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Heat Transfer in a Tube Revolving about a Displaced Axis

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Heat Transfer in a Tube Revolving about a Displaced Axis

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SUMMARY

Consideration is given to heat transfer and fluid flow in the entry region of a tube revolving about a displaced axis parallel to the tube axis. Experimental results indicate that both inherent swirling flow and centrifugal buoyancy contribute to increases in heat transfer as a result of rotation.

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* Replaces A.R.C.30 561

List of Symbols

A	($= H\Omega^2/g$) Acceleration ratio
d	Tube diameter
g	Acceleration due to gravity
Grg	($= g\beta d^3 \Delta t/\nu^2$) Standardised Grashof number
Grr	($= A.Grg$) Rotational Grashof number
H	Radius of revolution of test section
L	Length of test section
Nu	Nusselt number
p	Pressure
P	($= p/\rho W^2$) Non-dimensional pressure
Pr	Prandtl number
r', θ, z'	Rotating cylindrical co-ordinates
r	($= r'/d$) Dimensionless co-ordinate
z	($= z'/d$) Dimensionless co-ordinate
Re	($= Wd/\nu$) Reynolds number
S	($= H\Omega/W$) Non-dimensional parameter
t	Temperature
tw	Mean wall temperature
t_b	Mean bulk fluid temperature
Δt	($= tw - tb$) Temperature difference
T	($= t/\Delta t$) Dimensionless temperature
u', v', w'	Components of velocity
u, v, w	($= u'/W, v'/W, w'/W$) Dimensionless velocity components
W	Mean axial velocity
Ω	Angular velocity
ρ	Density
ν	Kinematic viscosity
β	Coefficient of volume expansion

Introduction/

1. Introduction

Attention is directed towards the problem of heat transfer and fluid flow in the entry region of a heated tube which revolves about a displaced and parallel axis. The idealised model is shown in Fig. 1 and experimental results for the particular case of air as coolant are presented. As the coolant flows in the tube so it experiences a centrifugal force which in the presence of density differences gives rise to secondary flow. Moreover, at the tube inlet the coolant may exhibit a swirling tendency as the fluid tries to remain irrotational, the degree of swirl depending not only on the angular velocity but also on the geometric configuration prior to the tube inlet. In turn, the heat transfer and friction characteristics of the tube are modified from the familiar stationary case.

The increasing demand for efficient cooling techniques for rotating machinery clearly indicates the practical significance of the geometry under study; as service conditions become more arduous so a more accurate understanding and prediction of heat transfer in such passages becomes more important. However, to date only a limited study has been made of the problem. Morris[1] and Mori and Nakayama[2] have made theoretical studies for the case of fully-developed laminar flow and Nakayama[3] considered the fully-developed turbulent flow case. In Refs. 1,2,3, inlet swirl is neglected and it is shown that the effect of rotation is to cause significant increases in both Nusselt number and friction factor, the effects being larger in laminar flow than in turbulent flow. On the experimental side, LeFeuvre[4] considered a system in which rotational effects were predominantly due to inlet swirl and recorded large increases in Nusselt number. For the same geometry Webb[5] recorded increases in the adiabatic pressure drop along the duct. Humphreys et al[6] presented some initial results for the present geometry in which both swirl and rotational buoyancy were important.

In the present paper a general inspection of the problem is made and experimental results are used to produce a qualitative appraisal of heat transfer in tubes rotating about a parallel axis.

2. Theoretical Aspects

A theoretical analysis of the hydrodynamic and thermal fields existing in the tube has proved impossible for even the simpler laminar flow regime. The conservation equations as applied to the problem are strictly three-dimensional and not amenable to analytical or computational solution. Nevertheless, the basic equations may be employed as a means of deriving the important non-dimensional parameters concerning the problem and at the same time give some insight into the problem. For simplicity the laminar flow equations will be used.

The radial and tangential accelerations which occur as a result of rotation are, in terms of Fig. 1 :-

$$\text{Radial:} \quad -(r' + H\cos\theta)\Omega^2 - 2\Omega v'$$

$$\text{Tangential:} \quad H\Omega^2 \sin\theta + 2\Omega u'.$$

Following Refs. 1,2,3, the Coriolis terms may be neglected and for H>>d the radial acceleration term reduces to $-H\Omega^2\cos\theta$. Further, an order-of-magnitude analysis permits the exclusion of certain other terms and considering density to be everywhere constant except in the evaluation of buoyancy terms the non-dimensionalised momentum and energy equations in cylindrical co-ordinates become:-

$u\frac{\partial u}{\partial u}$/

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial P}{\partial r} + \frac{1}{Re}\frac{\partial^2 u}{\partial r^2} + \frac{Grr}{Re^2} T \cos\theta \qquad \ldots (1)$$

$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = -\frac{\partial P}{\partial \theta} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right] - \frac{Grr}{Re^2} T \sin\theta \qquad \ldots (2)$$

$$u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] \qquad \ldots (3)$$

$$u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \theta} + w\frac{\partial T}{\partial z} = \frac{1}{RePr}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] \qquad \ldots (4)$$

Noting that the Nusselt number may be calculated from the temperature distribution then from (1), (2), (3) and (4) it is seen

$$Nu = f\,(\,Re,\ Pr,\ Grr\,).$$

Noting also that the geometry requires three characteristic lengths to specify it we may write

$$Nu = \psi(\,Re,\ Pr,\ Grr,\ L/d,\ H/d\,).$$

This equation expresses the hydrodynamic, thermal, and geometric similarity between systems with the same boundary conditions and neglecting Coriolis forces. If inlet swirl occurs to a significant extent then the boundary conditions of the test section are different at each rotational speed and consequently an additional, unaccounted for, effect is present. Further, since swirl is dependent on both upstream geometry and angular velocity the definition of a suitable parameter to describe it is difficult. However, for one intake geometry and one fluid the parameter $S = \frac{H\Omega}{W}$ may be arbitrarily chosen to be identified with swirl. Thus for a defined inlet geometry we may write:-

$$Nu = f\,(\,Re,\ Grr,\ Pr,\ S,\ L/d,\ H/d\,). \qquad \ldots (5)$$

Inspection of equation (5) reveals some interesting and far-reaching properties of this class of problem. Firstly, two geometric parameters occur in the general correlation equation and consequently changes in test section diameter, for example, cause changes in two parameters in the general correlation. Secondly, rotational speed must be catered for in two groups which cannot be varied independently without incurring changes in Reynolds number. Thus a correlation of experimental data cannot be readily obtained but rather a trial-and-error process is indicated when a sufficiently large amount of data becomes available. For present purposes the non-dimensional groups are used to obtain a general qualitative assessment of the problem.

In/

In the graphical presentation of the experimental data it is convenient to be able to compare rotational and non-rotational results on the same figure. However, in the event of plotting Nusselt number against rotational Grashof number it is evident that this is not possible. To overcome this the rotational Grashof number may be standardised as regards the applied body force by noting that

$$Grr = \frac{H\Omega^2}{g} \cdot \frac{g\beta d^3 \Delta T}{\nu^2}$$

$$= A. \; Grg$$

where A is the tube centre-line acceleration ratio.

Results may thus be plotted in terms of a standardised or gravitational Grashof number by the introduction of the parameter A, an artifice to facilitate ease of comparison.

3. Apparatus and Procedure

The apparatus is shown schematically in Fig. 2 and consisted basically of a built-up rotor shaft mounted between self-aligning bearings. Two steel support arms mounted the test section together with its inlet and outlet chambers parallel to the central shaft and at the required displacement. The rotor was driven by a variable speed electric motor and air was circulated through the test section via internal passages in the central shaft and radial connecting tubes.

The test section was made from brass tubing nominally 12 in. long by 1 in. internal diameter and was electrically heated by uniformly wound resistance wire. The resistance wire was insulated from the tube by glass fibre tape and Tufnol flanges were attached to both ends of the heater section. The wall temperature distribution was measured at seventeen axial locations and air inlet and outlet temperatures were measured by thermocouples located in the plenum and mixing chamber respectively. All thermocouples were made of copper-constantan and the signals were taken from the rotor via miniature instrumentation slip rings.

The experimental procedure was as follows. The motor speed was adjusted to give the desired test section rotation and the blower was throttled to give the required air flow. Tests were then performed over a range of heat fluxes, the maximum of which was required not to produce a tube wall temperature in excess of 400°F. This constraint was in order to avoid insulation breakdown. Steady conditions were reached in about one hour and then readings of tube wall temperatures, air inlet and outlet bulk temperatures, ambient temperature and pressure, air flow rate, rotational speed, and heater power were taken.

4. Results and Discussion

The experimental results are presented as mean data for the tube as a whole. The wall heat flux was calculated from the air bulk temperature rise and the mass flow rate, and the characteristic temperature difference used in the Nusselt and Grashof numbers is that difference between the integrated mean wall temperature and the arithmetic mean bulk temperature of the air between inlet and outlet to the test section. Air properties were evaluated at the mean bulk temperature.

Referring to Fig. 3, results are plotted as mean Nusselt number against the standardised Grashof number for two values of Reynolds number and a radius of

rotation/

rotation, H, of 12 in. At both Reynolds numbers substantial increases in Nusselt number are seen to occur due to rotation, those at the lower Reynolds number having the greater significance. The changes may be analysed in terms of a step increase occurring at the lower end of the Grashof number range (low heat flux) together with a subsequent continuous increase with increase in Grashof number. The step changes occur at very low heating rates and thus it would seem that they can hardly be associated with buoyancy. It is more likely that the effect is due to swirling flow at the inlet caused by the rotation. This being so, then the addition of flow straighteners immediately at the inlet to the test section could be expected to reduce the magnitude of the steps.

The results using flow straighteners are shown in Fig. 4, other conditions being the same as in Fig. 3. The general level of the results is lower than in Fig. 3 due to the sensitivity of entry region heat transfer to small modifications in inlet detail as noted by Boelter et al[7]. The step changes are now greatly reduced thus substantiating the ideas concerning swirl and it is also noticeable that the buoyancy effect is slightly more pronounced. An increase in rotational buoyancy has been gained at the expense of swirl, the overall effect being a reduction in Nusselt number. The complex interaction between swirl and centrifugal buoyancy is emphasised by inspection of Fig. 5. Here it is apparent that in one sector of the tube cross section swirl is tending to aid the buoyancy secondary flow whilst the opposite is true in another sector. Immediately at entry to the tube it would be expected that the axi-symmetric swirl would be predominant but that centrifugal buoyancy would modify this to a complex non-symmetric flow at stations downstream of inlet. At some station it may be that a certain portion of the flow exhibits virtually no secondary flow whilst another portion exhibits considerable secondary flow. At stations far downstream the secondary flow would be that due to centrifugal buoyancy only.

Fig. 6 compares results pertaining to different radii of revolution, H, and two Reynolds numbers. The values of H used were 6 in. and 12 in. and rotational speeds were such that the same values of acceleration ratio existed at the tube centre-line. Flow straighteners were used in both instances to reduce the inlet swirl. It is seen that the rotational Grashof number is adequate to describe the effects of buoyancy, results for the same values of A and Re falling on the same line. It is interesting to note that as H decreases with respect to the tube diameter, d, so the distribution of centrifugal force over a cross-section becomes less uniform and a considerably different flow pattern may exist in which the dependence of Nu on Grr may be quite different from that depicted by the present results. In the cooling of gas turbines and such, H/d values similar to the present, and greater, are the more likely.

From the preceding paragraphs it is apparent that a complex flow exists in the real problem of heat transfer in a tube rotating about a displaced axis. A much wider study is necessary before design criteria can be established, the present results being only intended to highlight some of the difficulties and probable rewards of a fuller investigation. The important question of dual pressure drop has not been considered but it has been noted by Webb[5] in a similar geometry that this is caused to increase by rotation. On the other hand it has been shown by Kreith and Margolis[8] that certain types of swirling flow can result in a reduction in pressure drop. Thus the question of whether or not to encourage swirl in the present tube geometry, from the point of view of an economic optimization of heat transfer, is a matter of open speculation. The over-riding factor at the moment is that heat transfer in internal ducts of rotating machinery is significantly greater than in the well-studied stationary duct case and that the subject warrants further investigation.

5. Conclusions

Experimental results regarding heat transfer in a tube rotating about a displaced axis have been presented and discussed on a general basis. It is shown that even with a limited range of rotational speed and "g" loading significant increases in heat transfer can occur, these being due to inherent inlet swirl and centrifugal buoyancy. No attempt has been made to correlate the data into a single equation since the complex interaction between flow conditions and upstream geometry preclude a generally applicable result. Rotational speeds and centrifugal accelerations much higher than those encountered here can occur in present-day rotating machinery and the accompanying increases in heat transfer are likely to be large, thus indicating the desire for further study of this new class of problem.

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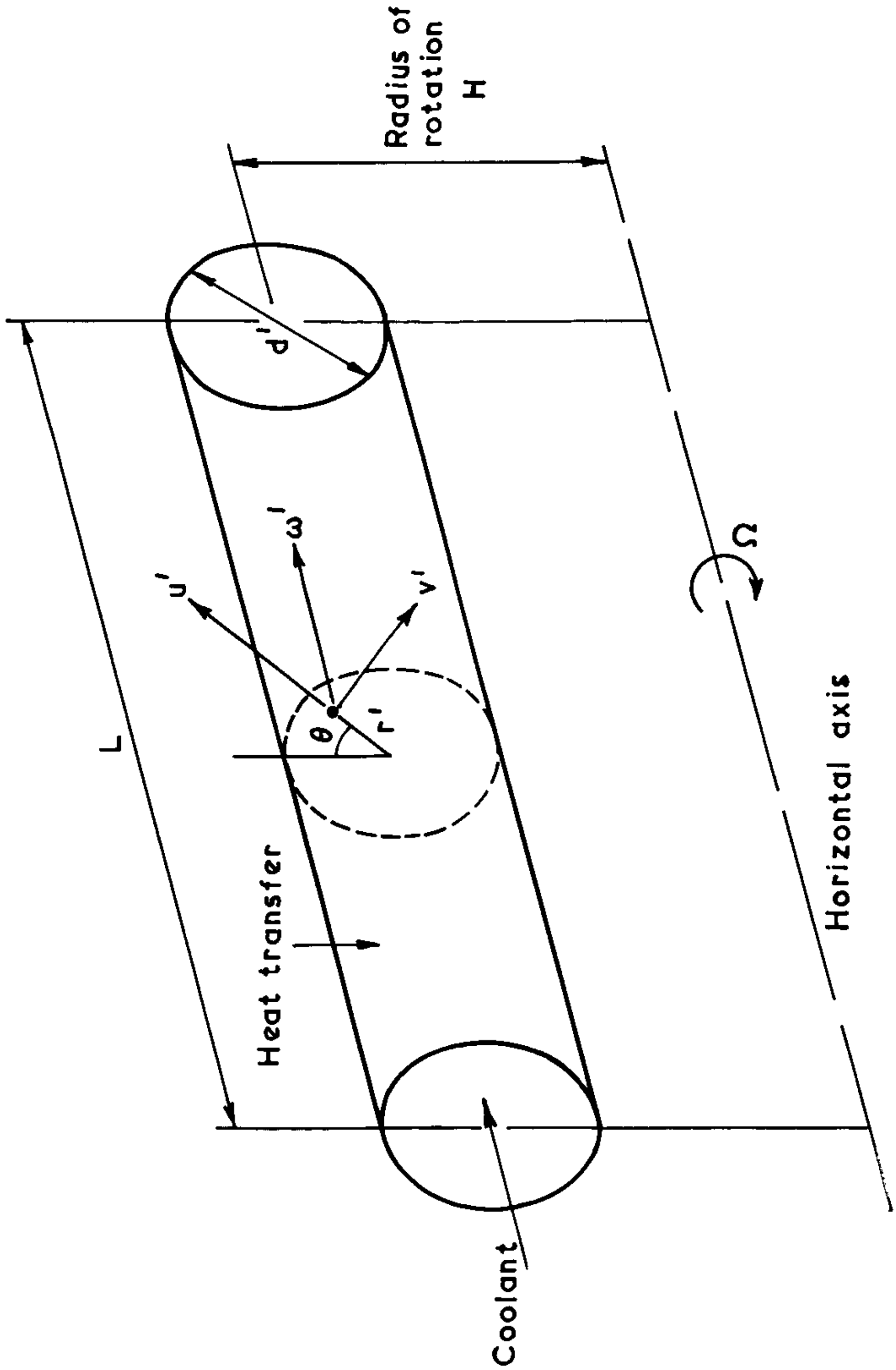
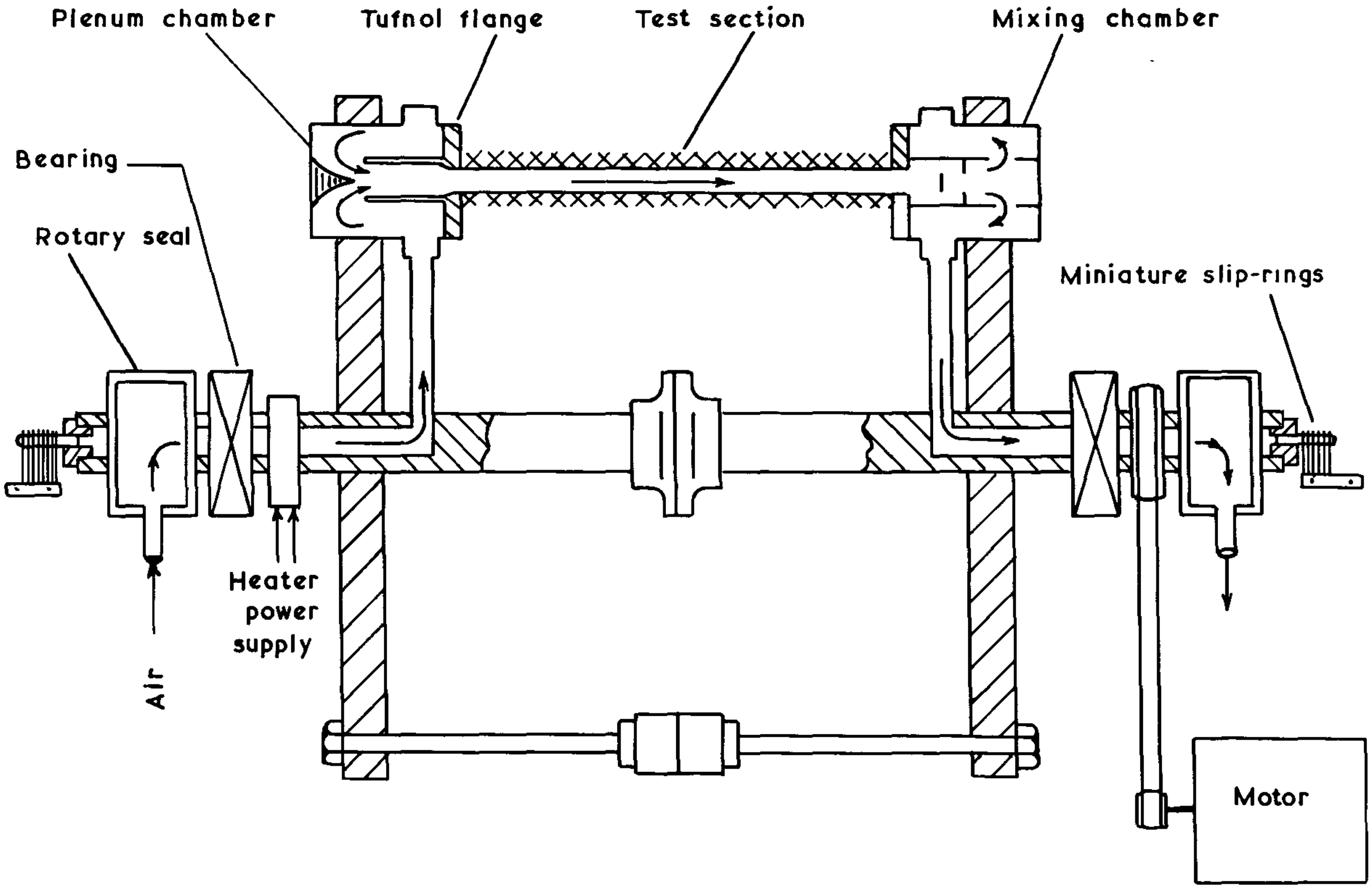
FIG. 1



Idealized model

FIG. 2



Apparatus — diagrammatic

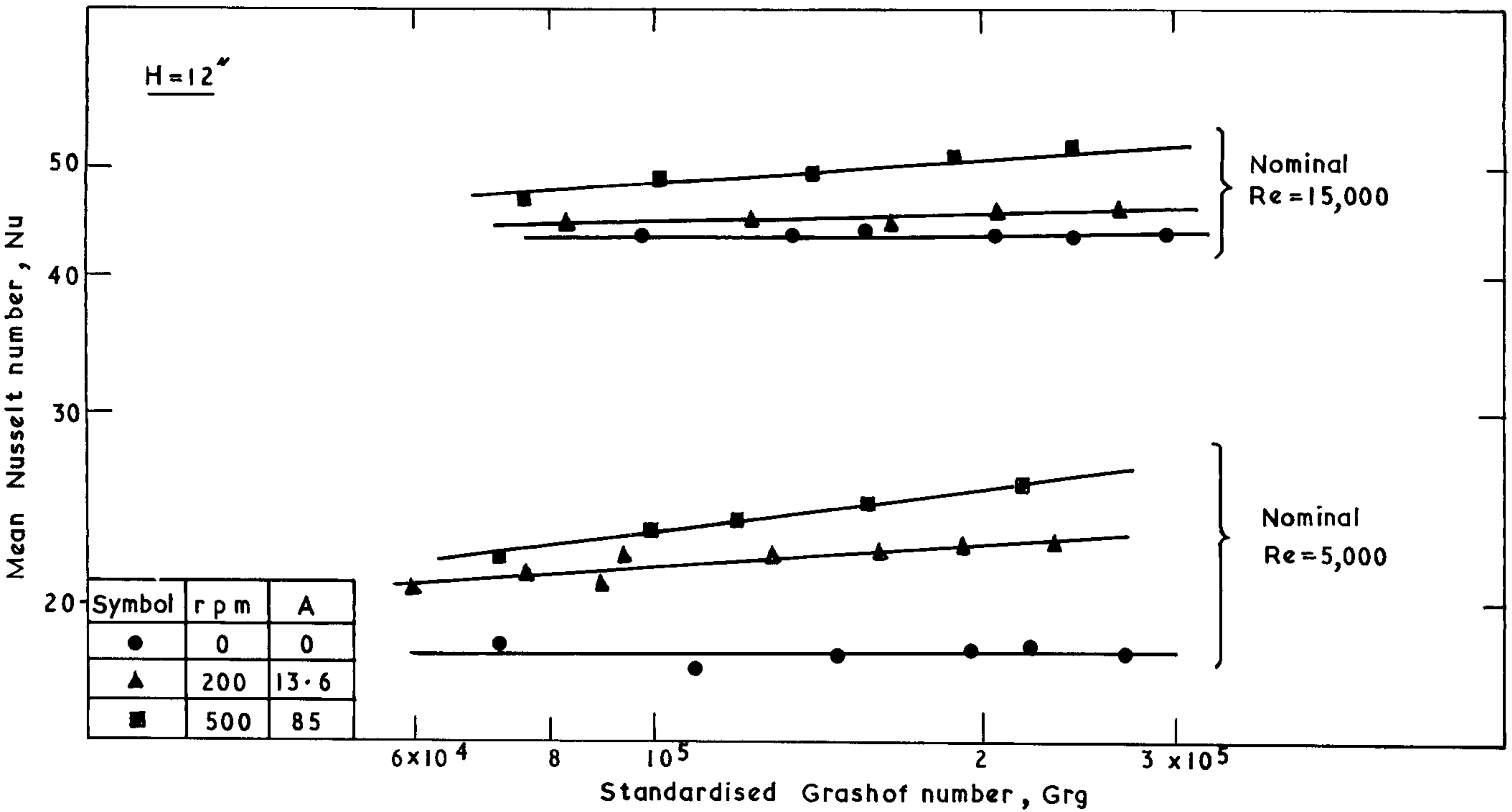
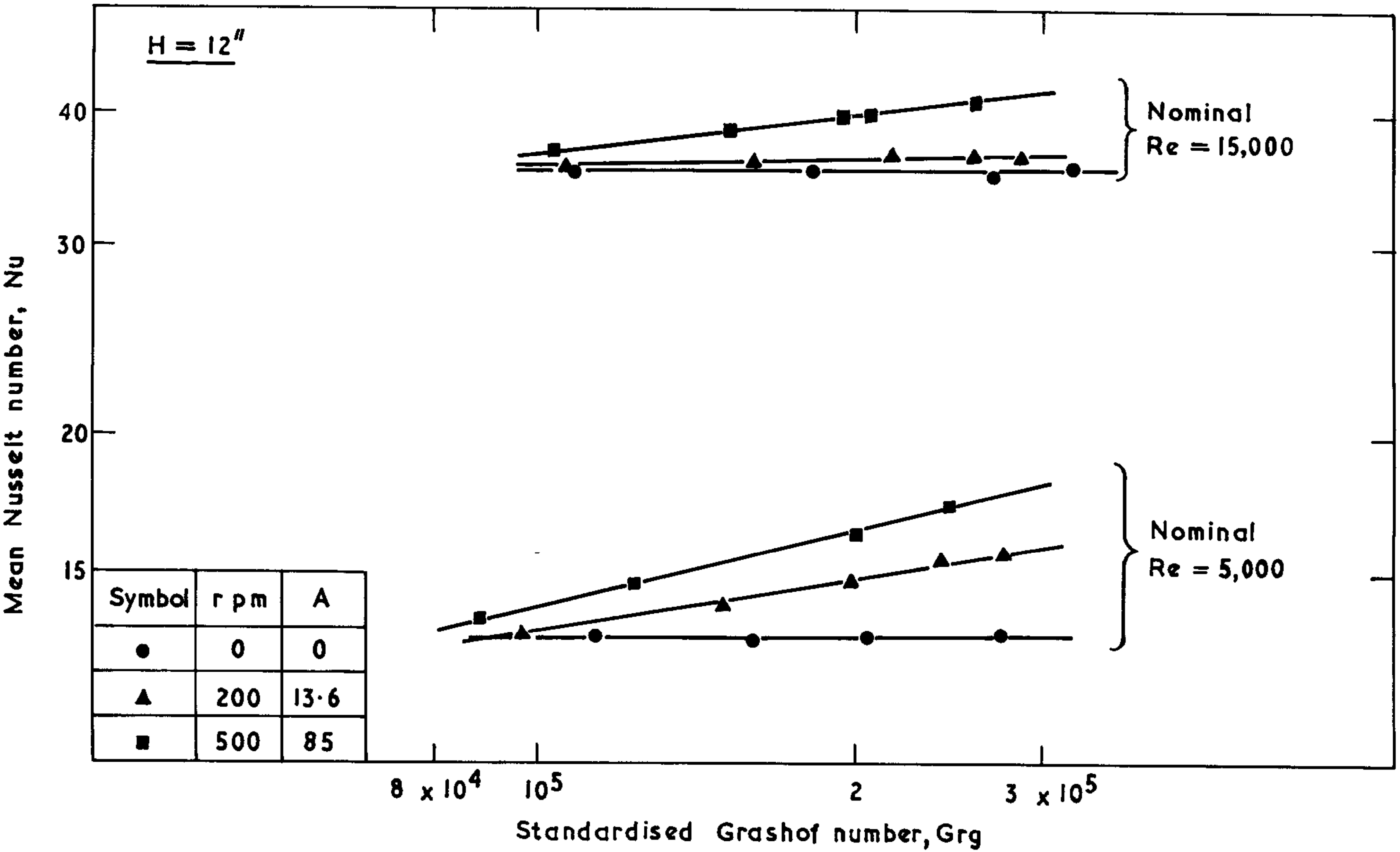


FIG. 3

Variation of Nusselt number with Grashof number – maximum swirl condition

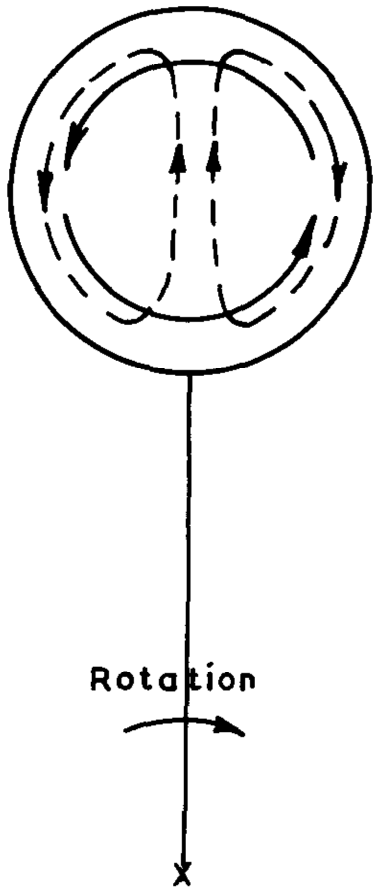


Symbol	r p m	A
●	0	0
▲	200	13·6
■	500	85

FIG. 4

Variation of Nusselt number with Grashof number – minimum swirl condition

FIG. 5



Basic secondary flows in a revolving tube

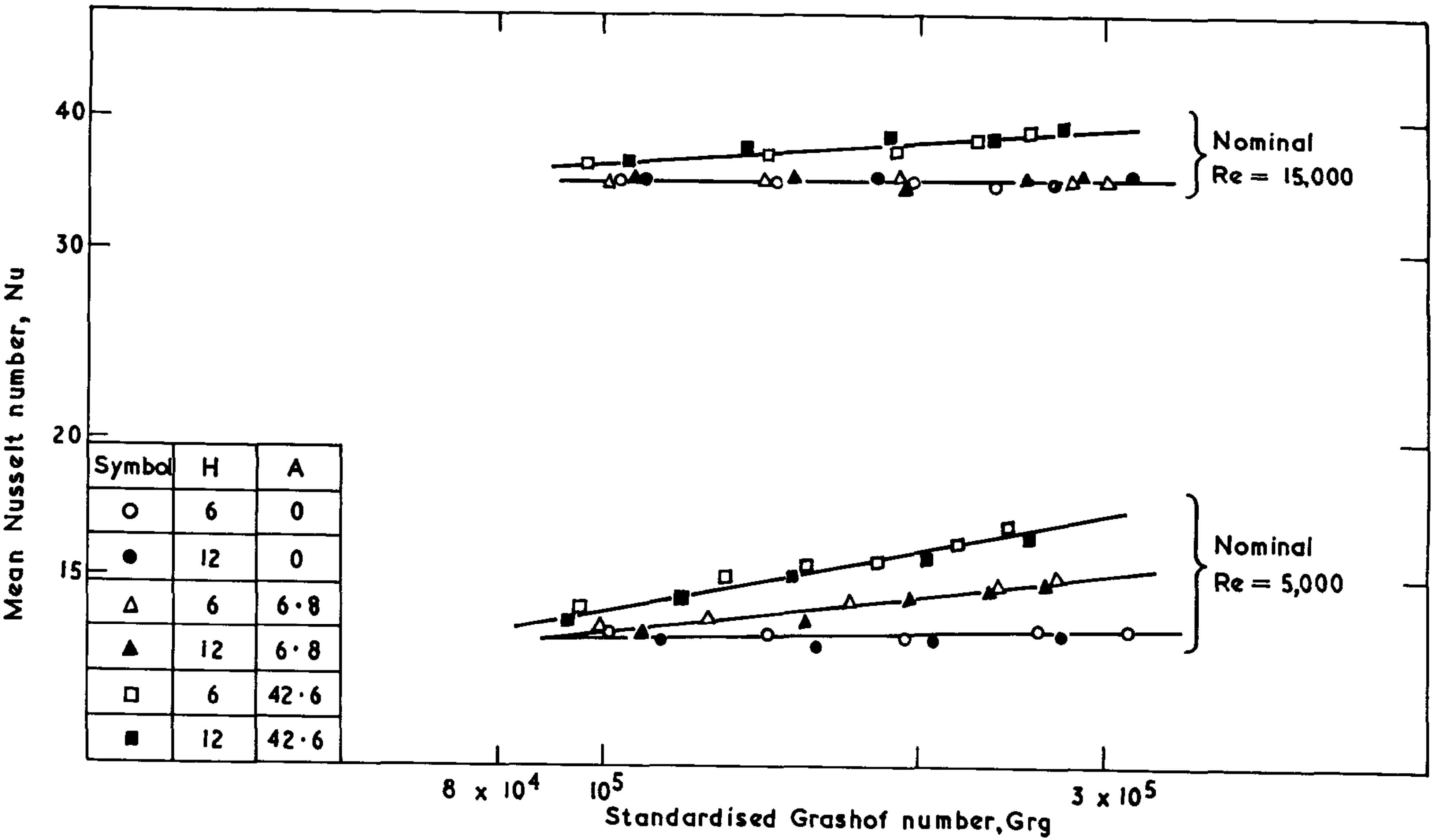


FIG. 6

Variation of Nusselt number with Grashof number for different radii of revolution

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