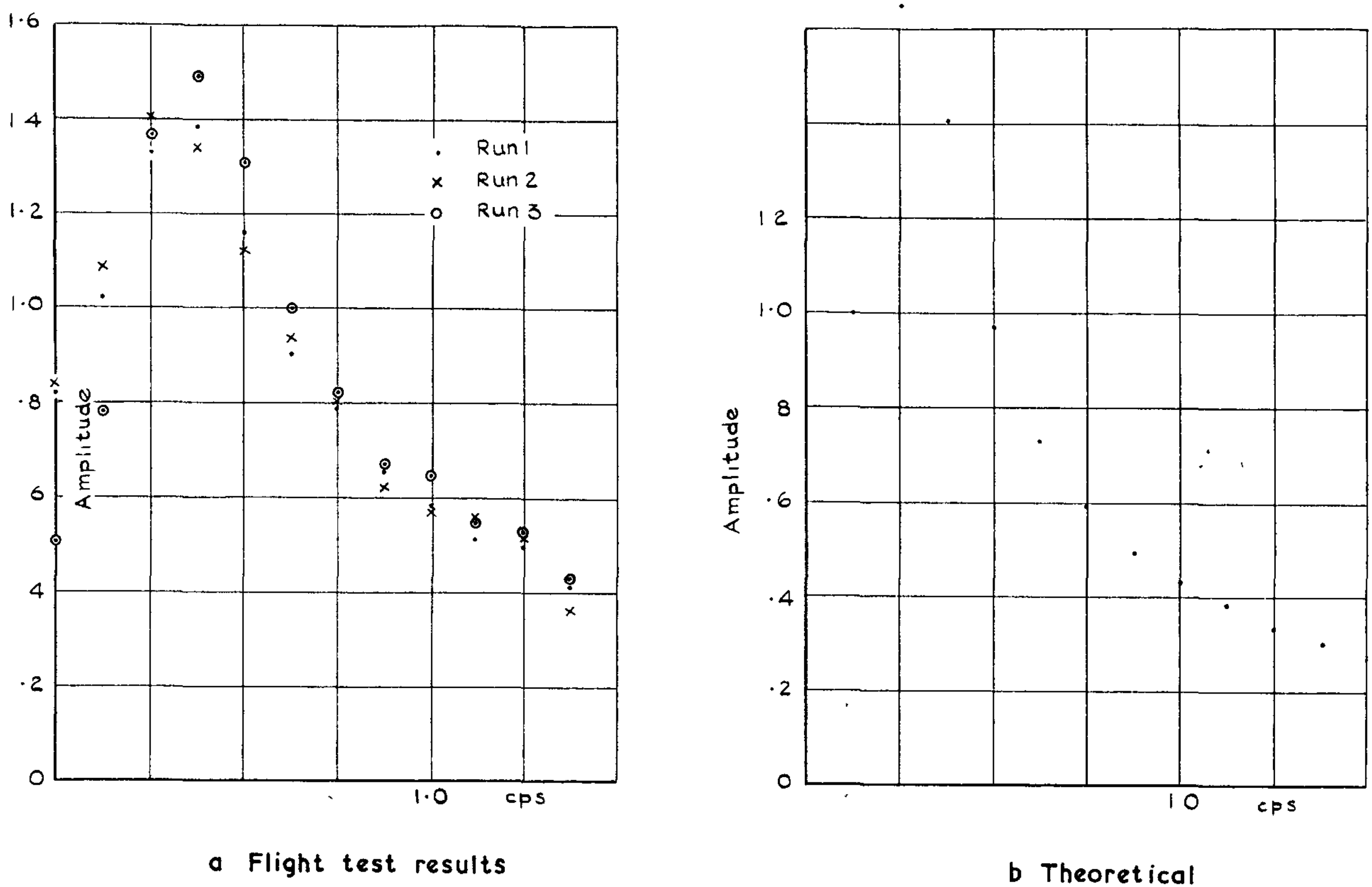


Fig. 5 a & b Amplitude of transfer function, q/η derived from $S_{yy}(\omega)/S_{xx}(\omega)$
Flight case 1 M = 0·3, 6000 ft (1800 m)

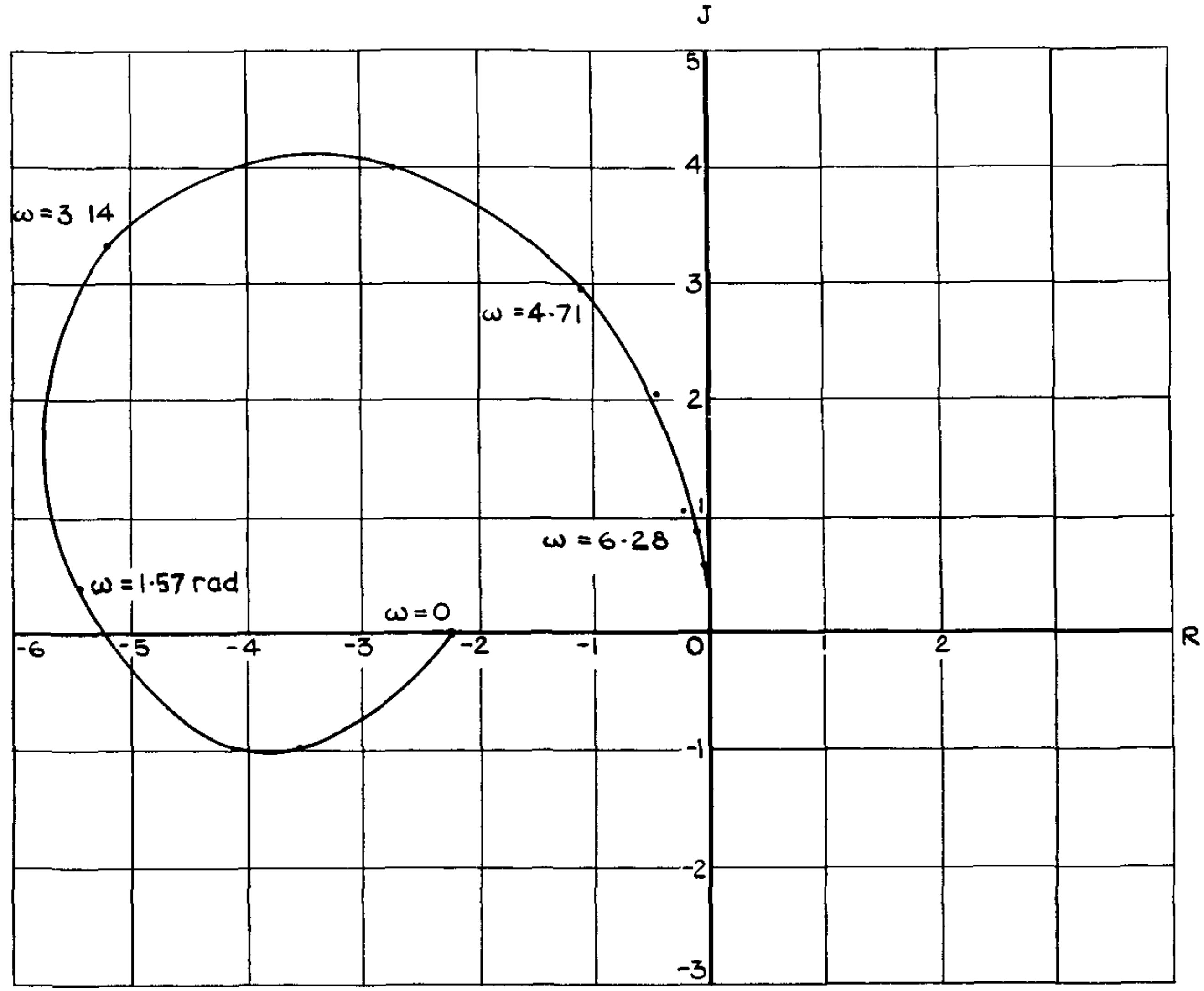


Fig. 6 Nyquist diagram of transfer function
M=0·3, 6000 ft (1800m)

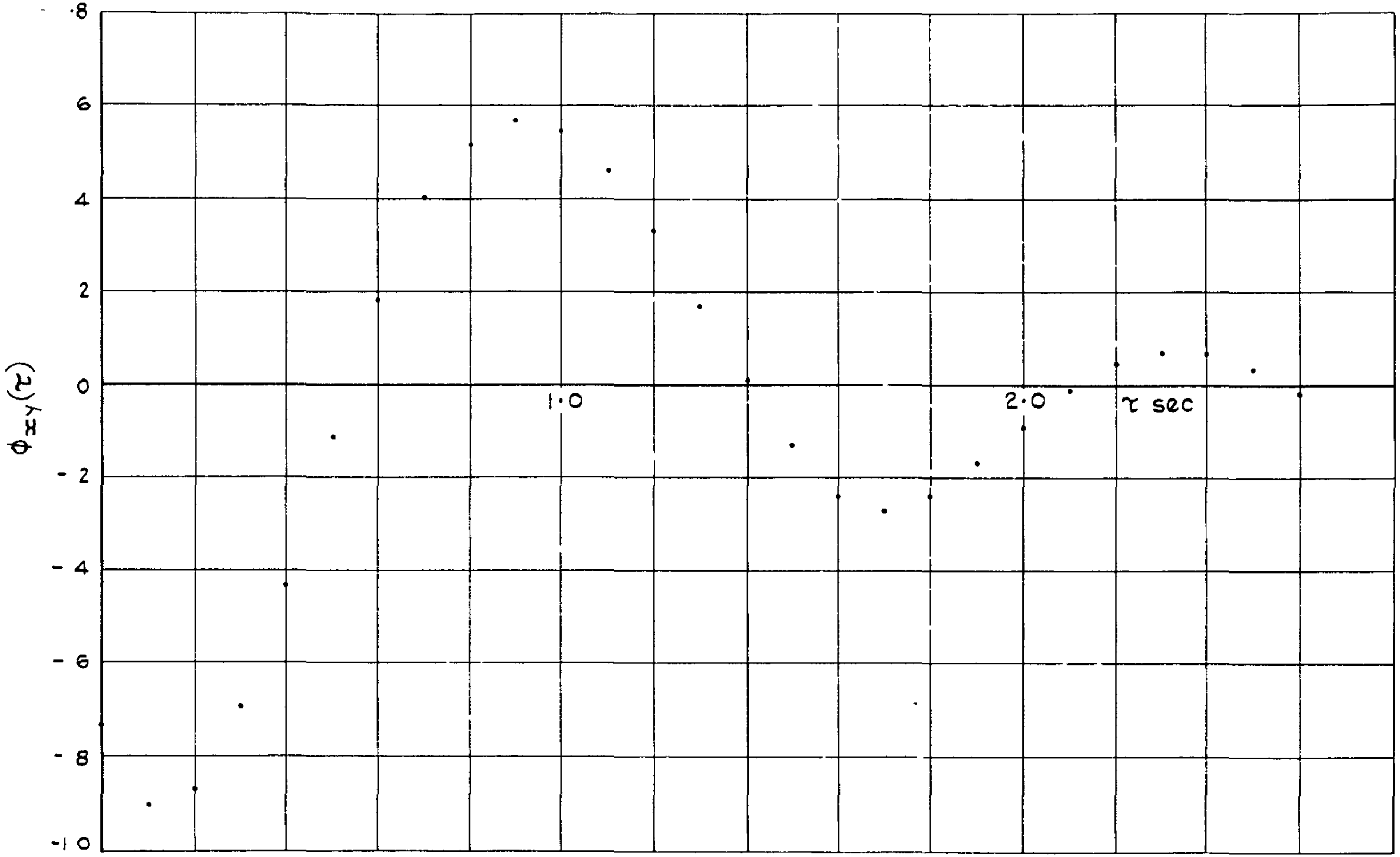
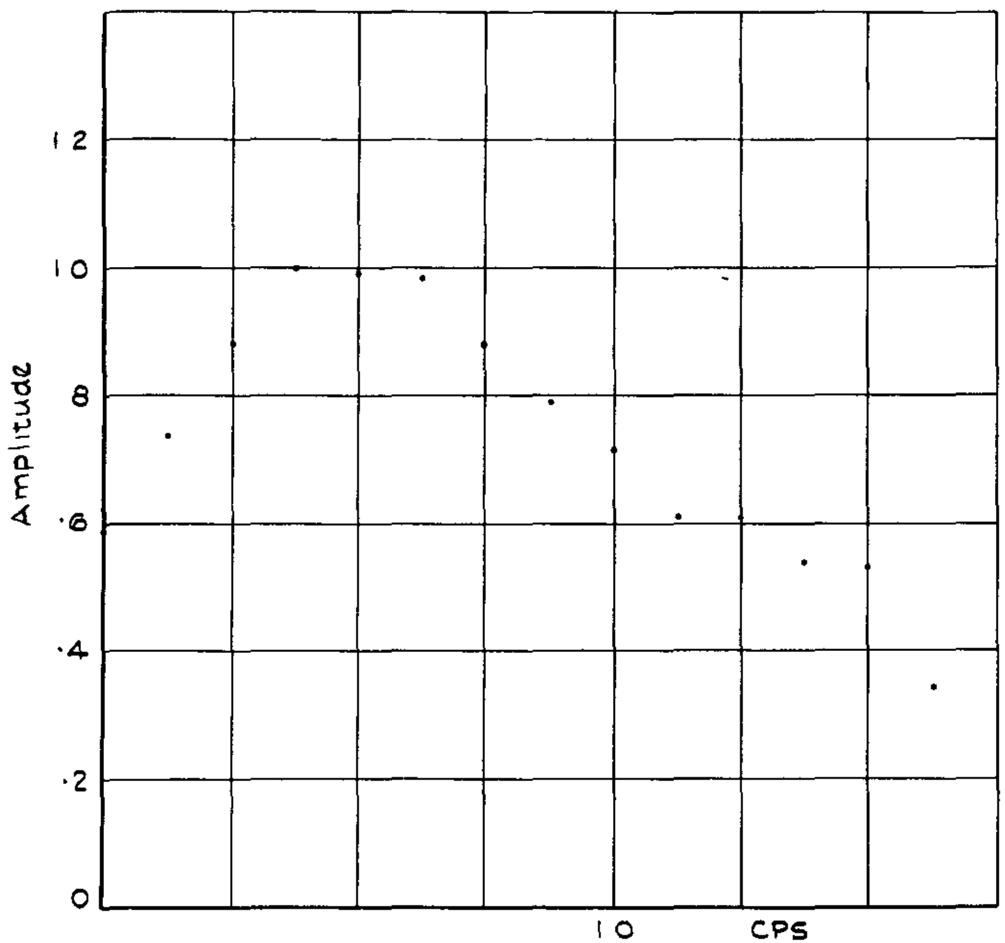


Fig. 7 Impulse response
Flight case 2, M = 0·6, 6000 ft (1800 m) pitch rate



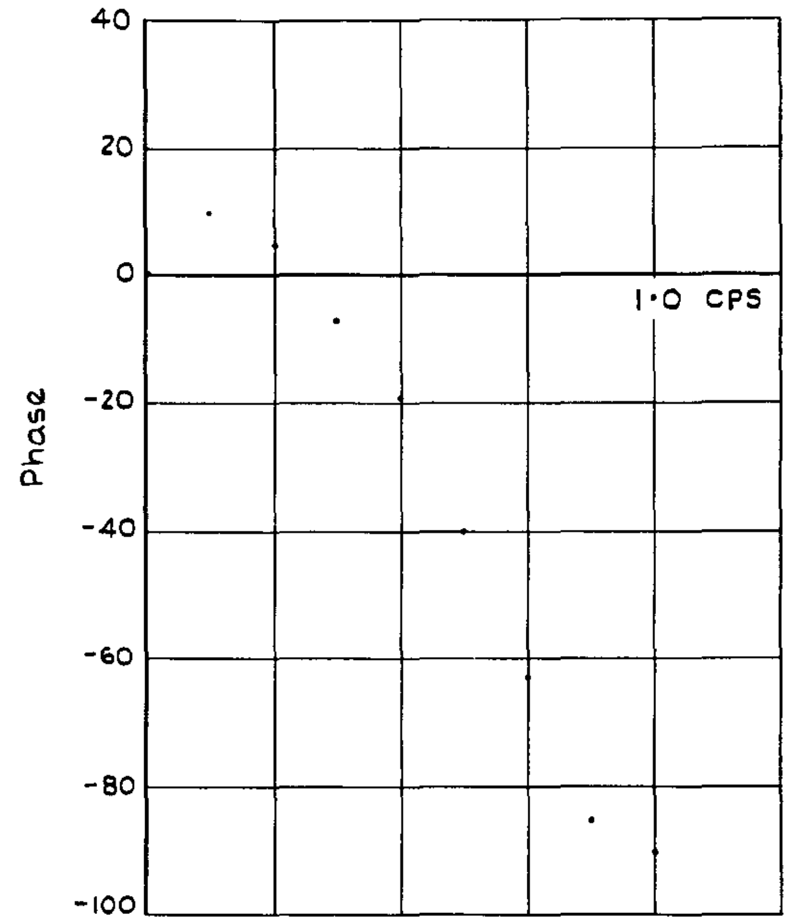


Fig 8 Amplitude and phase of transfer function, q/η derived from $S_{xy}(i\omega)/S_{xx}(\omega)$
Flight case 2, M = 0·6, 6000 ft (1800 m)

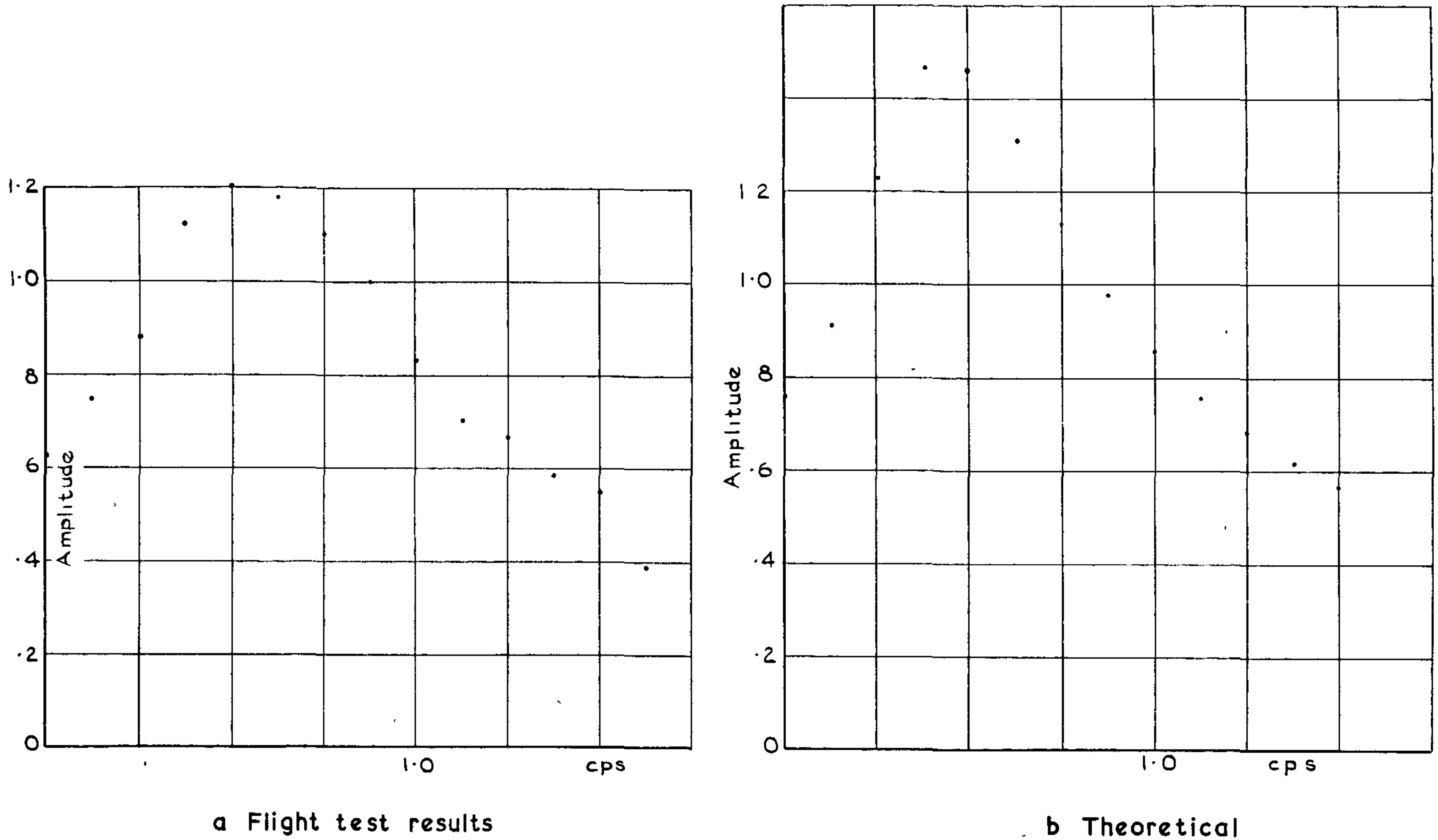


Fig. 9 a & b Amplitude of transfer function, q/η derived from $S_{yy}(\omega)/S_{xx}(\omega)$
Flight case 2, M = 0·6, 6000 ft (1800 m)

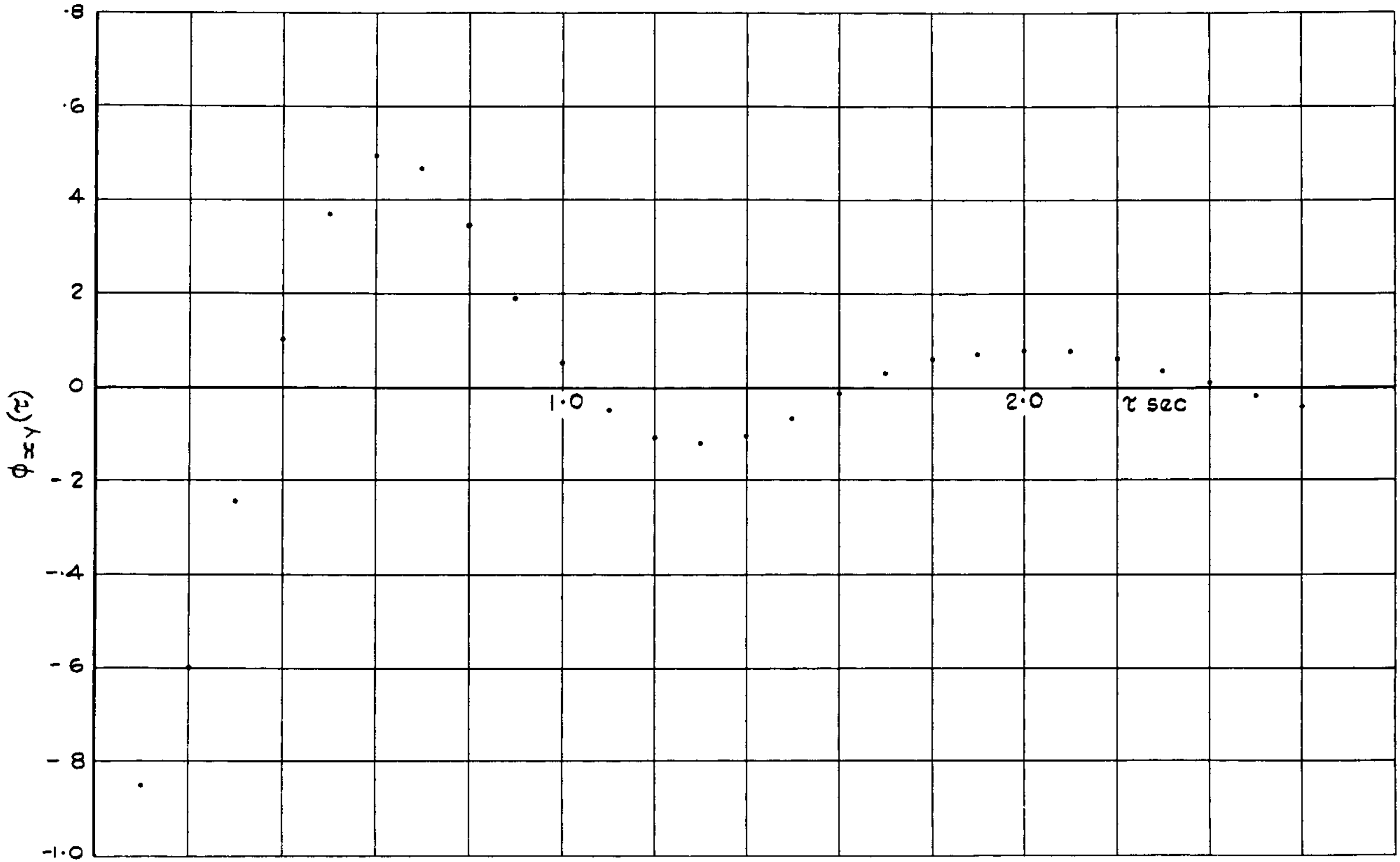


Fig 10 Impulse response, q, flight case 3
M = 0·9, sea level

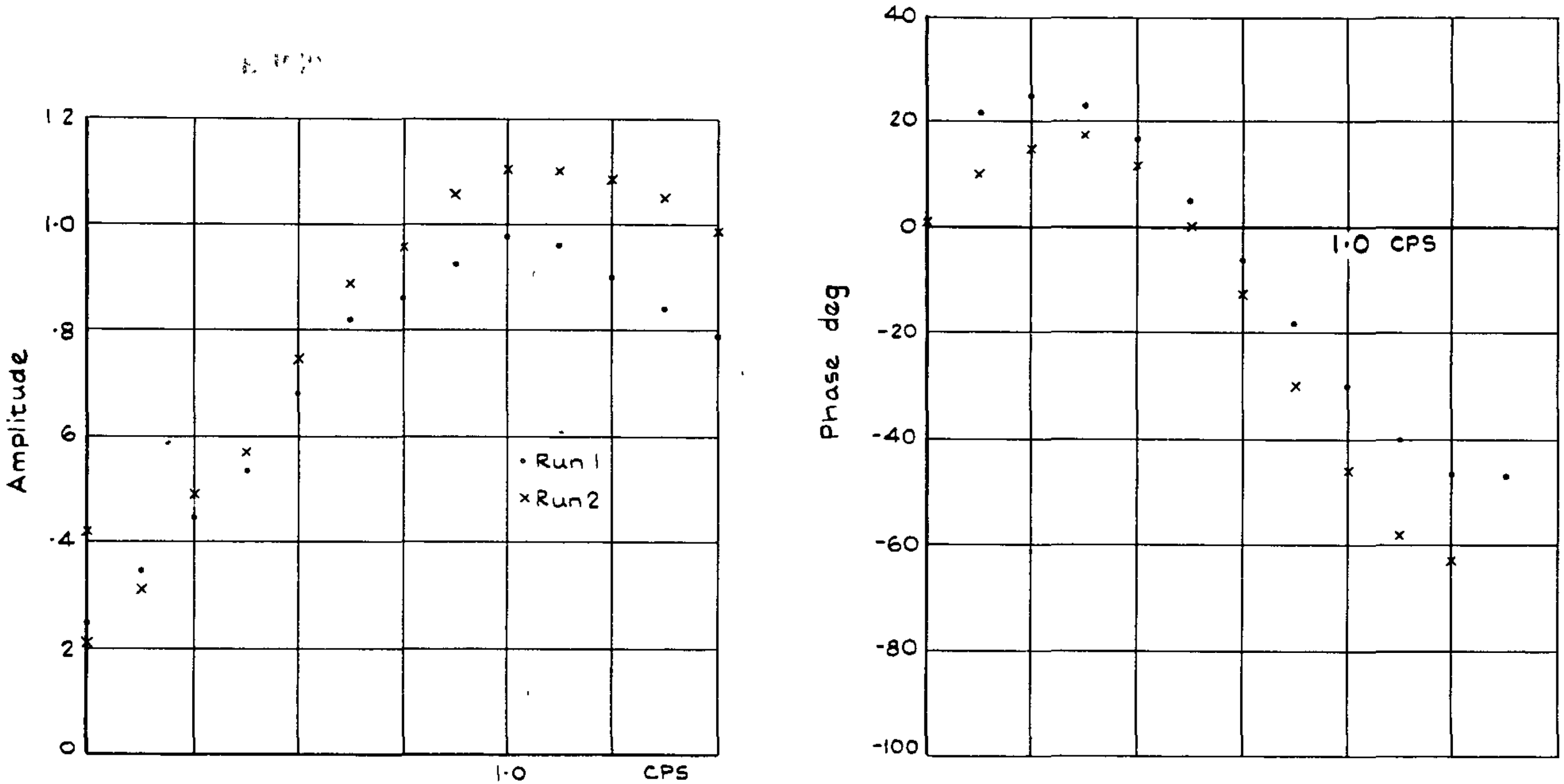
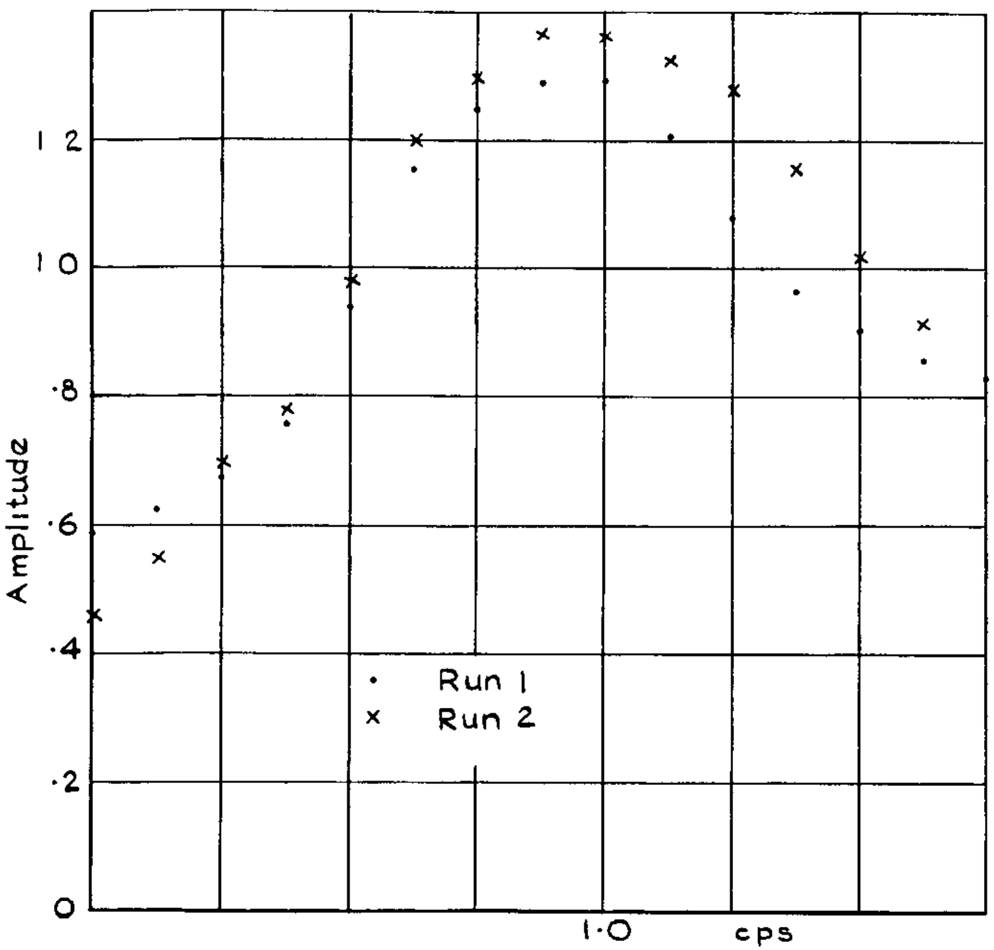
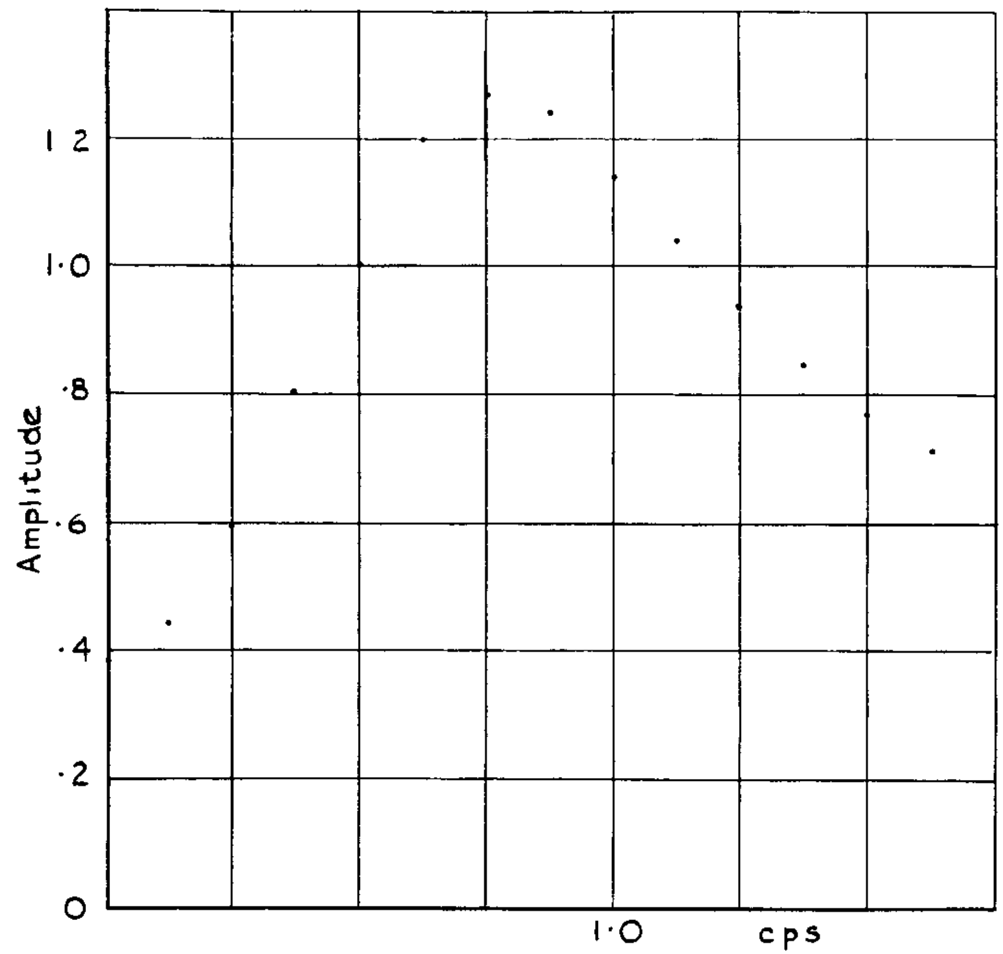


Fig. 11 Amplitude and phase of transfer function, q/η derived from $S_{xy}(i\omega)/S_{xx}(\omega)$

Flight case 3, M = 0·9, sea level



a Flight test results



b Theoretical

Fig. 12 a & b Amplitude of transfer function, q/η derived from $S_{yy}(\omega)/S_{xx}(\omega)$

Flight case 3, M = 0·9, sea level

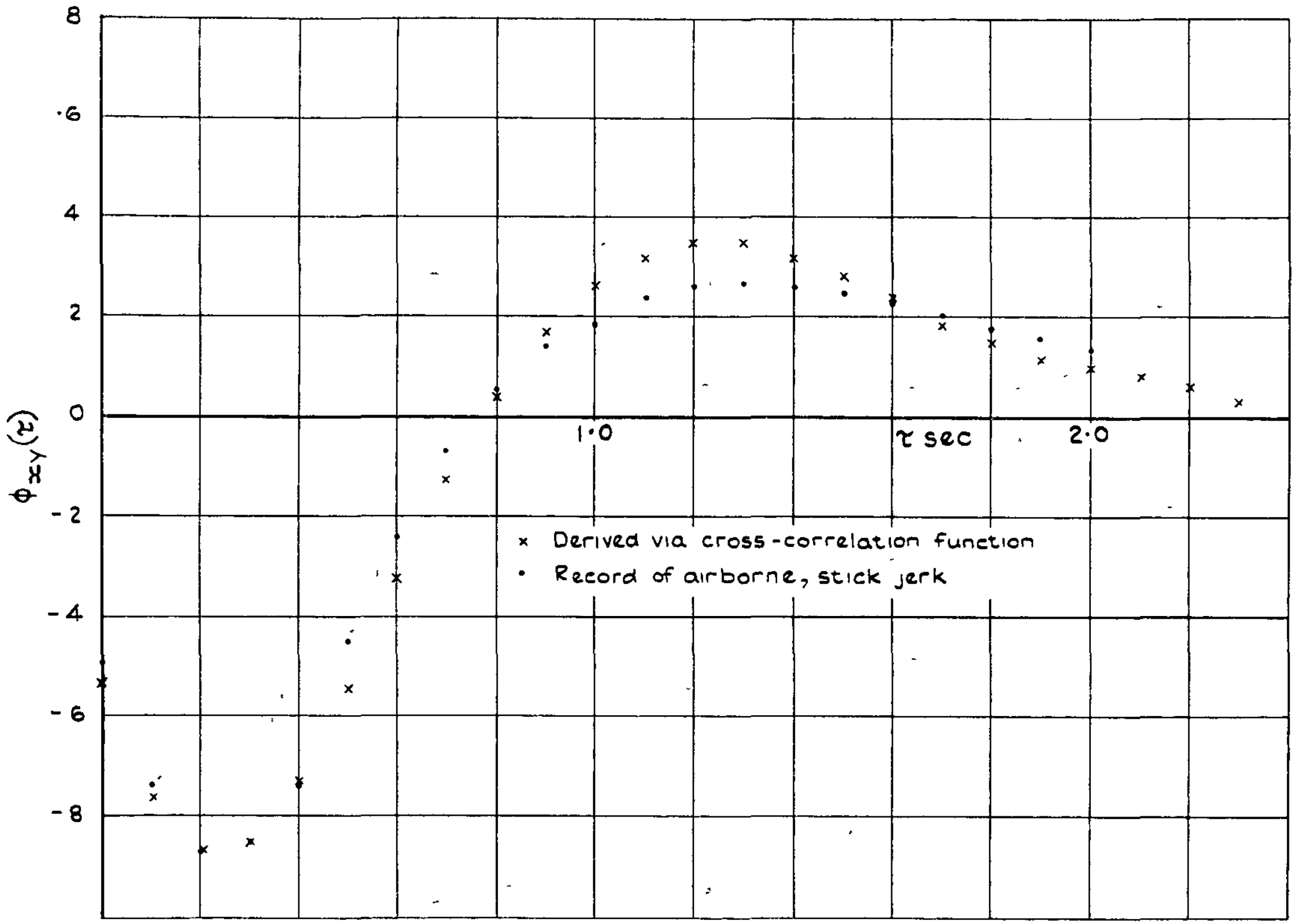


Fig. 13 Impulse response
Flight case 4, pitch rate M=0·9, 40000 ft, run 1, (12000 m)

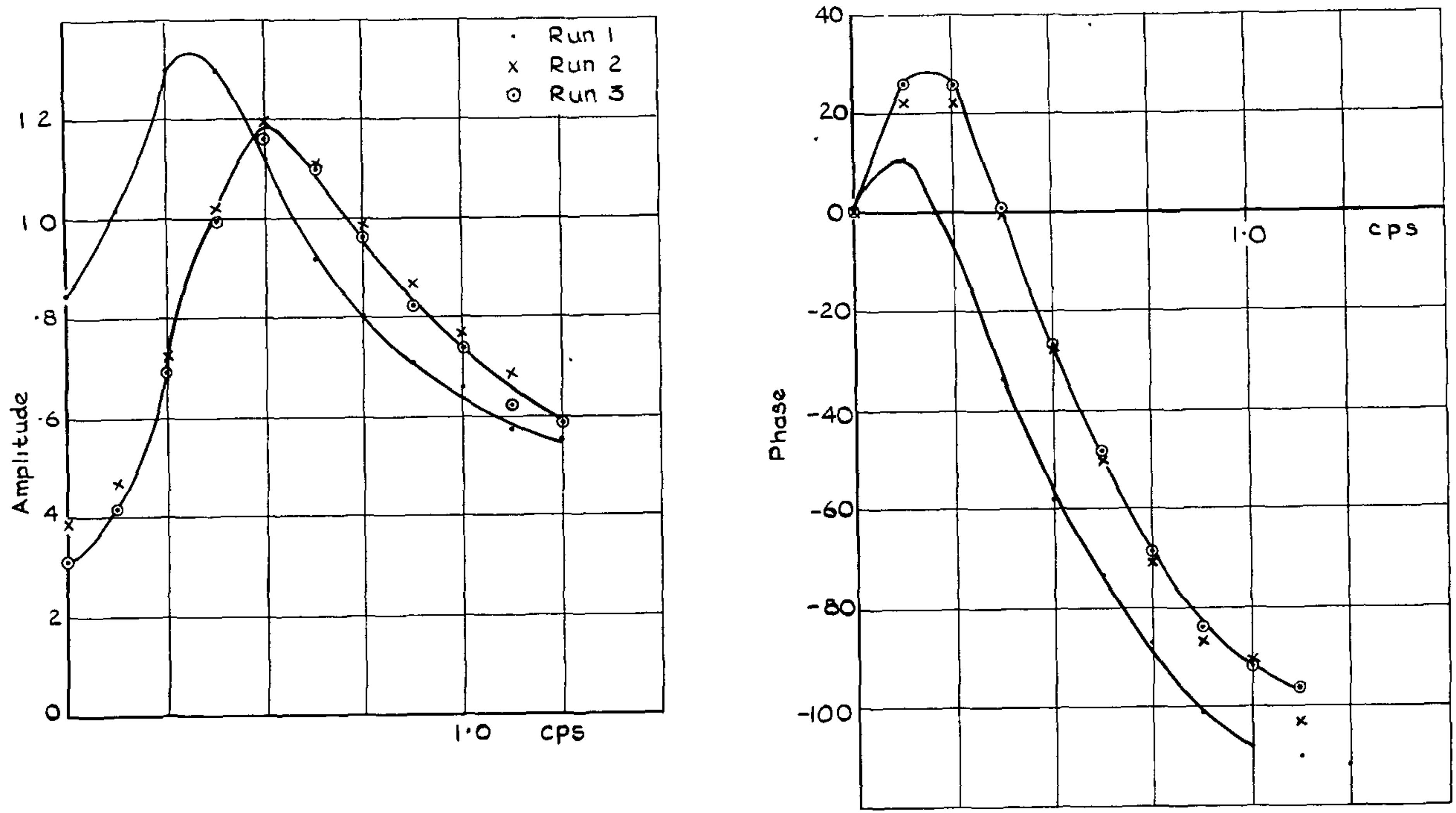


Fig. 14 Amplitude and phase of transfer function, q/η derived from $S_{xy}(i\omega)/S_{xx}(\omega)$
Flight case 4, M=0·9, 40000 ft (12000 m)

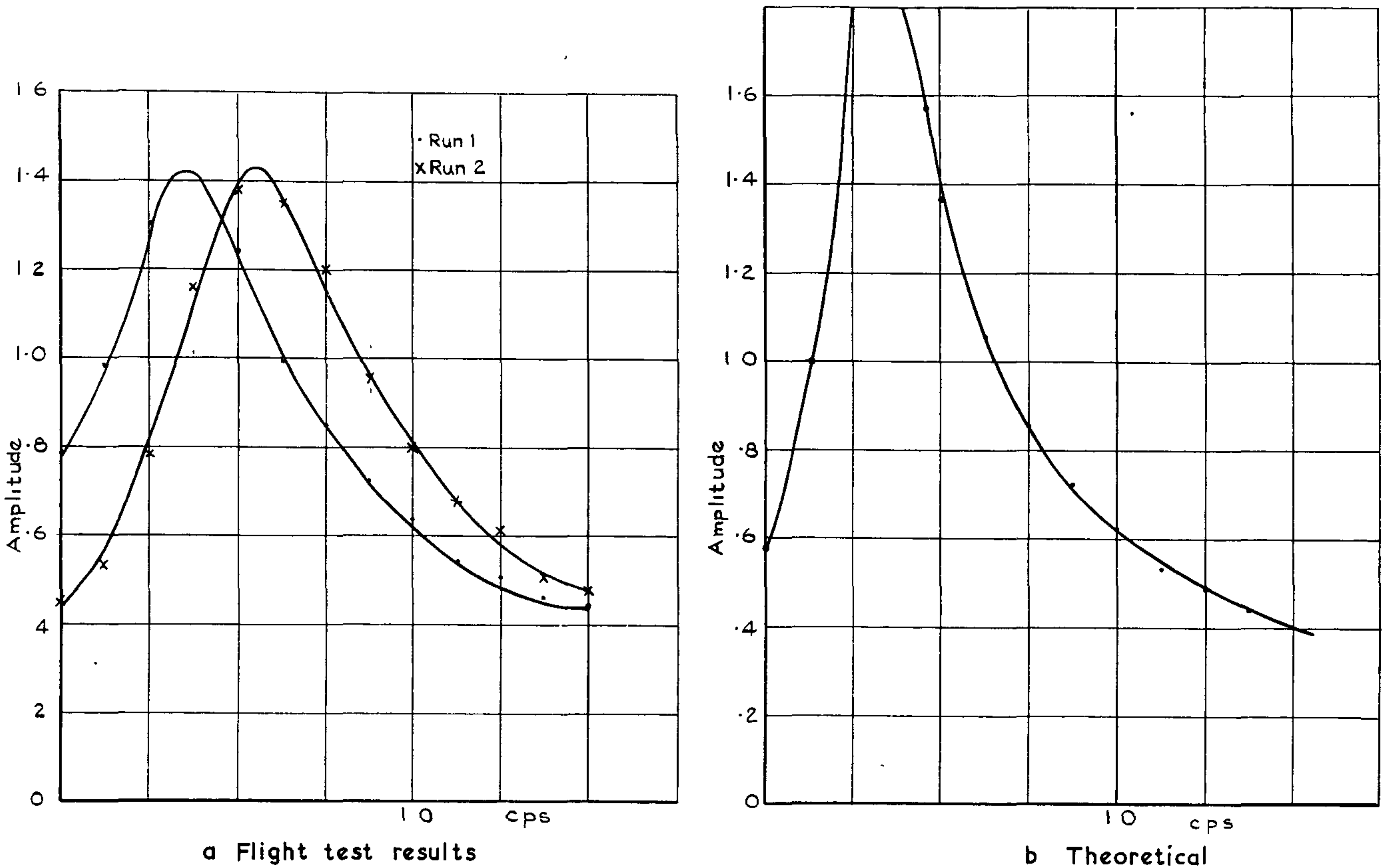


Fig. 15 a & b Amplitude of transfer function, q/η derived from $S_{yy}(\omega)/S_{xx}(\omega)$
Flight case 4, M = 0·9, 40000 ft (12000 m)

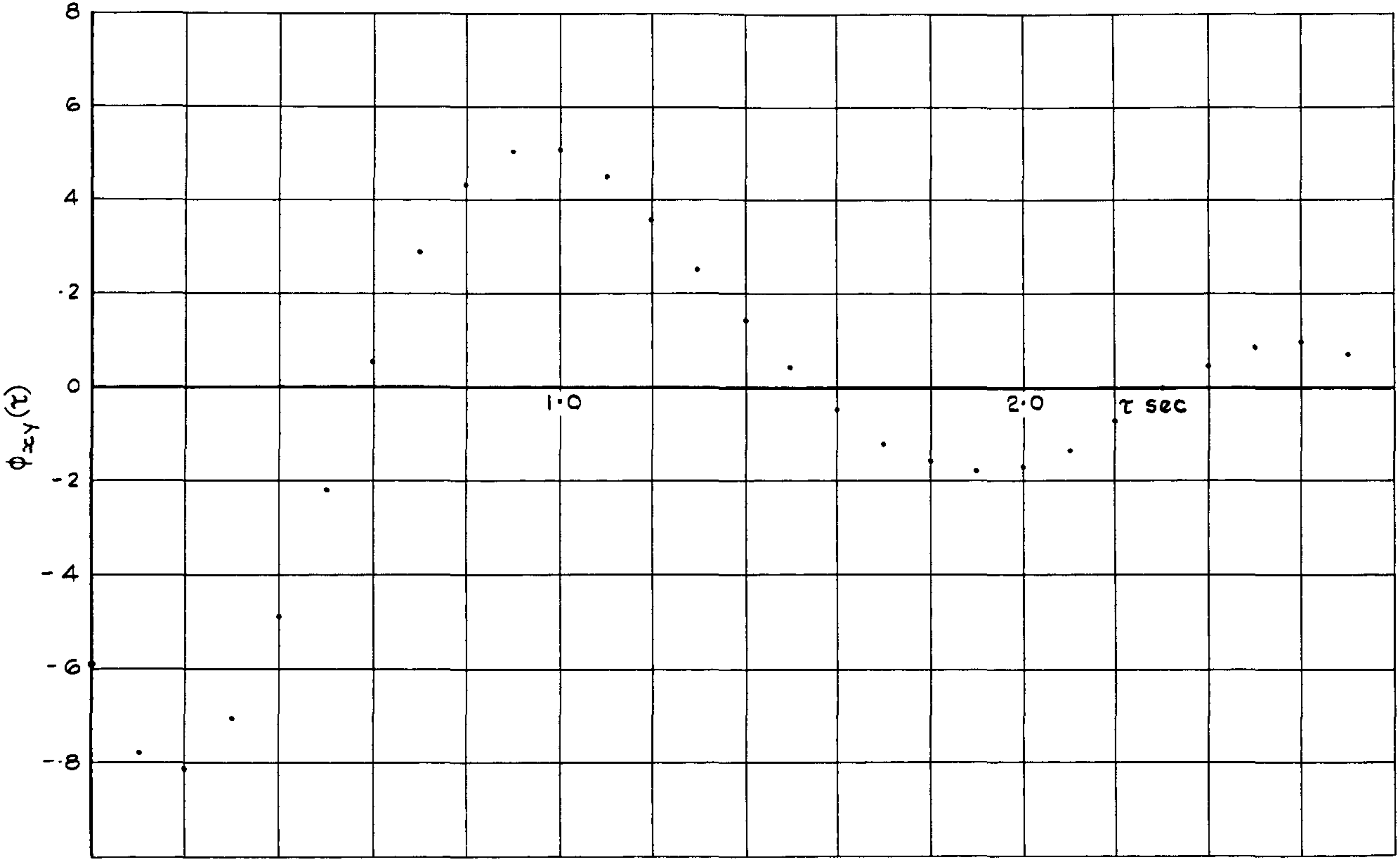


Fig. 16 Impulse response, q, flight case 4
M = 0·9, 40000 ft, run 2, (12000 m)

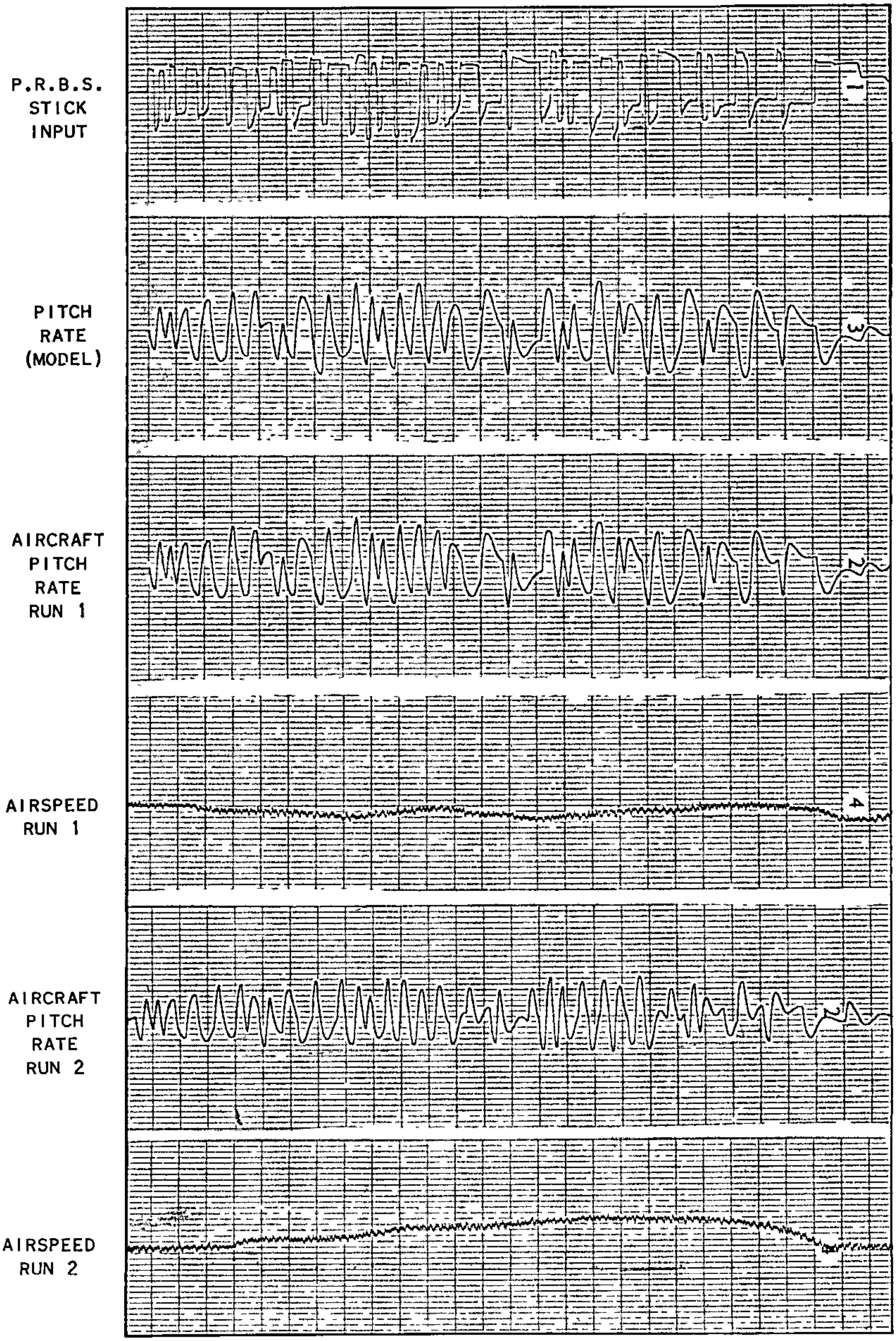


Fig.17. Effect of speed change on pitch-rate response. Psuedo-random noise input, flight case 4, M=0.9, 40,000ft (12,000m)

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621.374.32

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