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A Semi-Empirical Theory for the Growth and Bursting of Laminar Separation Bubbles

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SUMMARY

A simple pressure gradient criterion for the determination of the conditions under which re-attachment of a turbulent shear layer can occur is proposed. Application of this criterion to the laminar separation bubble problem, together with a simple bubble model and an approximate method of calculation of the momentum thickness growth over the bubble, leads to a method of prediction of the bubble growth. It is found that for a given imposed pressure distribution there exists a Reynolds number at separation below which re-attachment is impossible; this is associated with the so-called 'bursting' phenomenon. The predicted bursting parameters are in good agreement with experimental observations; in particular, the value of Crabtree's pressure rise parameter is found to be weakly dependent upon the boundary-layer Reynolds number at separation, varying between the limits 0.27 to 0.36 over the range of practical significance. It is concluded that bursting occurs as a failure of the re-attachment process, as suggested by Woodward.

1. Introduction

The investigations of McGregor¹, Gaster² and Woodward³ at Queen Mary College into the structure and behaviour of laminar



* Replaces A.R.C.29 185

separation bubbles have shown clearly that the simple criterion originally proposed by Owen and Klanfer⁴, which states that a bubble is short or long according to whether the boundary layer Reynolds number at separation, R_{δ^*s} , is greater or less than about 450, is by itself inadequate to determine the conditions under which bursting occurs. The hypothesis associated with this criterion, that a fundamental change in the stability of the separated laminar shear layer causes considerably delayed transition below this critical Reynolds number and hence much more extensive lengths of separated flow, has been shown by Woodward to be incorrect since he found transition to occur in very nearly the same physical position in bubbles just before and just after bursting. This observation, together with the discontinuous nature of the bursting phenomenon, suggested to Woodward that bursting occurs as a sudden failure of the shear layer to re-attach to the surface even though it is turbulent. This suggests that an examination of the conditions governing the re-attachment of a turbulent shear layer might be helpful in gaining an insight into the physical mechanism causing bubble bursting, and into the behaviour of separation bubbles in general.

2. A Simple Criterion for Turbulent Re-attachment

A criterion for turbulent re-attachment, analogous to the laminar and turbulent separation criteria of Thwaites⁵ and Buri⁶, may be derived by considering the behaviour of the momentum integral equation together with either the kinetic energy integral equation.

or Head's⁷ entrainment equation. We consider here only the former case.

The momentum integral and kinetic-energy integral equations for turbulent flow are, omitting the terms involving the normal Reynolds stresses,

$$\frac{d\theta}{dx} + (H + 2) \frac{\theta}{u_e} \frac{du_e}{dx} = \frac{1}{2} C_f, \dots\dots\dots (1)$$

$$\text{and } \frac{d\epsilon}{dx} + 3 \frac{\epsilon}{u_e} \frac{du_e}{dx} = C_d, \dots\dots\dots (2)$$

$$\text{where } H = \frac{\delta^*}{\theta}, \quad C_f = \frac{\tau_w}{\frac{1}{2} \rho u_e^2}, \quad C_d = \frac{2}{\rho u_e^3} \int_0^{\infty} \tau \frac{\partial u}{\partial z} \cdot dz, \dots\dots (3)$$

$$\left. \begin{aligned} \text{and } \delta^* &= \int_0^{\infty} \left(1 - \frac{u}{u_e}\right) dz, \text{ the displacement thickness} \\ \theta &= \int_0^{\infty} \frac{u}{u_e} \left(1 - \frac{u}{u_e}\right) dz, \text{ the momentum thickness} \\ \epsilon &= \int_0^{\infty} \frac{u}{u_e} \left(1 - \frac{u^2}{u_e^2}\right) dz, \text{ the energy thickness.} \end{aligned} \right\} \dots (4)$$

Introducing now the energy shape parameter

$$H_\epsilon = \epsilon/\theta,$$

equation 2 may be written in the form

$$\frac{d\theta}{dx} + \frac{\theta}{H_\epsilon} \cdot \frac{dH_\epsilon}{dx} + 3 \frac{\theta}{u_e} \frac{du_e}{dx} = \frac{C_d}{H_\epsilon} \cdot \dots\dots\dots (5)$$

Elimination of $d\theta/dx$ between equations (1) and (5) leads to Truckenbrodt's²³ shape-parameter equation:-

$$\theta \frac{dH_\epsilon}{dx} = (H - 1) H_\epsilon \cdot \frac{\theta}{u_e} \frac{du_e}{dx} + C_d - \frac{1}{2} H_\epsilon \cdot C_f \cdot \dots\dots\dots (6)$$

Let us now examine the behaviour of equation (6) at a point of re-attachment. At such a point we have by definition that the skin friction is zero, so equation (6) becomes

$$H_\epsilon (H - 1) \frac{\theta}{u_e} \frac{du_e}{dx} = \theta \frac{dH_\epsilon}{dx} - C_d \cdot \dots\dots\dots (7)$$

Now as can be seen from Fig. 1., H_ϵ becomes virtually independent of H for the high values of H (i.e. $H = 3$) associated with re-attachment. (These curves, due to Thompson, were derived for conventional attached boundary layers, but the experimental points included on the figure indicate that the curves are equally applicable to re-developing boundary layers after re-attachment.) Thus unless dH/dx is exceedingly large, we should expect dH_ϵ/dx to be small at re-attachment. Some measurements of the streamwise variation of H_ϵ near re-attachment are shown in Fig. 2, which indicate that H_ϵ passes through a minimum at the re-attachment point;

$$\text{i.e. } \left(\frac{dH_\epsilon}{dx} \right)_R = 0 .$$

Thus at re-attachment, equation 7 reduces to the equation

$$\frac{\theta}{u_e} \frac{du_e}{dx} = - \frac{C_d}{H_\epsilon (H - 1)} \cdot \dots\dots\dots (8)$$

It follows that if the velocity and shear stress profiles at re-attachment are universal, then $\left(\frac{\theta}{u_e} \frac{du_e}{dx}\right)_R$ is a function of Reynolds number only.

Some evidence for the assumption of universality of velocity profiles is presented in Fig. 3; re-attachment profiles behind steps measured by Mueller⁹ and Tani¹⁰ are shown together with a number of profiles measured in swept separation bubbles. The presence of cross-flows in the latter case eliminates the inaccuracies occurring in the two-dimensional measurements due to the non-linear response of the hot-wire anemometer. The resulting mean profile is virtually identical to the self-preserving wake profile of Bradbury¹¹; this lends support to the idea that re-attachment has a wake-like character, as is inherent in Coles'¹² hypothesis, although the profile is rather more full than that of Coles. The profile is of a different form from that of turbulent separation profiles, being much less full.

This wake-like character of the re-attachment process suggests that the assumption of constant eddy viscosity through the layer may be valid; indeed Clauser¹³ has shown that the outer 80-90% of turbulent boundary layers in general may be considered to have constant eddy viscosity, only the wall region being excluded. In the case of re-attachment, the wall region will be absent. For equilibrium boundary layers Clauser¹³ has found that the eddy viscosity μ_t is given by

$$\mu_{\tau} = k\rho u_e \delta^* , \dots\dots\dots (9)$$

where μ_{τ} is defined by $\tau = \mu_{\tau} \frac{\partial u}{\partial z}$, and $k = 0.018$. Shear stress measurements in re-attaching layers by Mueller⁹ and Tani¹⁰ suggest that k should be 0.020. This implies that a turbulent Reynolds number R_T defined by $R_T = \rho u_e \delta^* / \mu_{\tau}$ has the constant value $R_T = 50 = 1/k$.

$$\text{Now } C_d = \frac{2}{R_T} \int_0^{\infty} \left(\frac{\partial \tilde{u}}{\partial \eta} \right)^2 d\eta , \dots\dots\dots (10)$$

where $\tilde{u} = \frac{u}{u_e}$ and $\eta = \frac{z}{\delta^*}$.

But for the mean velocity profile of Fig. 3. we have that

$$\int_0^{\infty} \left(\frac{\partial \tilde{u}}{\partial \eta} \right)^2 d\eta = 0.554 , \dots\dots\dots (11)$$

so that with $R_T = 50$ we get that at re-attachment

$$C_d = 0.0222. \dots\dots\dots (12)$$

The use of a constant value of R_T leads to the result that C_d is independent of Reynolds number. For the ~~mean~~ re-attachment profile we have that $H = 3.50$ and $H_e = 1.51$ so that from equation 8 we get

$$\left(\frac{\theta}{u_e} \frac{du_e}{dx} \right)_R = - 0.00590. \dots\dots\dots (13)$$

Recently Fiedler and Head¹⁴ have found that the rate of

entrainment into a re-attaching boundary layer is higher than for the corresponding attached layer, and the same may be expected to be true of the dissipation coefficient C_d . Accordingly, the value of $\left(\frac{\theta}{u_e} \frac{du_e}{dx}\right)_R$ predicted by equation 13 may be expected to be too low; the actual form of the criterion, with its independence of Reynolds number, may nevertheless be expected to be correct.

A number of experimental determinations of $\left(\frac{\theta}{u_e} \frac{du_e}{dx}\right)_R = \Lambda_R$ are presented in Fig. 4; the rather large scatter is probably mainly attributable to the difficulty of measuring du_e/dx , which changes rapidly in the re-attachment zone. No definite Reynolds number effect is apparent, and the mean value of Λ_R is -0.0082 with a standard deviation of $.0016$; the distribution of points about the mean follows approximately a normal curve, lending credence to the idea that the scatter is mainly due to experimental error.

This mean experimental value, viz.

$$\Lambda_R = \left(\frac{\theta}{u_e} \frac{du_e}{dx}\right)_R = -0.0082, \dots\dots\dots (14)$$

will be used in the ensuing theory.

3. A Simple Model of the Short Bubble

Laminar separation bubbles are essentially a first-order interaction phenomenon; that is, the perturbation of the inviscid velocity distribution due to the presence of a bubble is first order, rather than second order as is the case with attached boundary-layers

in incompressible flow, so that the external velocity distribution should strictly be calculated to be compatible with the displacement effect of the bubble. In subsonic flow however this is rather a formidable problem, but fortunately some experimental observations of the general nature of the perturbed velocity distributions in the presence of short bubbles enable us to make use of a simple assumed form of perturbed velocity distribution, in which the total bubble length is essentially a free parameter which may be varied according to conditions. Numerous investigations have determined the following essential facts (see Fig. 5):

(1) The perturbation to the inviscid velocity distribution is negligible except over the length of the bubble itself. Thus, the separation point may be calculated from the inviscid velocity distribution by the usual laminar boundary layer methods and separation occurs at close to the corresponding inviscid value of u_e ; re-attachment takes place at some value of u_e lying on the inviscid velocity distribution curve.

(2) The pressure, and hence external velocity, over the laminar part of the bubble is constant, to a good approximation.

(3) The external velocity falls nearly linearly between the transition and re-attachment points.

There is a discontinuity in du_e/dx at the re-attachment point if we assume a linear fall of external velocity between transition and re-attachment; in practice of course there is a blending-in of the two curves, but it is found that this occurs after re-attachment so that the value of du_e/dx at re-attachment may be taken as that of the linear velocity drop between transition and re-attachment.

The length l_1 of laminar separated flow is obviously an important variable in the problem, but can only be determined experimentally. From dimensional considerations it can be argued that, provided the level of fluctuations in the boundary layer at separation is small, l_1/θ_s should be a function of R_{θ_s} . The results of careful experiments by McGregor¹, Gaster² and Woodward³ are shown in Fig. 6, and it is found that the formula $l_1/\theta_s = 4 \times 10^4 / R_{\theta_s}$ correlates the results quite well. This formula is of the same form as that suggested by von Doenhoff¹⁵, but with a different constant (von Doenhoff's value being 5×10^4).

Under the above assumptions it is evident that the value of (du_e/dx) at re-attachment depends only upon the velocity drop over the bubble and the length l_2 of turbulent flow. In order to determine the re-attachment according to equation 14, the value of θ_R is required; in the next section a simple method of calculating this in terms of θ_s will be given.

4. Calculation of Shear Layer Development

We consider the laminar and turbulent parts of the bubble separately.

(1) The Laminar Part

As we have seen, the pressure is essentially constant in the laminar part of the bubble, and as we might therefore expect it is found that the reverse flow velocities under the laminar shear layer are exceedingly small. The skin friction in this region is therefore negligible, so from the momentum equation we get simply $d\theta/dx = 0$,

$$\text{i.e. } \theta_T = \theta_S, \dots\dots\dots (15)$$

where θ_T is the momentum thickness at transition.

(2) The Turbulent Part

Methods based on the Crocco-Lees¹⁶ mixing equation, the momentum-integral equation and the energy-integral equation have been compared, and it is found that the energy-integral equation method is preferable in the existing conditions (strong adverse pressure gradient), as this method is considerably less sensitive to the value of the empirically-determined constants occurring in all three methods.

The energy integral equation 2 may be written as

$$\frac{1}{u_e} \frac{d}{dx} (u_e^3 H_\epsilon \cdot \theta) = C_d, \dots\dots\dots (16)$$

so that, integrating between transition T and re-attachment

R we have that

$$\left(H_\epsilon \theta u_e^3 \right)_T^R = \int_T^R C_d \cdot u_e^3 \cdot dx. \dots\dots\dots (17)$$

Now from the experimental results shown in Fig. 2 we see that at T, $H_\epsilon = 1.48$ and at R, $H_\epsilon = 1.52$, and it is inferred that in general to a good approximation we may use a mean value of

$H_{\epsilon_m} = 1.50 = \text{constant}$. Writing also $\bar{u}_e = u_e/u_{e_s} = u_e/u_{e_T}$, $\bar{\theta} = \theta/\theta_s = \theta/\theta_T$, $\bar{x} = x/\theta_s$, we get

$$\bar{\theta}_R \bar{u}_{eR}^3 - 1 = \frac{1}{H_{\epsilon_m}} \int_{\bar{x}_T}^{\bar{x}_R} C_d \cdot \bar{u}_e^3(\bar{x}) \cdot d\bar{x}. \dots\dots\dots (18)$$

The contribution from the right-hand side of this equation is not usually large, and accordingly the assumption of a constant overall value of C_d may be expected to yield results of acceptable accuracy. Making this approximation, equation 18 becomes

$$\bar{\theta}_R \bar{u}_{eR}^3 - 1 = \frac{C_{d_m}}{H_{\epsilon_m}} \int_{\bar{x}_T}^{\bar{x}_R} \bar{u}_e^3(\bar{x}) d\bar{x}, \dots\dots\dots (19)$$

where C_{d_m} is the mean overall value of C_d .

5. Growth and Bursting Theory

We are now in a position to obtain a closed solution for the length of a bubble by combining the re-attachment criterion, equation 14, with equation 19, assuming the external velocity distribution model previously described.

Consider the turbulent part of the bubble alone; the velocity is assumed linear, and the ratio of the velocity at re-attachment to that at transition is $u_{eR}/u_{eT} = u_{eR}/u_{eS} = \bar{u}_{eR}$, so that in non-dimensional form we have

$$\left(\frac{d\bar{u}_e}{d\bar{x}}\right)_R = -\frac{(1 - \bar{u}_{eR})}{\bar{\ell}_2}, \dots\dots\dots (20)$$

where $\bar{\ell}_2 = \ell_2/\theta_S$.

Now from equation 14 we have at re-attachment

$$\Lambda_R = \left(\frac{\theta}{u_e} \frac{du_e}{dx}\right)_R = \left(\frac{d\bar{u}_e}{d\bar{x}}\right)_R \cdot \frac{\bar{\theta}_R}{\bar{u}_{eR}}, \dots\dots\dots (21)$$

so that from equations 20 and 21,

$$\bar{\theta}_R = -\Lambda_R \cdot \frac{\bar{u}_{eR} \bar{\ell}_2}{(1 - \bar{u}_{eR})} \dots\dots\dots (22)$$

Also the external velocity distribution between T and R is

$$\bar{u}_e = 1 - (1 - \bar{u}_{eR}) \left[\frac{\bar{x} - \bar{\ell}_1}{\bar{\ell}_2}\right], \dots\dots\dots (23)$$

which upon substitution into equation 19 gives

$$\bar{\theta}_R = \frac{1}{\bar{u}_{eR}^3} + \frac{C_{dm}/H_{\epsilon m}}{\bar{u}_{eR}^3} \int_{\bar{x}_1}^{\bar{\ell}_1 + \bar{\ell}_2} \left[1 - (1 - \bar{u}_{eR}) \left(\frac{\bar{x} - \bar{\ell}_1}{\bar{\ell}_2}\right)\right]^3 \cdot d\bar{x} \dots\dots (24)$$

On integration this gives

$$\bar{\theta}_R = \frac{1}{\bar{u}_{eR}^3} + \frac{C_{d_m}}{4H_{\epsilon_m}} \cdot \frac{\bar{l}_2 (1 - \bar{u}_{eR}^4)}{\bar{u}_{eR}^3 (1 - \bar{u}_{eR})} \dots \dots \dots (25)$$

Combining equation 22 and 25 leads to a relationship between \bar{u}_{eR} and \bar{l}_2 :-

$$\bar{u}_{eR}^4 = \frac{\frac{C_{d_m}}{4H_{\epsilon_m}} + \frac{(1 - \bar{u}_{eR})}{\bar{l}_2}}{\frac{C_{d_m}}{4H_{\epsilon_m}} - \Lambda_R} \dots \dots \dots (26)$$

Equation 26 therefore provides a relationship between the ratios of the external velocities at re-attachment and separation, $\bar{u}_{eR} = u_{eR}/u_{eS}$, and the non-dimensional length of turbulent separated flow, $\bar{l}_2 = l_2/\theta_S$.

A more familiar parameter than \bar{u}_{eR} for expressing the velocity drop over a bubble is Crabtree's¹⁷ parameter σ , where

$$\sigma = \frac{P_R - P_S}{\frac{1}{2}\rho u_{eS}^2} = 1 - \left(\frac{u_{eR}}{u_{eS}}\right)^2 = 1 - \bar{u}_{eR}^2 \dots \dots \dots (27)$$

Fig. 7 shows the calculated variation of σ with \bar{l}_2 , together with experimental results of McGregor¹, Gaster² and Woodward³. The three curves correspond to calculations with various values of C_{d_m} ; the curve for $C_{d_m} = 0.0182$, which is the value for a turbulent half-jet with Liepmann and Laufer's²² value of spread parameter $\sigma_S = 11$, correlates the results more satisfactorily than the other values, and hence this value will be adopted in the ensuing theory.

By making the substitutions

$$\left. \begin{aligned} \bar{l}_2 &= \bar{l} - \bar{l}_1 \\ \text{and } \bar{l}_1 &= \frac{4 \times 10^4}{R\theta_S} \end{aligned} \right\} \dots\dots\dots (28)$$

in equation 26, we can obtain the relationship between the bubble length \bar{l} , the velocity ratio \bar{u}_{eR} and the separation Reynolds number $R\theta_S$. Now since S and R lie on the inviscid velocity distribution, there exists an additional relationship between \bar{l} and \bar{u}_{eR} . Also, $\theta_S = f_n(R\theta_S)$, so that the variation of \bar{l} and \bar{u}_{eR} with $R\theta_S$ may be determined uniquely.

Making the substitution 28 in equation 26 we get

$$\bar{u}_{eR}^4 = \frac{C_d/4H_\epsilon + \frac{(1 - \bar{u}_{eR})}{\bar{l} - \frac{4 \times 10^4}{R\theta_S}}}{C_d/4H_\epsilon - \Lambda_R} \dots\dots\dots (29)$$

The curves of \bar{l} against \bar{u}_{eR} for various $R\theta_S$ so determined may be described as loci of possible re-attachment points.

As an example let us find the growth behaviour for a linear inviscid velocity distribution given by

$$\bar{u}_e = 1 - x/c, \text{ when } c \text{ is a reference length, or,}$$

putting $x^* = x/c$, $\bar{u}_e = 1 - x^*$,(30)

Assume for the sake of argument that

$$\left(\frac{\theta_S}{c}\right)^2 \cdot \frac{u_{eS}c}{v} = 0.1,$$

$$\text{i.e.} \quad R_{\theta_S} = \frac{0.1}{\theta_S^*},$$

$$\text{where } \theta_S^* = \theta_S/c.$$

The result is shown in Fig. 8a, loci of possible points of re-attachment being shown for values of R_{θ_S} between 160 and 250. The points of intersection of these loci with the inviscid velocity distribution determine the re-attachment points. It will be seen that a progressive expansion in the bubble with reduction of R_{θ_S} occurs until the curves become tangential at $R_{\theta_S} = 175$; below this Reynolds number re-attachment is impossible, and we may associate this with the bursting condition. As a result of this tangency condition, the growth rate with reduction of R_{θ_S} at bursting becomes infinite, as shown in Fig. 8b.

Let us extend the above analysis to the case of a general linear imposed velocity distribution; since Gaster has found that a good correlation exists at bursting between R_{θ_S} and a parameter P expressing the average velocity gradient over the bubble, we may expect that such a linearised approach will be a good approximation for most bubbles. For this linear type of velocity distribution, the bursting condition is that the line joining the points of separation and re-attachment in the $\bar{u}_e \sim \bar{x}$ plane becomes tangential to the locus of possible points of re-attachment. This leads to the result that at bursting

$$4B\bar{u}_{eR}^3(1 - \bar{u}_{eR})^2 = \frac{4 \times 10^4}{R_{\theta S}} (\bar{u}_{eR}^4 - C)^2 \quad \left. \vphantom{\frac{4 \times 10^4}{R_{\theta S}}} \right\} \dots\dots\dots (31)$$

where $B = \frac{1}{C_d/4H_\epsilon - \Lambda_R}$, $C = \frac{C_d/4H_\epsilon}{C_d/4H_\epsilon - \Lambda_R}$.

Substitution of the resulting values of \bar{u}_{eR} at bursting, for various $R_{\theta S}$, into equation 29 then gives the non-dimensional bubble length at bursting. Hence the values of Gaster's parameter

$$P = \frac{R_{\theta S} \cdot (\bar{u}_{eR} - 1)}{\bar{l}} \quad \dots\dots\dots (32)$$

may be calculated. Also values of Crabtree's parameter $\sigma = 1 - \bar{u}_{eR}^2$ may be obtained. The growth curves \bar{l} against $R_{\theta S}$ at constant P are shown in Fig. 9; it will be seen that at bursting $d\bar{l}/dR_{\theta S}$ is infinite.

Comparisons of the resulting theoretical curves of σ , \bar{l} and P against $R_{\theta S}$ at bursting with experimental results are shown in Figs. 10, 11 and 12. The agreement with experiment is quite good, some of the scatter of points being attributable to departures from linearity of the imposed velocity distributions, and some to inaccuracies in the formula used to predict the length of laminar flow. The theoretical curve of \bar{l}_B against $R_{\theta S}$ follows the curve $\bar{l}_B = 6 \times 10^4/R_{\theta S}$ quite closely, in quite good agreement with the curve $\bar{l}_B = 6.4 \times 10^4/R_{\theta S}$ suggested by Young¹⁸ to be the best curve through the experimental points. The value of Crabtree's parameter σ at bursting is found to be only weakly dependent on $R_{\theta S}$, varying between the limits 0.27 to 0.36 over the range $100 < R_{\theta S} < 500$; this compares

favourably with Crabtree's¹⁷ suggested constant value of 0.35. As shown in Fig. 13, the predicted value of σ_B is not particularly dependent upon the value of the constant in the formula defining the length of separated laminar flow, and the shape of the loci of possible re-attachment points is such that quite large departures from non-linearity of the imposed velocity distribution cause only small changes in σ_B . Thus for most purposes Crabtree's hypothesis that σ_B is a constant appears to be quite a good approximation; however, in order to find σ for a given velocity distribution, it is necessary to know the bubble length and this quantity is strongly dependent upon the length of laminar flow (see Fig. 14) and the curvature of the velocity distribution.

From equation 26 it may be seen that as \bar{x}_2 tends to infinity, \bar{u}_{eR} tends to a minimum value given by

$$\bar{u}_{eR \min}^4 = \frac{C_d/4H_\epsilon}{C_d/4H_\epsilon - \Lambda_R},$$

which leads to a maximum attainable value of $\sigma_{\max} = 0.48$. That part of the analysis leading to this result may be considered to be equally applicable to long as well as short bubbles, so this limiting value of σ may be expected to relate to long bubble separations. The value is close to that derived for long bubbles by Norbury & Crabtree¹⁹.

6. Discussion

The good qualitative and fair quantitative agreement with experiment of the present simple approach strongly indicates certain essential features of the mechanism of bursting. This may be stated in the following terms. The total velocity drop along the turbulent shear layer is related to the length of turbulent separated flow, and these two quantities are dictated by the length of laminar flow and the imposed velocity distribution; bursting occurs when expansion of the turbulent part of the shear layer with decrease of Re_S cannot supply a sufficient pressure rise (velocity drop) to satisfy the requirements of the imposed velocity distribution whilst at the same time attaining the requisite value of the re-attachment parameter.

The most important aspect of the present analysis is the correlation between \bar{l}_2 and σ , from which the growth and bursting theory immediately follows. The satisfactory prediction of this correlation by means of the re-attachment criterion lends additional support to the utility of the criterion.

The lack of sensitivity of σ_B to both Reynolds number and length of laminar flow indicates that the pressure rise over the bubble is the major factor determining bursting.

7. Conclusions

- (1) A simple criterion of the form

$$\left(\frac{\theta}{u_e} \frac{du_e}{dx} \right)_R = -0.0082$$

appears sufficient to determine under what conditions a turbulent shear layer will re-attach.

(2) There exists a correlation between the non-dimensional length of laminar separated flow, $\bar{\ell}_1 = \ell_1/\theta_S$, and R_{θ_S} such that

$$\bar{\ell}_1 = \frac{4 \times 10^4}{R_{\theta_S}} .$$

(3) There exists a correlation between the non-dimensional length of turbulent separated flow, $\bar{\ell}_2 = \ell_2/\theta_S$ and the ratio of the external velocities at re-attachment and transition, u_{eR}/u_{eT} ; and hence between $\bar{\ell}_2$ and σ .

(4) Bubble growth and bursting behaviour may be predicted to a reasonable degree of accuracy by making use of conclusions 2 and 3.

(5) The value of Crabtree's parameter σ at bursting varies only slightly with R_{θ_S} , between limits 0.27 and 0.36.

(6) The non-dimensional bubble length at **bursting** (for linear inviscid velocity distributions) may be approximated by the curve $\bar{\ell} = 6 \times 10^4/R_{\theta_S}$.

(7) Bubble bursting occurs as a fundamental breakdown of the re-attachment process.

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The measurements in swept separation bubbles used herein are described in Part II of this thesis.

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SYMBOLS NOT DEFINED IN THE TEXT

- x, z Co-ordinates measured along and normal to the aerofoil surface
- $l = x_R - x_S$ Total length of bubble
- $l_1 = x_T - x_S$ Length of laminar separated flow
- $l_2 = x_R - x_T$ Length of turbulent separated flow
- p Static pressure
- u Streamwise velocity
- $R_\theta = \frac{\theta u_e}{\nu}$ Reynolds number based on momentum thickness
- $R_{\delta^*} = \frac{\delta^* u_e}{\nu}$ Reynolds number based on displacement thickness
- ν Kinematic viscosity
- ρ Density
- τ_w Wall shear stress
- $\Lambda = \left(\frac{\theta}{u_e} \frac{du_e}{dx} \right)$ Pressure gradient parameter
- Denotes lengths and velocities non-dimensionalised by θ_S and u_{eS} respectively.
- β Geometric parameter in swept bubble experiments.

Suffices

- B denotes conditions at bursting
- e denotes conditions at the edge of the viscous layer
- S, T, R denotes conditions at the points of separation, transition and re-attachment respectively
- ∞ denotes conditions in the undisturbed stream.

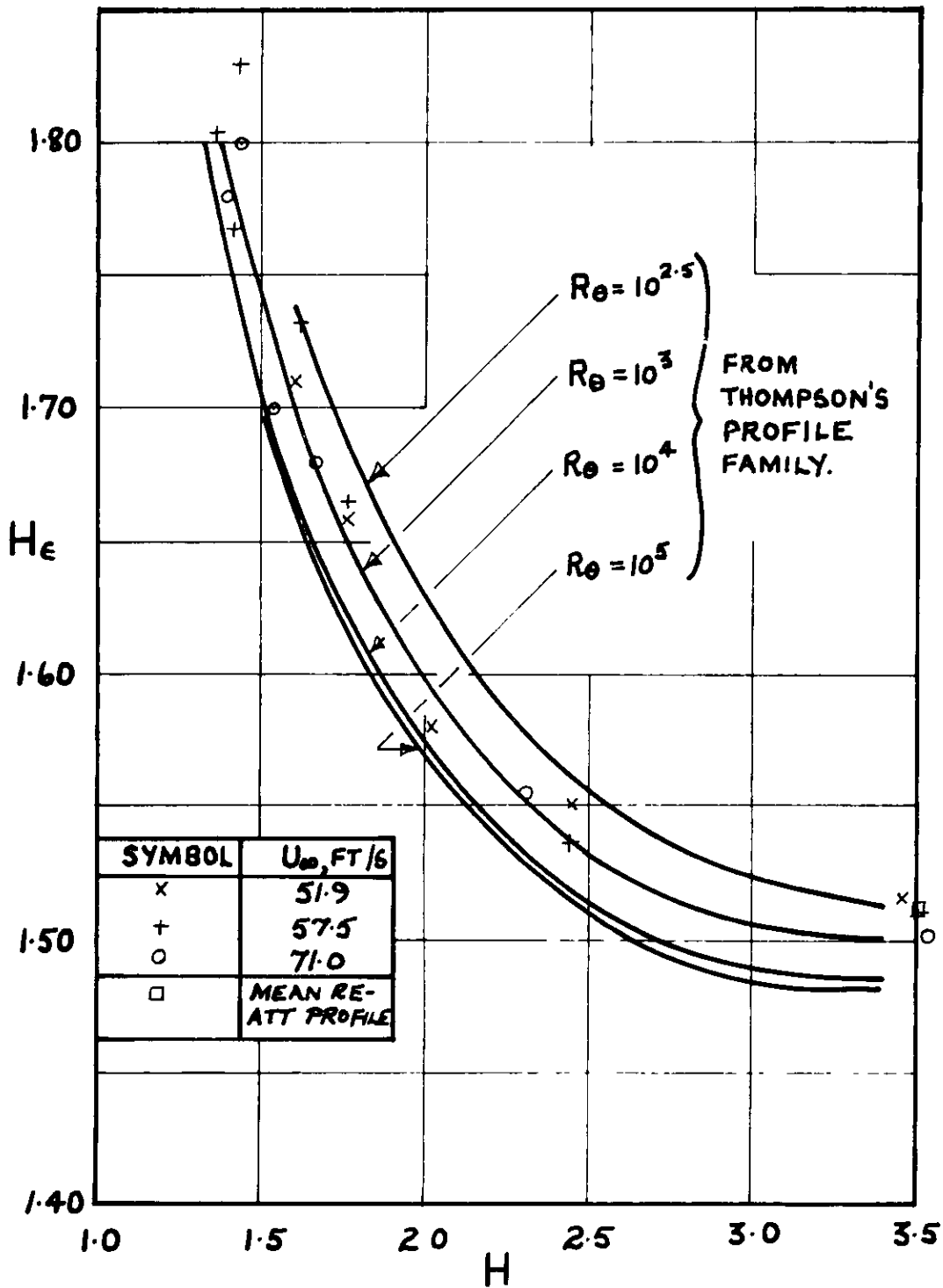


FIG.1.

VARIATION OF ENERGY SHAPE PARAMETER H_e WITH CONVENTIONAL SHAPE PARAMETER H ACCORDING TO THOMPSON, TOGETHER WITH MEASURED VALUES IN 3-D RE-DEVELOPING LAYERS. THE SHAPE PARAMETERS ARE STREAMWISE VALUES, AND VALUES OF Re ARE $O(10^3)$.

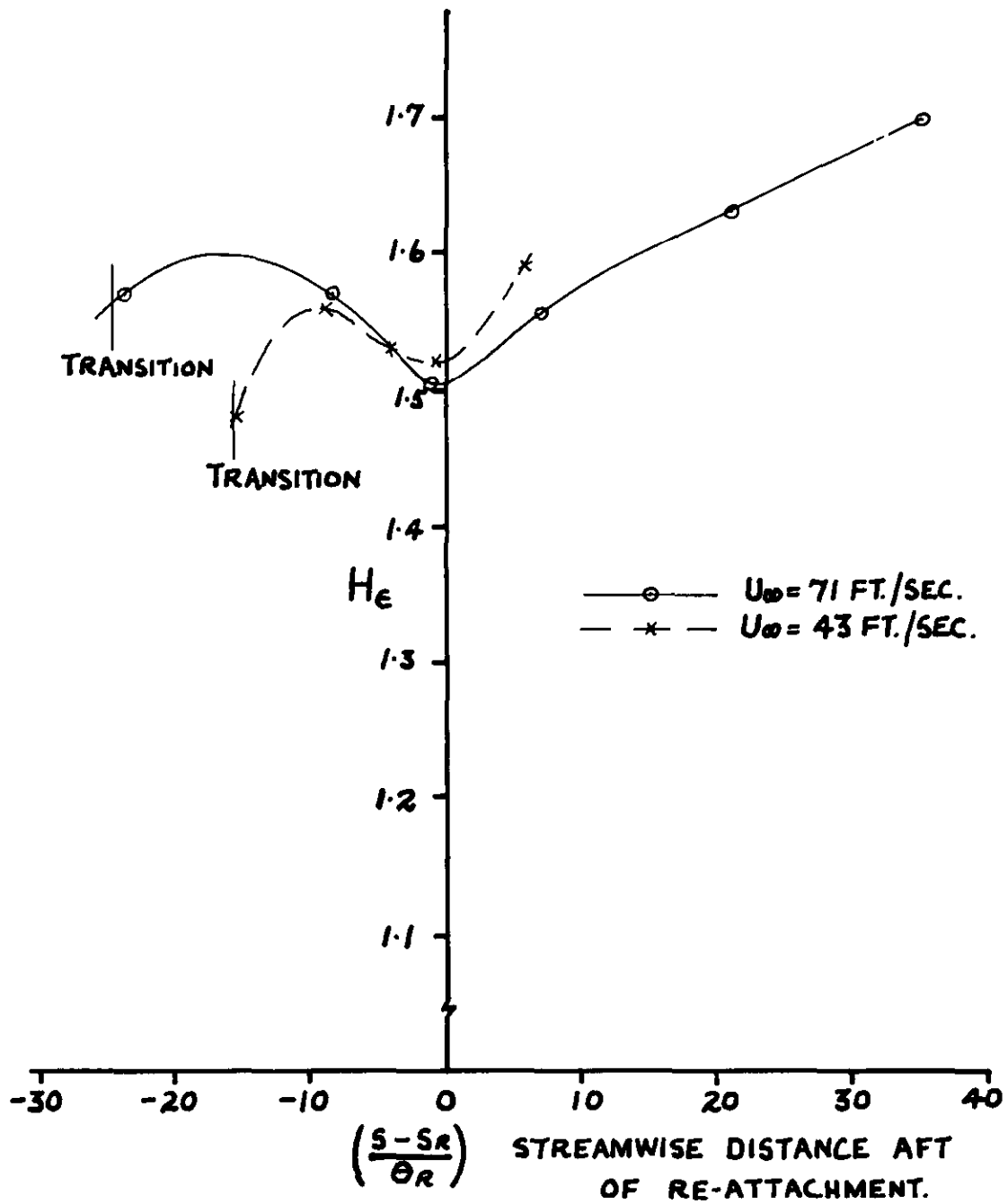


FIG. 2.

VARIATION OF STREAMWISE ENERGY SHAPE PARAMETER IN THE VICINITY OF RE-ATTACHMENT FOR TWO SWEEP SEPARATION BUBBLES.

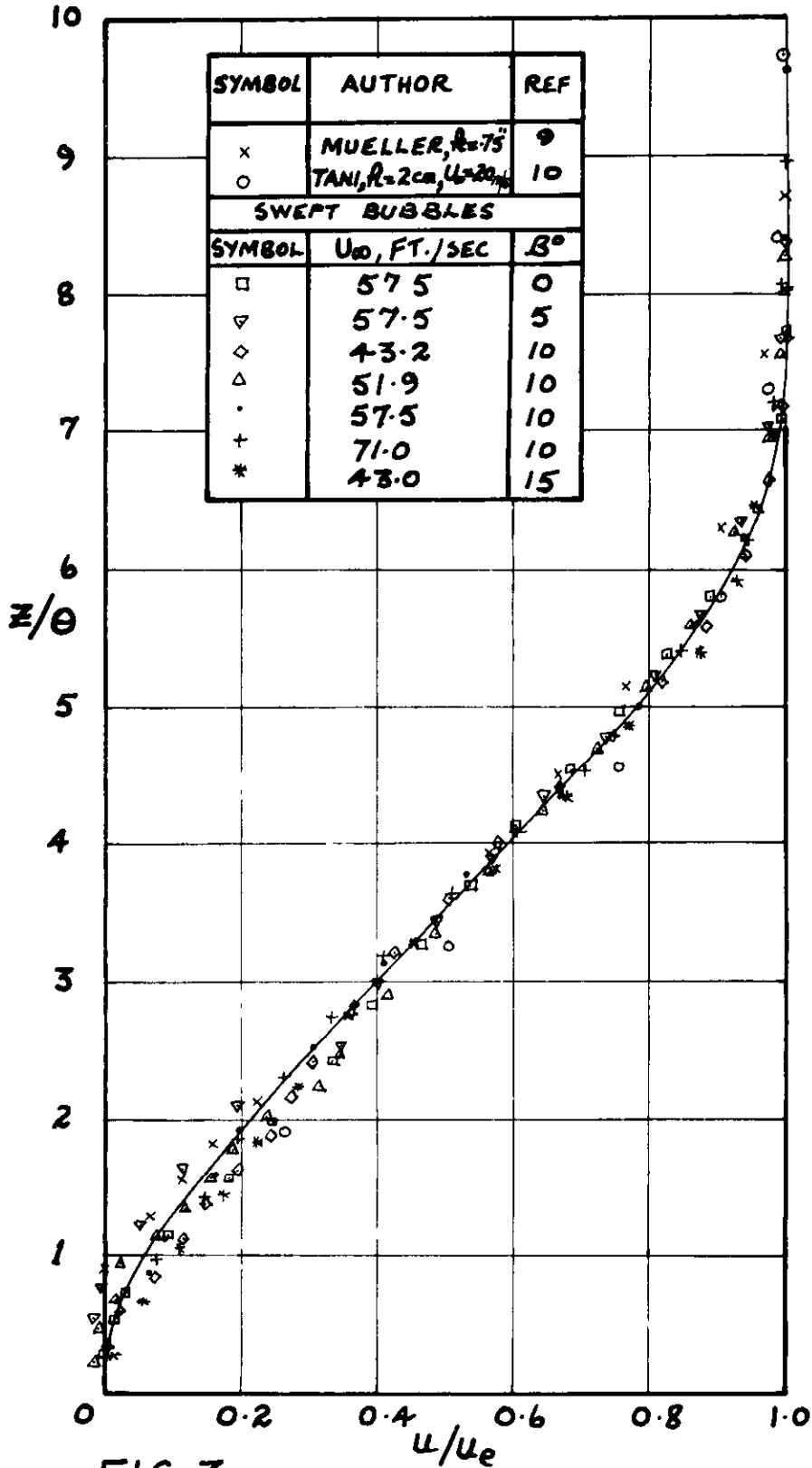


FIG. 3.

COMPARISON OF RE-ATTACHMENT PROFILES.
 (FOR THE MEAN PROFILE $H=3.5$, $H_1=4.0$, $H_c=1.51$, $\delta/\theta=7.5$).

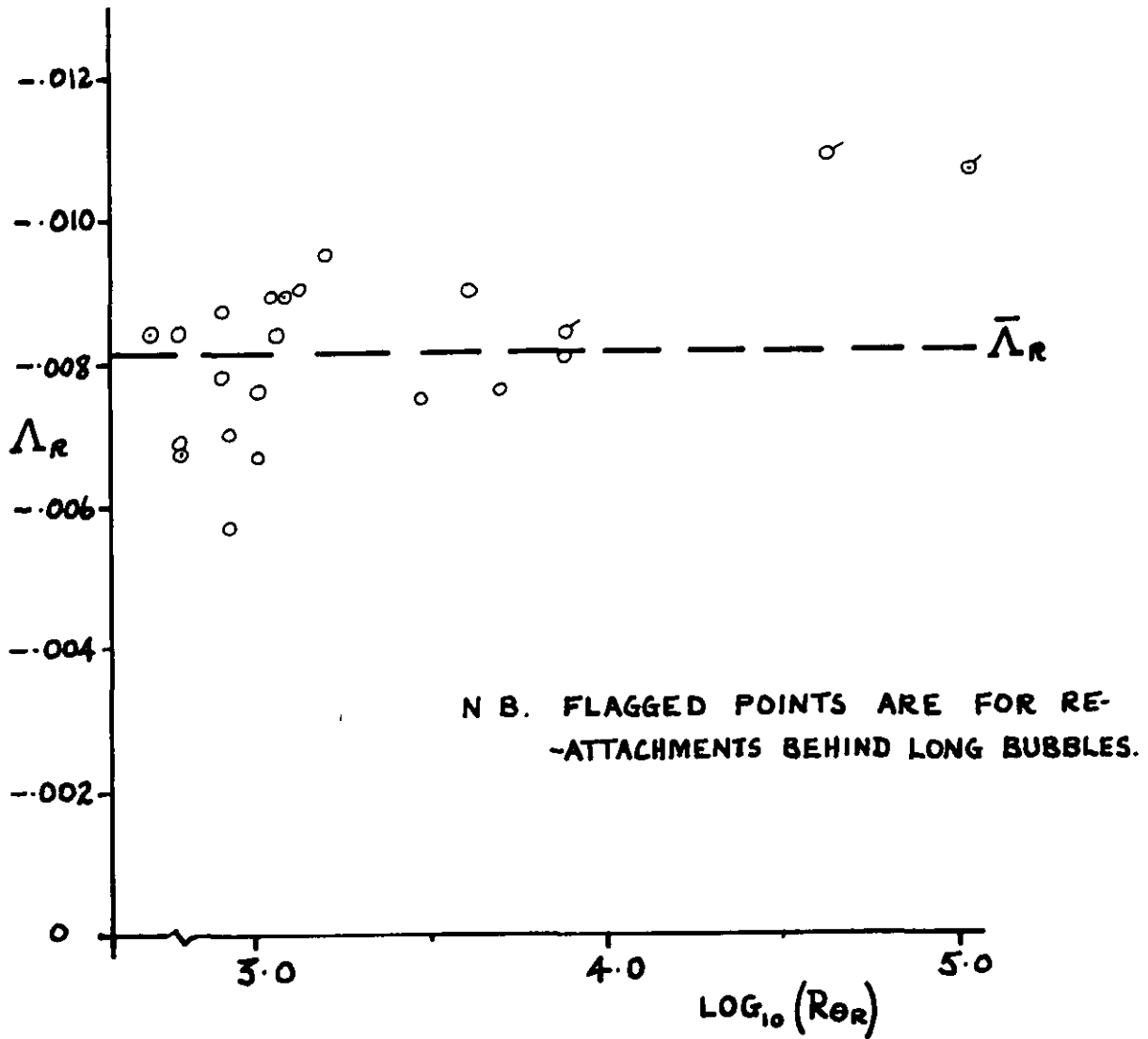


FIG. 4.

EXPERIMENTAL DETERMINATIONS OF THE PRESSURE GRADIENT PARAMETER Λ AT RE-ATTACHMENT.

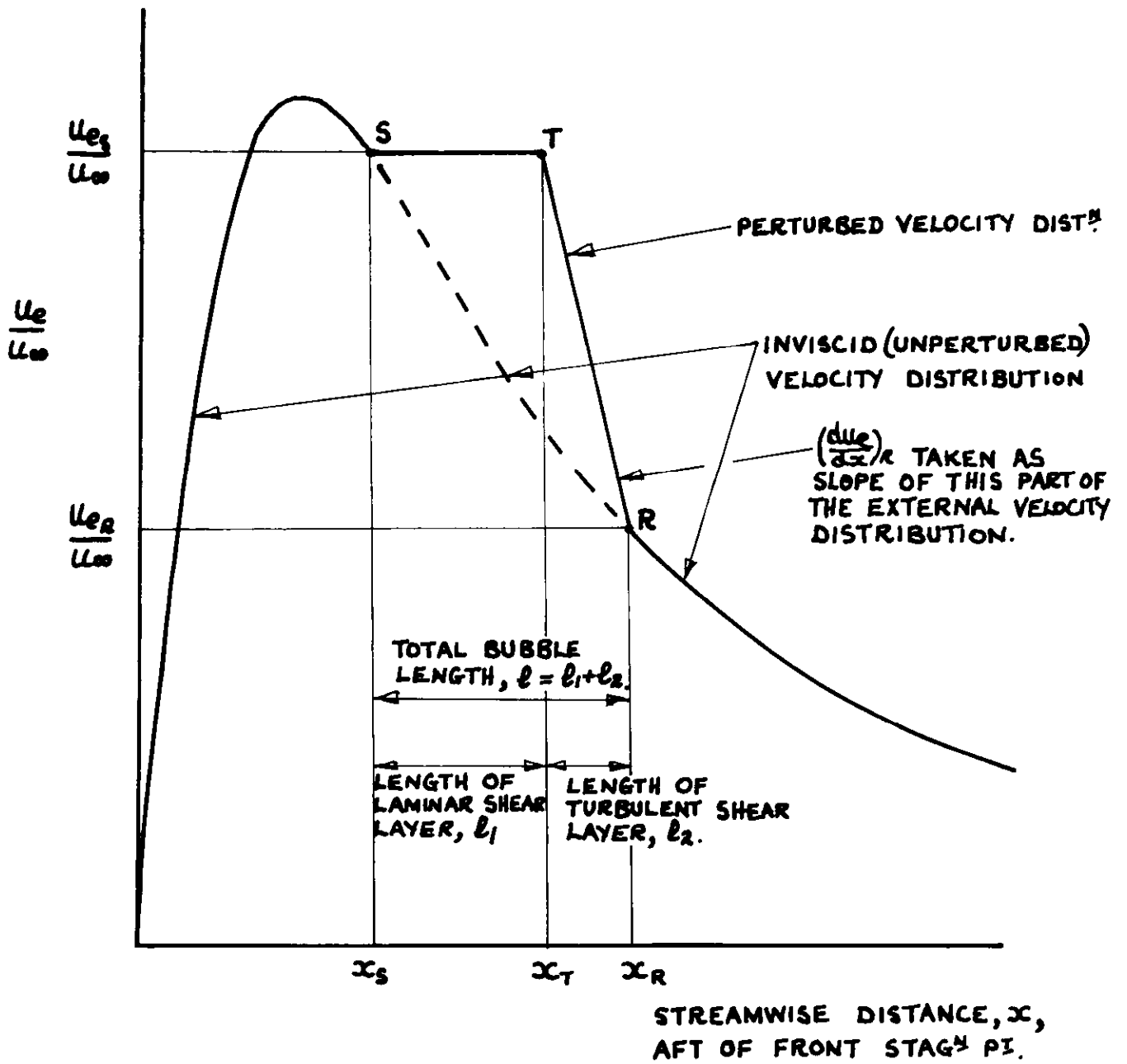


FIG.5.

SIMPLIFIED MODEL OF SHORT LAMINAR SEPARATION BUBBLE.

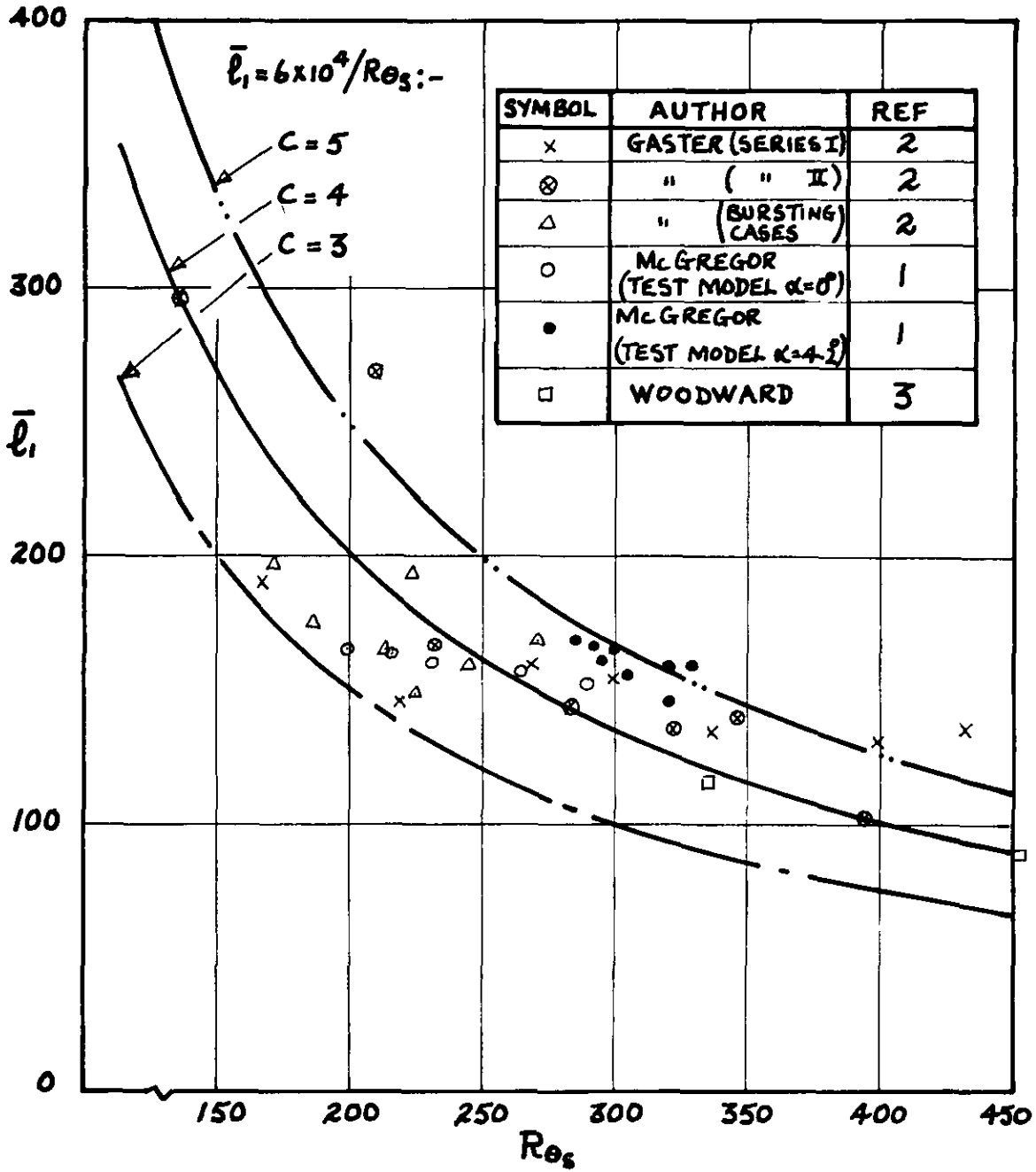


FIG. 6.

VARIATION OF NON-DIMENSIONAL LENGTH OF SEPARATED LAMINAR FLOW WITH SEPARATION REYNOLDS NUMBER.

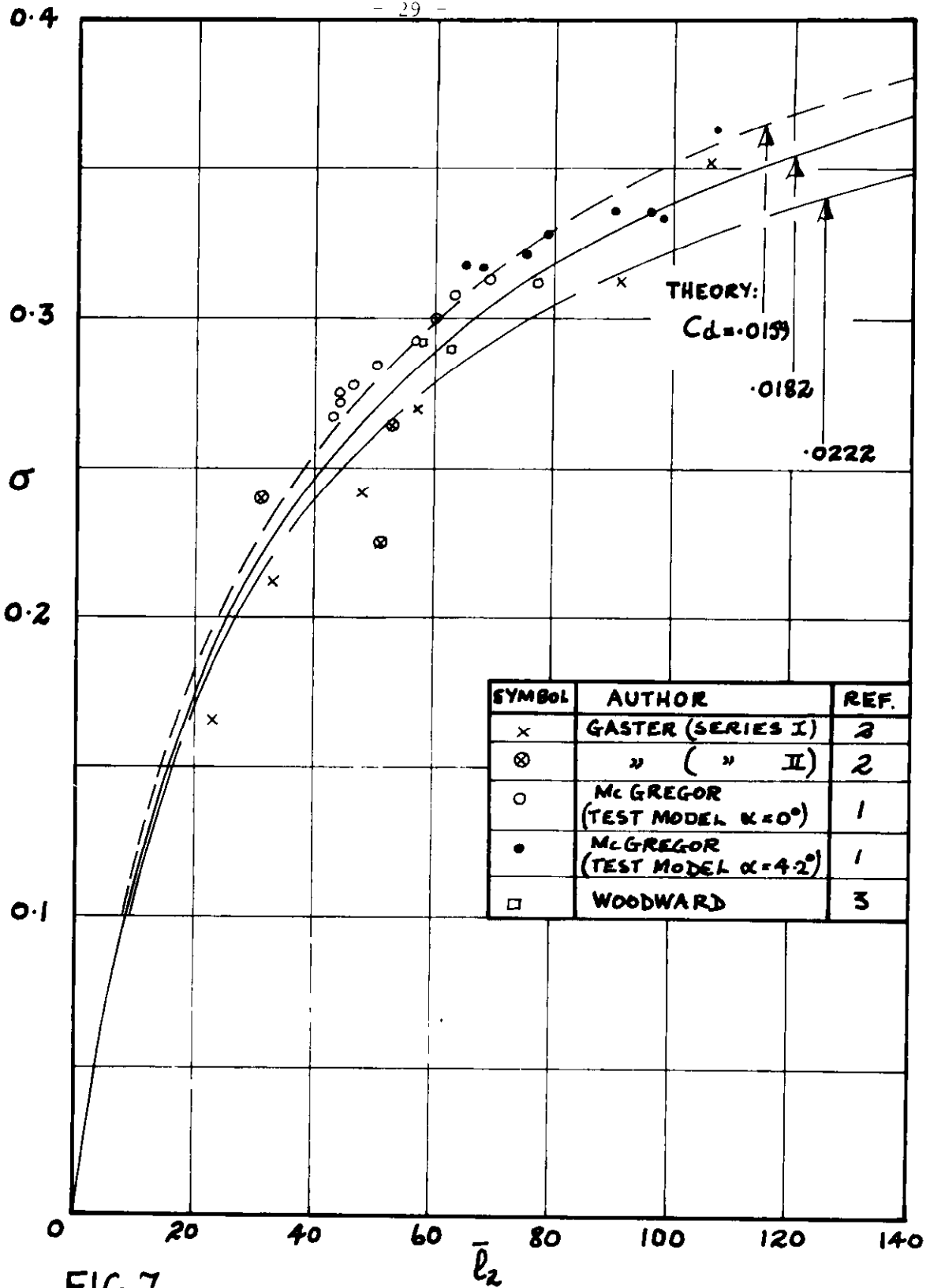


FIG.7

COMPARISON OF CALCULATED AND MEASURED VARIATION OF PRESSURE RECOVERY PARAMETER WITH LENGTH OF TURBULENT PART OF THE BUBBLE.

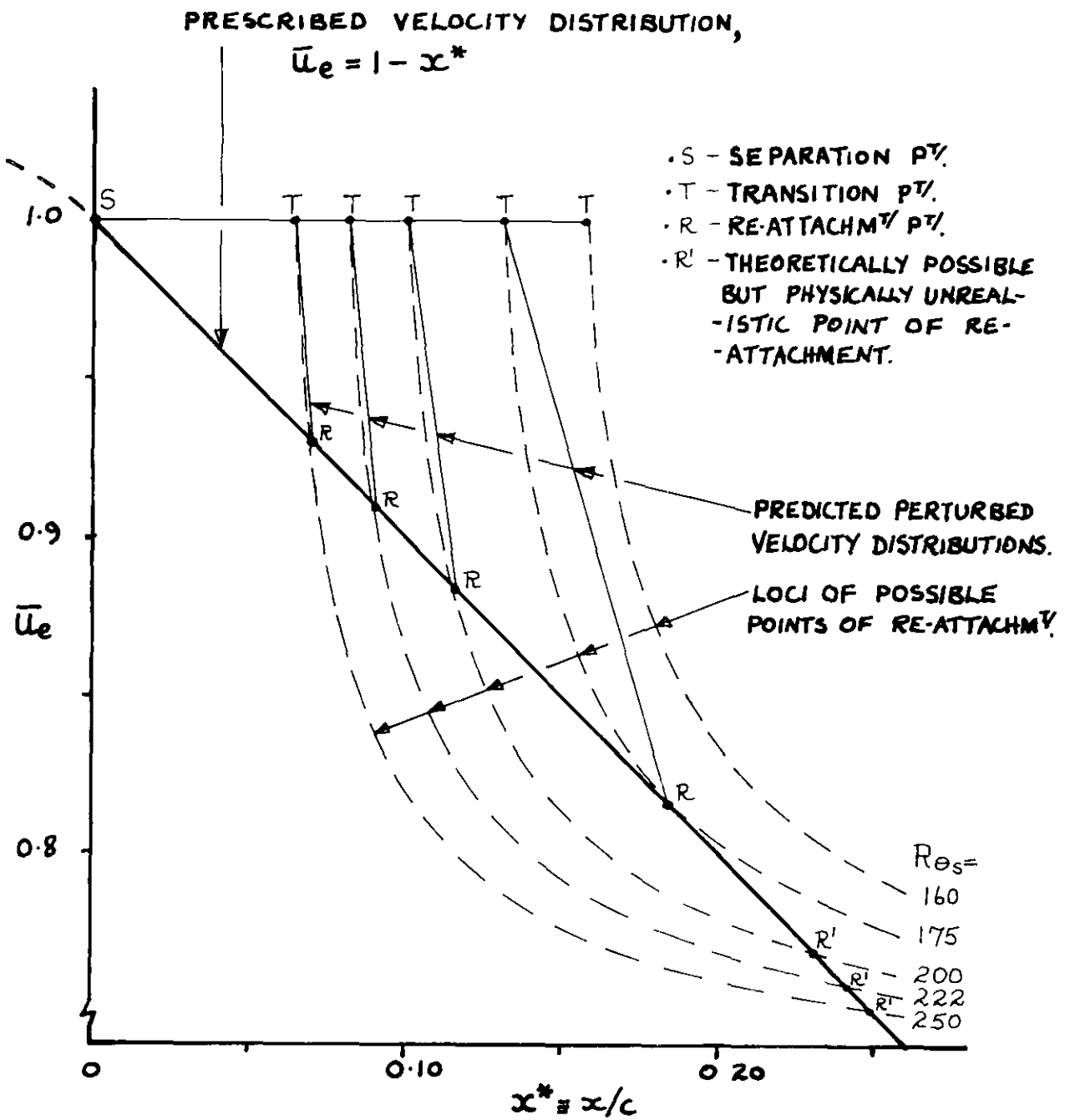


FIG. 8a

EXAMPLE OF GRAPHICAL PREDICTION OF BUBBLE GROWTH AND BURSTING.

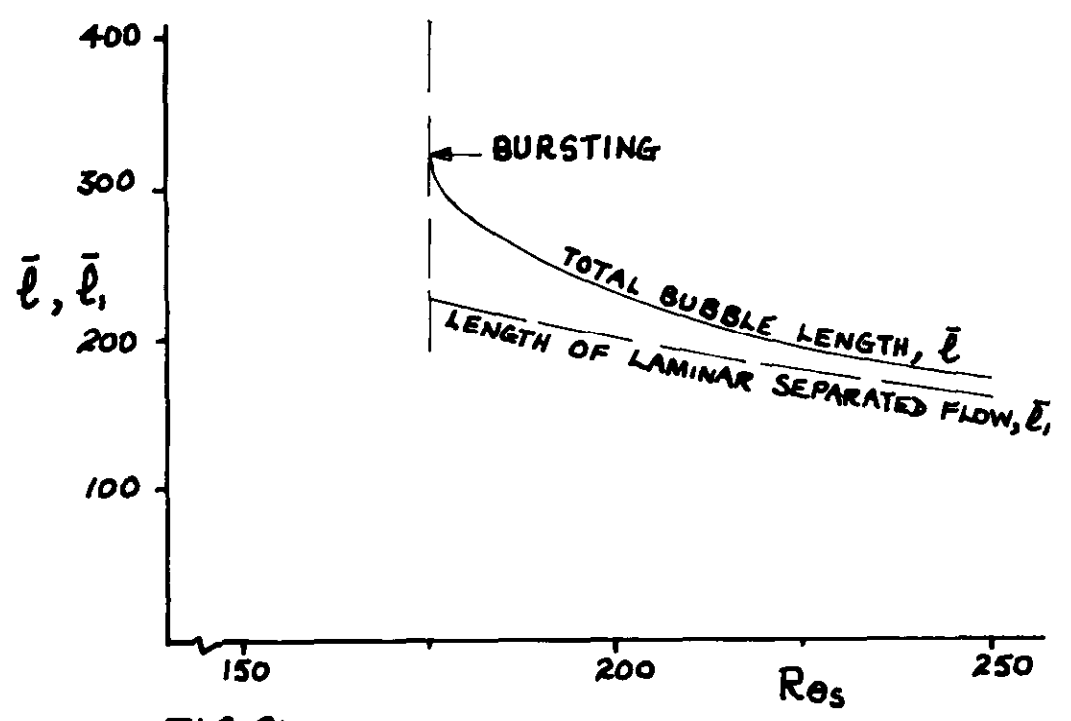
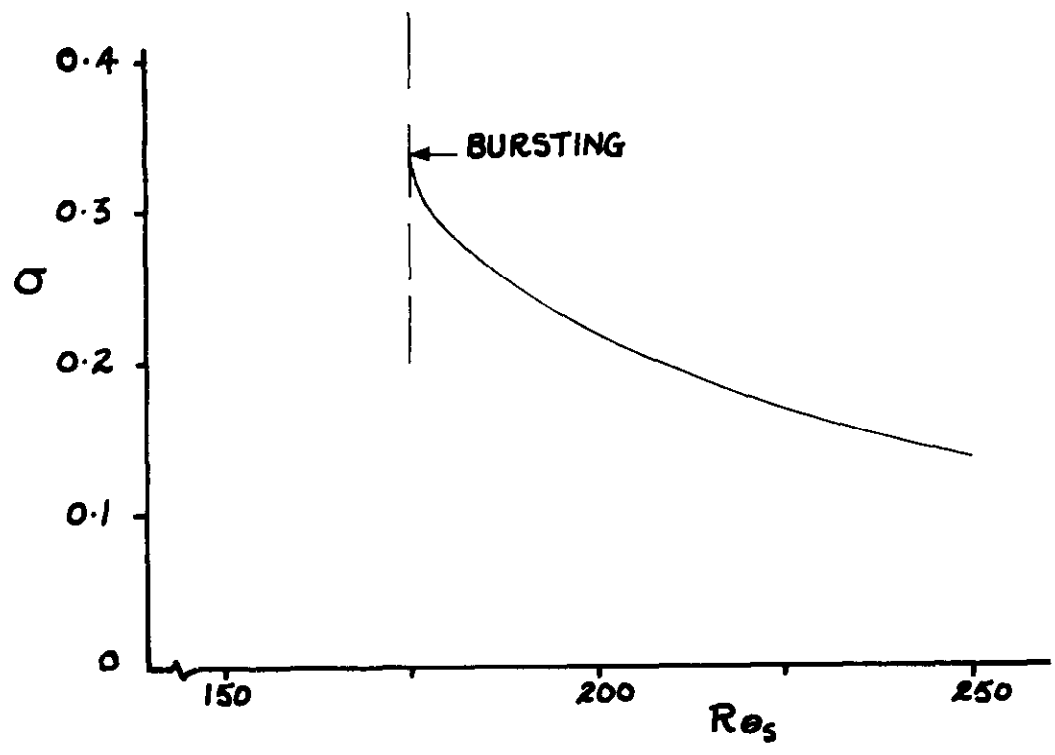


FIG. 8b.
VARIATION OF CRABTREE'S PRESSURE RECOVERY PARAMETER, AND OF TOTAL BUBBLE LENGTH, WITH SEPARATION REYNOLDS NUMBER, FOR THE GRAPHICAL EXAMPLE.

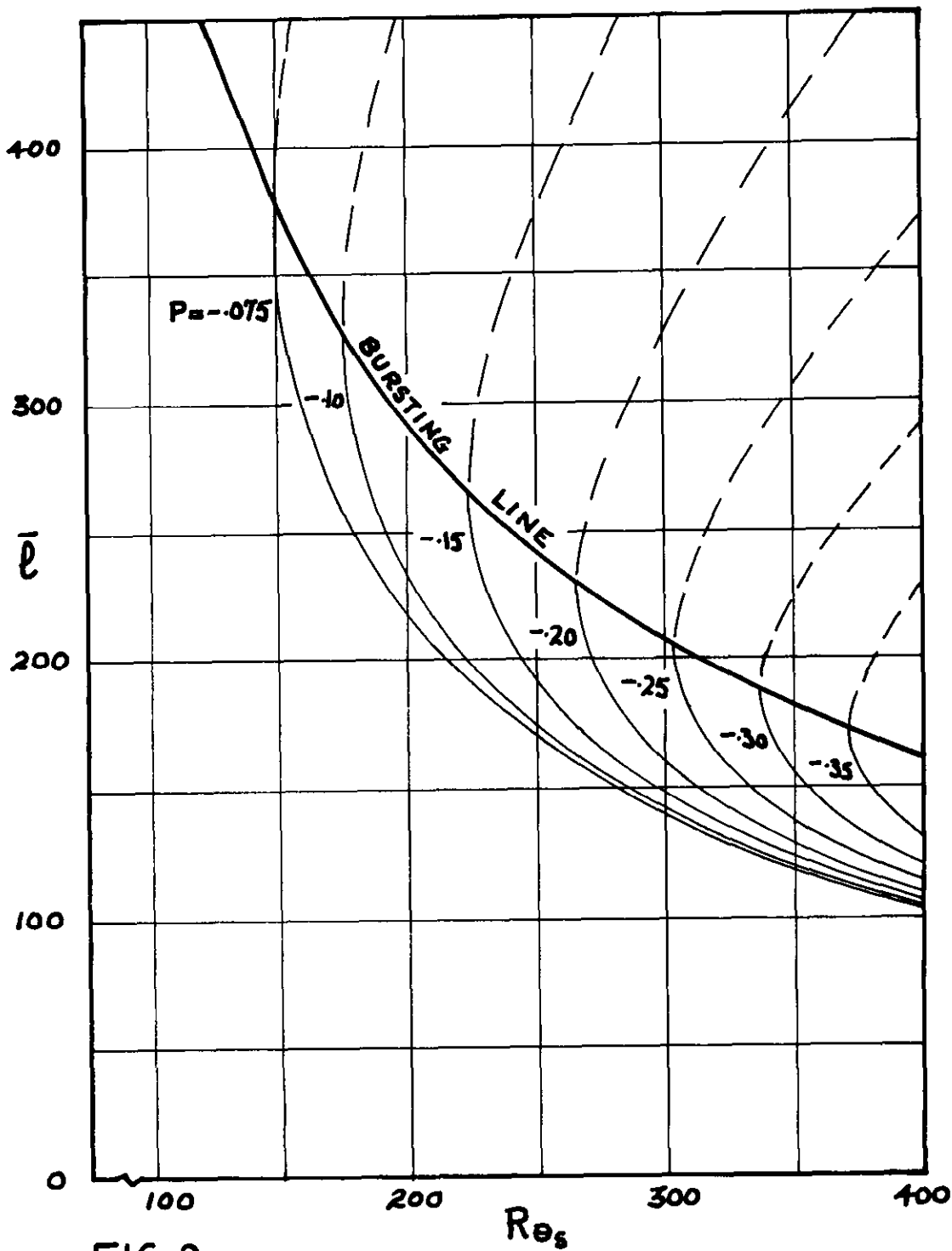


FIG. 9.

BUBBLE GROWTH LINES AT CONSTANT P.

- GROWTH LINES CORRESPONDING TO PHYSICALLY REALISTIC SOLUTION.
- - - PHYSICALLY UNREALISTIC GROWTH LINES.

SYMBOL	AUTHOR	REF.
x	GASTER	2
o	MCGREGOR	1
□	WOODWARD	3
△	CRABTREE	17
+	MCCULLOUGH AND GAULT	20,21

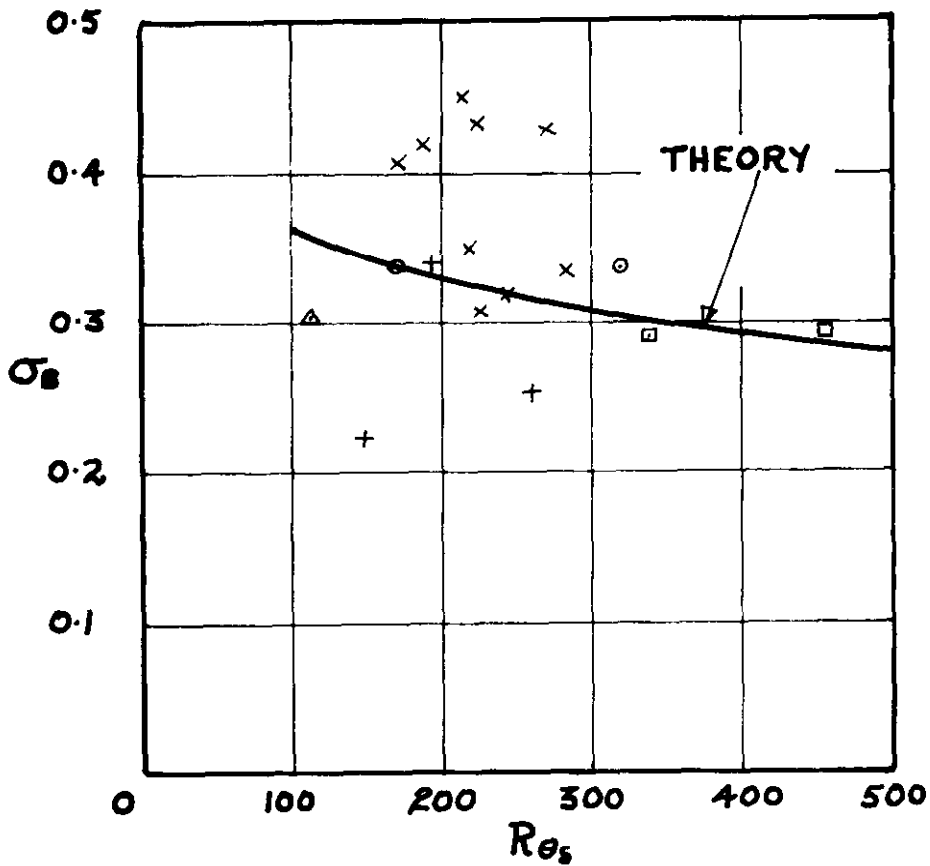


FIG. 10.

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL
PRESSURE RECOVERY PARAMETERS AT BURSTING.

FOR LEGEND SEE FIG. 10.

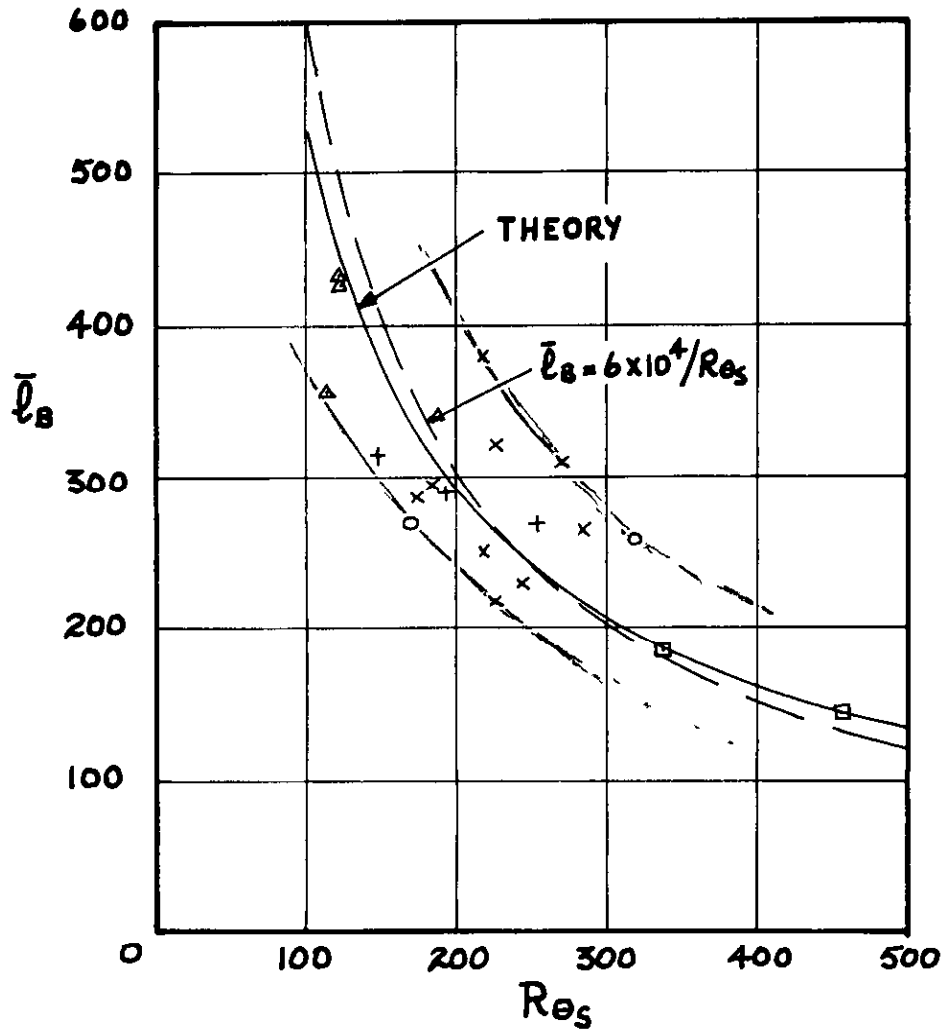


FIG. II.

COMPARISON OF THEORETICAL AND EXPERIMENTAL BUBBLE LENGTHS AT BURSTING.

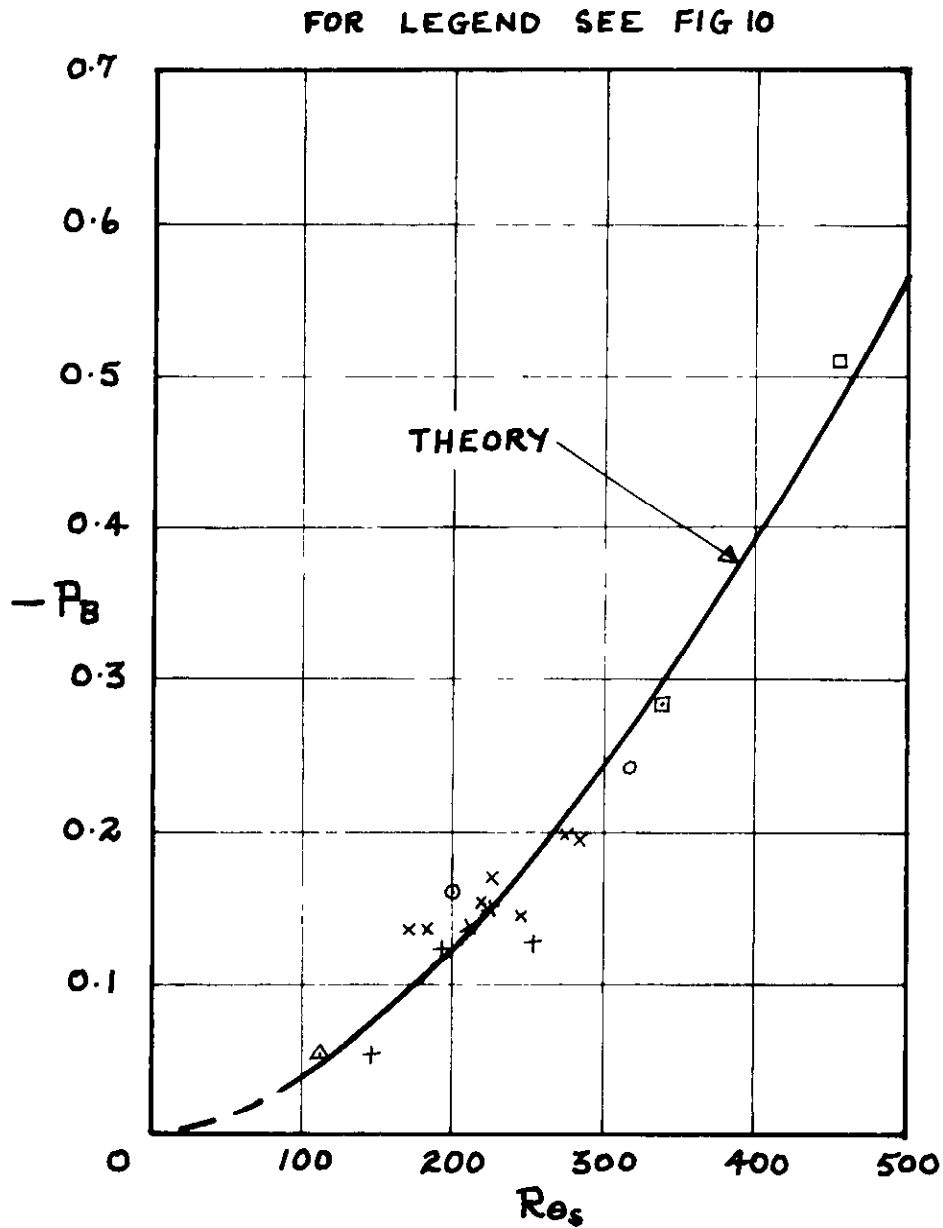


FIG.12.

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES OF GASTER'S PARAMETER P AT BURSTING.

(EXPERIMENTAL VALUES OF P CALCULATED USING MEASURED VALUES OF U_{er} AND U_{es}).

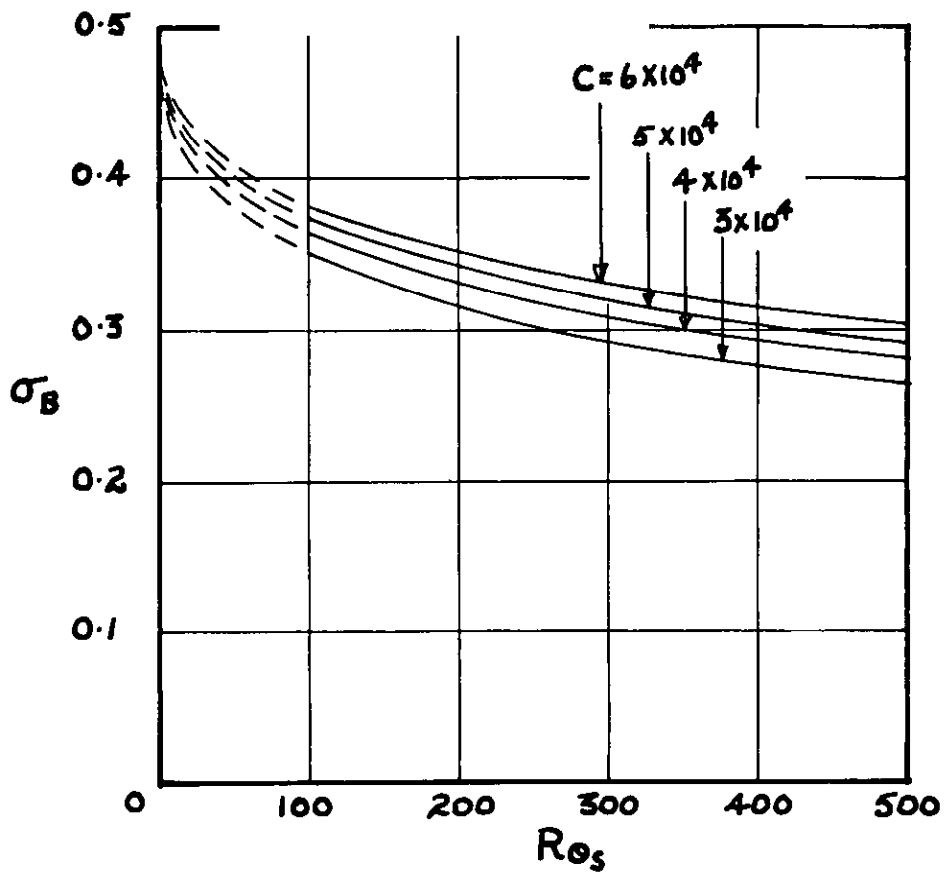


FIG.13.

EFFECT OF VARIATION OF TRANSITION LAW UPON THE PRESSURE RECOVERY PARAMETER AT BURSTING.

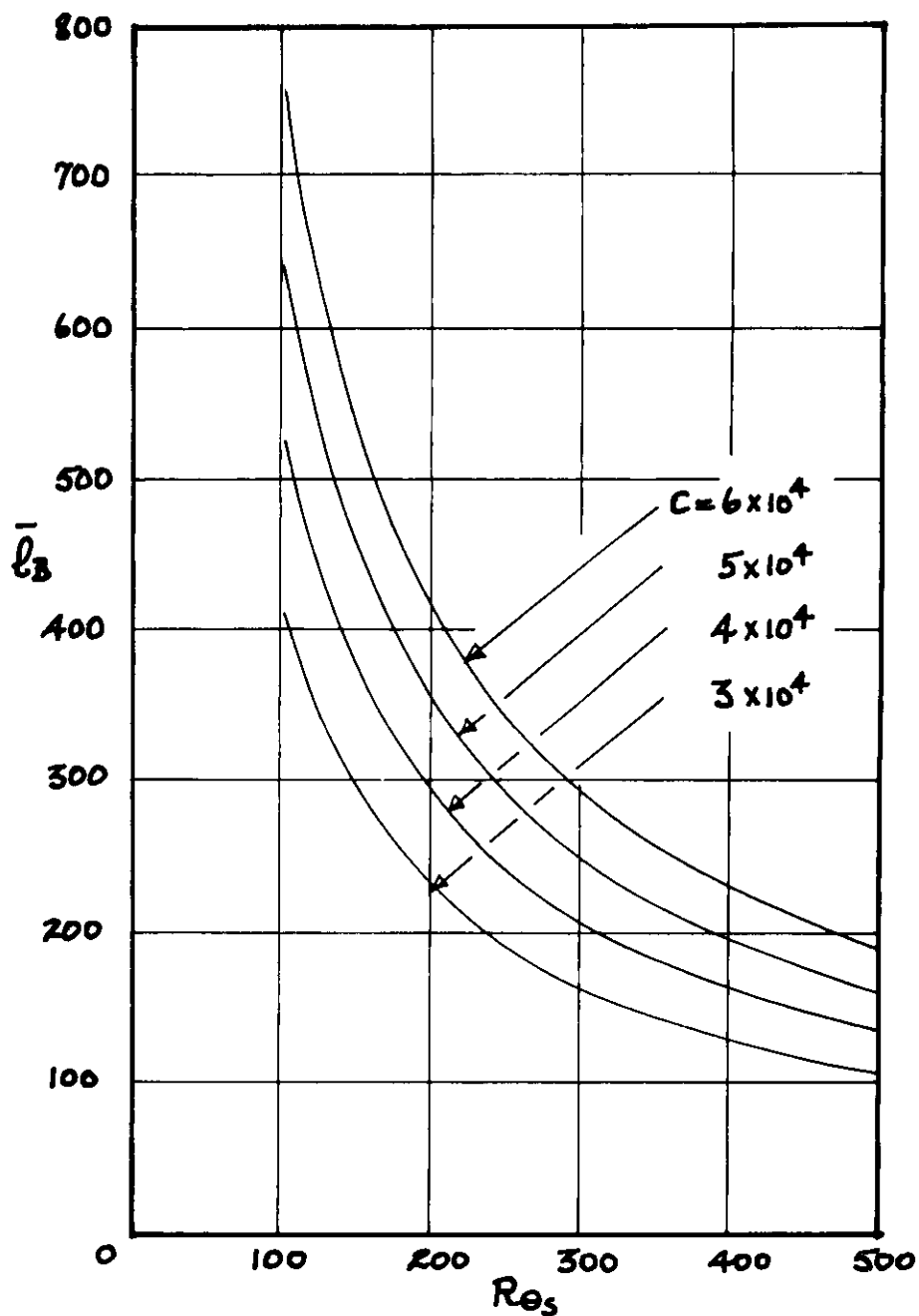


FIG.14.

EFFECT OF VARIATION OF TRANSITION LAW UPON BUBBLE LENGTH AT BURSTING.

<p>A.R.C. C.P. 1073 June 1967</p> <p>Horton, H. P.</p> <p>A SEMI-EMPIRICAL THEORY FOR THE GROWTH AND BURSTING OF LAMINAR SEPARATION BUBBLES</p> <p>A simple pressure gradient criterion for the determination of the conditions under which re-attachment of a turbulent shear layer can occur is proposed. Application of this criterion to the laminar separation bubble problem, together with a simple bubble model and an approximate method of calculation of the momentum thickness growth over the bubble, leads to a method of prediction of the bubble growth. It is found that for a given imposed pressure distribution there</p> <p>(Over)</p>	<p>A.R.C. C.P. 1073 June 1967</p> <p>Horton, H. P.</p> <p>A SEMI-EMPIRICAL THEORY FOR THE GROWTH AND BURSTING OF LAMINAR SEPARATION BUBBLES</p> <p>A simple pressure gradient criterion for the determination of the conditions under which re-attachment of a turbulent shear layer can occur is proposed. Application of this criterion to the laminar separation bubble problem, together with a simple bubble model and an approximate method of calculation of the momentum thickness growth over the bubble, leads to a method of prediction of the bubble growth. It is found that for a given imposed pressure distribution there</p> <p>(Over)</p>
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