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by

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FLIGHT MEASUREMENTS OF WING-TIP VORTEX MOTION NEAR THE GROUND

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SUMMARY

Tests have been made to measure the movement of the wing tip vortices from a Hunter aircraft flying at 170 knots approximately 35 feet above a runway, in a variety of wind conditions. Measurements were limited to a maximum time of 20 seconds after vortex generation. During this period the theoretical predictions presented are in good general agreement with the observed motions; however significant differences did occur. There was no clear indication as to whether the vortices decayed more rapidly in the presence of the ground and atmospheric turbulence, than would have been expected from earlier measurements away from the ground in calm air. Limited tests were also made to study the vortex mutual interaction away from the ground.

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* Replaces R.A.E. Technical Report 68007 - A.R.C. 30345.

CONTENTS

1 INTRODUCTION

Previous flight tests[1] have established the behaviour of the vortex trail behind aircraft flying at low speed away from the ground in essentially calm air conditions. Since one of the critical regions for wake penetrations by other aircraft is in the airport terminal area[2], information is required on the behaviour of vortices close to the ground.

Simple potential flow theory (Appendix A) suggests that as the trailing vortices descend towards the ground their vertical velocity decreases and they begin to travel horizontally over the ground away from each other. In practice several effects may be expected to modify the behaviour of the vortices close to the ground. Because wind varies with height, so does its effect on the motion of the vortices. Compared with the earlier measurements at 10000 feet and above, atmospheric turbulence increases at lower altitudes and this, together with ground friction effects, should dissipate the vortices more rapidly than in the case of calmer air conditions away from the ground.

To obtain a better understanding of these effects some flight tests have been made at R.A.E. Bedford airfield using a Hunter aircraft with different coloured smoke injected into its two wing tip vortices. In the tests the overall motion of the vortices was observed, but measurement of local velocities was not attempted. This Report presents the results of these tests and some comparison with theory together with suggestions for further theoretical analysis.

2 TEST AIRCRAFT

A Hunter 6 aircraft (see Table 1) was fitted with special racks on the outer wing pylons, the port rack carrying 4 red, and the starboard, 4 yellow smoke grenades. The grenades burned for about 15 seconds, and could be fired by the pilot in any combination to mark each trailing vortex distinctively during the test runs. No recording instrumentation was fitted in the Hunter.

A Whirlwind helicopter, with an observation hatch in the cabin floor, was used as an airborne photographic station to record the vortex motions.

3 DESCRIPTION OF TESTS

3.1 Tests away from the ground

During earlier tests with a Comet aircraft away from the ground[1], using a similar method of visualising the trailing vortices, a mutual interaction between the two vortices was observed. Initial tests were therefore made to determine the behaviour of the Hunter's trailing vortex system away from the ground. The aircraft was flown at 170 kn eas at an altitude of 1000 feet, and the vortex pattern photographed by a ground-based cine-camera. The results of these tests permitted a height to be selected for the Hunter flights near the ground, such that the vortex motion would be significantly affected by the ground before there was any likelihood of vortex mutual interaction occurring.

3.2 Tests near the ground

3.2.1 Test method

The tests were made over the airfield at R.A.E. Bedford, which is situated in flat countryside. There are few buildings on or near the airfield, and none within half a mile of the test area. This area is shown diagrammatically in Fig.1 and contains the intersection of two runways at right angles, along one of which the Hunter was flown. The other runway was used as a camera base, with the camera located 925 feet from the test runway centre-line.

A second camera was carried in the Whirlwind helicopter, which at the commencement of a test hovered at about 500 feet above the runway intersection A vertical measuring plane, at right angles to the track of the test aircraft, contained the ground camera position, and a marker board in a corner between the two runways.

The Hunter was flown at 170 kn in all cases, and at an altitude of about 35 feet above the runway. Before traversing the test area, the pilot ignited the smoke grenades, and maintained smooth, steady flight across the test area. The time required for the aircraft to make a complete circuit was deemed to be sufficiently long to allow the disturbance caused by the previous run either to settle, or be blown away, before the next test. Visual observation of the Hunter's smoke trails, with and without the helicopter in position, suggested that the effects of helicopter rotor downwash could be neglected, since no difference in the behaviour of the smoke trail could be detected. Also, with the helicopter in position, no difference in trail behaviour could be detected along the length of the smoke trail, which extended about 2000 feet either side of the measuring plane.

The two cine-cameras, loaded with colour film and running at 16 frames per second, were used to record continuously the vortex positions as marked by the red and yellow smoke trails. The cameras were started before the

Hunter passed through the measuring plane, and stopped only after the smoke had dispersed. The ground camera was fixed, and recorded the heights of the intersections of the smoke trails with the measuring plane, while the airborne camera, at about 500 feet altitude, recorded the lateral positions of the intersections. During the tests, the helicopter pilot endeavoured to maintain position in the measuring plane, and vertically above the trails, enabling the camera operator to aim his camera vertically through the open hatch in the floor of the cabin. The marker board, which was initially in the field of view of both cameras, defined a reference point subsequently used in the analysis of the tests. The film records were synchronized by identifying on both films the instant at which the Hunter passed through the measuring plane.

Thirty-eight runs were made in a variety of wind conditions, with speeds varying between zero and 15 kn and directions between 0 and 90° to the Hunter's track. Helicopter piloting problems did not allow tests in higher winds.

3.2.2 Measurement of vortex positions

Selected frames of the cine-film records were projected, and the vertical and lateral positions of the intersections of the smoke trail with the measuring plane were measured, and scaled appropriately to yield displacements in feet from their initial positions immediately after generation. The scale factors were established for the ground camera from the image length of the Hunter aircraft, and for the airborne camera from the image size of the 10-foot square concrete slabs of the runway surface.

A correction was applied to the measurements from the ground camera film to compensate for perspective distortion, which caused the apparent trail height above ground to vary with distance from the camera. No correction was applied to the airborne camera records, since the likely errors due to imperfect positioning of the helicopter were assessed as being of a considerably greater magnitude than the perspective errors.

The film frames were analysed at 1-second intervals, corresponding to 1-second increments of wake age in still air. Since the presence of a head - or tail - wind component effectively moved a younger or older portion of wake into the measuring plane, the elapsed time has been multiplied by a factor* to yield the true age of the portion of the trail in the measuring plane. The headwind component was derived from knowledge of the local wind strength and direction measured at 30 feet above ground level, by a standard Meteorological office recording anemometer.

4 RESULTS AND DISCUSSION

4.1 Vortex mutual interaction away from the ground

During earlier flights tests with a Comet aircraft[1], a mutual interaction was observed between the two vortices. A sinuous distortion developed, which increased until the smoke filaments marking the vortices almost touched at intervals along the trails. The culmination of this process was the rupture of the smoke filaments, and the linking of the free ends across the flight path to form a series of loops. The process of interaction is shown for increasing vortex ages in Fig. 2. The breaks in the smoke trails visible in the photographs at 38 seconds, and at the extreme left of the photograph at 81 seconds are due to trail penetrations by an instrumented aircraft[1]. For the Comet, flying at 150 kn eas and 10000 feet altitude, the time required for the formation of these loops was about 90 seconds, and it appeared that after little more than 120 seconds the vorticity had substantially decayed, although it cannot be necessarily assumed that a rapid vortex breakdown follows the interaction.

The Hunter flight tests at 1000 feet confirmed that the trailing vortex behaviour was similar to that of the larger aircraft, although in this case, the time taken for the loops to form was about 12 seconds.

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* True age $= \dfrac{V - u}{V} \times$ elapsed time

where V = airspeed of aircraft

u = headwind component along aircraft's track.

In the analysis it was assumed that u did not vary with height. Since $\left(\dfrac{V - u}{V}\right)$ was always near unity in these tests $\left(0.95 < \dfrac{V - u}{V} < 1.05\right)$ the error introduced by this assumption was small.

A reason for the shorter interaction time for the Hunter may be the smaller lateral separation of the vortex pair. If it is supposed that the system is adequately described at $t = 0$ by a vortex pair, and also that the individual vortex structure is always described by Squire's formula[3], with the eddy viscosity a universal constant, and further, that the time origin is suitably chosen to permit differences in core size, the initial conditions are fully specified by:-

(i) vortex strength, K,

(ii) lateral spacing, s ($= \frac{\pi b}{4}$ where b is geometric wingspan),

(iii) initial core size, d.

Further, if it is supposed that the effect of d is small, then dimensional analysis leads from $T = f(K, b)$ to

$$\frac{TK}{b^2} = \text{constant } C, \text{ say}$$

since this is the only non-dimensional group that can be formed from the parameters. If we now write

$$K = \frac{4L}{\pi\rho\, Vb} ,$$

where L = aircraft lift

ρ = air density

V = aircraft speed

b = geometric wingspan

we obtain,

$$T = C\,\frac{\rho\, Vb^3}{L} .$$

The value of C cannot be found precisely from the Comet and Hunter tests, but appears to be of the order 10 for both aircraft. More work would be required to establish C, and to determine whether other parameters, defining a more general initial structure, had a significant effect. For aircraft with slender wings, such as the Concorde, although vorticity is shed into the wake, the trailing vortex pattern is more complex than that shed by an aircraft with attached flow[1], and may be different in character. Consequently, the mechanism of the decay process for the Concorde may be different, and more flight tests would be required with a slender wing aircraft to investigate this. It is, nevertheless, interesting to note that for the Concorde, on the climb-out, at a weight of 350 000 lb $\frac{\rho Vb^3}{L} = 1.5$ seconds, and on the approach at a weight of 200 000 lb $\frac{\rho Vb^3}{L} = 1.9$ sec. If the Concorde vortices behaved as those of the Hunter and Comet aircraft, they would interact within 15-20 seconds, but as the decay process may be different, the vortices could persist for much longer than this time.

4.2 General features of vortex motion near the ground

Although, as will be seen later, the general motion of the vortices was as predicted by simple theory, various other features were frequently apparent.

Immediately after generation the lateral separation of the vortex pair was about 35 feet (the Hunter's wingspan is 33.7 feet), but during the first two seconds the separation decreased to about 27 feet, corresponding closely to the usually accepted value of $\frac{\pi}{4}$ (geometric wingspan). It may be that the process of entrainment of the smoke into the vortex core, or the rolling up of the vortex sheet, caused the initially larger separation.

The minimum height above ground of the vortices was normally of the order of 10 feet, and vortex mutual interaction never occurred. However, it was seen that occasionally one or both of the vortices broke into a series of roughly semi-circular arches with both ends perpendicular at the ground, as sketched in Fig.3. This suggests that the theoretical analogy of replacing the ground plane by a pair of mirror image vortices is valid, and that each vortex could be considered to be interacting with its mirror image in a manner resembling the vortex mutual interaction observed during the Comet tests away from the ground (Fig.2).

The vortices frequently moved in such a way that their heights became different from each other, and sometimes both became sinuous. The principal result of these effects was that the study was then of the motion of an asymmetric vortex arrangement. However, the sinuous vortices introduced a further effect; although the departure from a two-dimensional system was in itself normally small, the effect of any headwind component was to impart a spurious apparent motion to the measurements which were taken in a fixed plane.

Another feature noticed was that the vortices frequently "bounced", i.e. while they moved apart they reached a minimum height and then began to rise again.

4.3 Typical results measured near the ground

The horizontal and vertical co-ordinates of the intersections of the smoke trails with the measuring plane for each test are tabulated in Table 2 against true wake age in seconds. For each run the Meteorological Office wind velocity is quoted, together with the track of the test aircraft.

A limited selection of the results is shown in Fig.4, which presents the movements of the vortices in the measuring plane for wind conditions from calm (less than 2 knots) up to 16 knots. Figs.4a and 4b correspond to nominally calm conditions, and it will be seen that the upwind vortex descended almost vertically and remained very near the runway centreline. The very light cross-wind was sufficient to neutralise the upwind movement due to the vortex-induced velocities. This could present a hazard to following aircraft using the same runway. For the downwind vortex the two effects were additive. Fig.4c shows the vortex movement with a $6\frac{1}{2}$ knot cross-wind component, both the vortices being blown clear of the runway centreline quite rapidly. Again in Fig.4d, with an 8 knot cross-wind component, the lateral vortex motion was very rapid, and the vortices would have presented no hazard to following aircraft using the same runway but could present a possible hazard on nearby parallel runways.

Figs.5-8 present the results shown in Fig.4, together with theoretical predictions based on the initial conditions in each case. The expected paths of the vortices in still air were calculated from the expressions derived in the Appendix and an allowance made for the effects of cross-wind.

The actual cross-wind component during the first 3 seconds of recording was derived from the mean horizontal velocity of the vortices during this period. This cross-wind was then applied to the calculated still air motion, assuming a 1/7 power law for the variation of wind strength with height in the earth's boundary layer and using a step-by-step integration process.

It will be seen that the theoretically predicted paths are in reasonable agreement with the measured results, suggesting that the motion derived in the Appendix, together with a 1/7 power law for wind velocity, provides an adequate prediction for vortex motion near the ground.

Smoke dissipation or vortex mutual interaction limited the period of measurement in the present tests to less than 20 seconds. For large conventional aircraft the mutual interaction time appears to be considerably greater, so it is possible that their vortices would persist near the ground for considerable periods, as they do away from the ground. Thus any information on vortex decay near the ground would be valuable.

From the present tests, an attempt has been made to extract rates of decay at positions well outside the vortex core by comparison of measured vortex motions with the predictions of the Appendix. The time histories of the vertical and horizontal velocities of the individual vortices relative to the surrounding air have been derived from the recorded positions, with allowances for wind as described in sections 3.2.2 and 4.3.

The theoretical prediction of the velocity of each vortex has been based on the measured vortex separation, but to simplify the calculation the assumption has been made that the other vortex of the pair was at the same height as the one being studied. The measured and predicted horizontal velocities have been compared on the basis of $(\text{height})^{-1}$, and the vertical velocities on the basis of $(\text{half the horizontal separation})^{-1}$. These bases were chosen because when the horizontal velocity is largest it is primarily dependent on height, and when vertical velocity is largest it is primarily dependent on horizontal separation. Equations (4) and (5) in the Appendix show that for zero eddy viscosity the predicted velocities are given by:

$$\dot{y} = \frac{K}{4\pi Az^3} \quad \text{and} \quad \dot{z} = \frac{-K}{4\pi Ay^3}$$

where y is positive away from the vertical plane of symmetry between the vortices

and z is positive upwards.

The Appendix assumes a symmetrical disposition of the vortices, so for convenience the expression

$$z = \frac{-K}{4\pi As^3}$$

where s = half the horizontal separation of the vortices, has been used in the study of velocities.

Figs.9-12 show the comparison between measured velocities of the vortices and those predicted for zero eddy viscosity, for the four tests presented in Figs.4-8. The scatter in the measurements is large but it should be pointed out that in the analysis of the present tests, the wind velocity profile for any given test was assumed invariant with time, whereas in practice, atmospheric turbulence can cause appreciable fluctuations of wind velocity with time. Thus the horizontal and vertical velocity components of Figs.9-12 include any effects of random velocity fluctuations due to atmospheric turbulence. Because of this scatter there is no clear indication as to the decay in the translational velocities of the vortices, but it cannot be large.

Thus the vortex motion near the ground can probably be predicted adequately. However, no knowledge was gained of the velocity distributions within the vortex near the ground so it is not possible to predict how rapidly the peak velocities within the vortices will decay or assess whether the expression given by Squire[3], for velocities within the vortex still holds under these conditions.

5 CONCLUSIONS

Flight tests have been made to study the behaviour of wing-tip vortices near the ground. A Hunter aircraft was used, flying at 170 knots at a height of about 35 feet above a runway, in a variety of wind conditions.

Measurements were limited to a maximum time of 20 seconds after vortex generation. During this period the predictions of existing theory (see Appendix) are in good general agreement with the observed vortex motions. However, significant differences did occur, the two vortex heights frequently becoming different from each other, and the vortices sometimes descending to a minimum height and then rising again. A reaction between a single vortex and the ground, similar to the vortex mutual interaction that had been seen in earlier tests, was also occasionally observed.

There was no clear indication as to whether the vortices decayed more rapidly in the presence of the ground and atmospheric turbulence, than would have been expected from earlier measurements away from the ground in calm air.

All the measurements obtained in the tests are presented in this report, for the benefit of any reader who wishes to make a more complete analysis of the data.

Limited tests at 1000 feet altitude indicated that away from the ground the vortex trail developed into a series of closed loops and that the interaction time for this process was approximately 10 seconds compared with earlier measurements of about 90 seconds for a Comet. The results suggest that there may be a correlation between the interaction time and the quotient (vortex separation)/(velocity induced at one vortex centre by the other). Although this result cannot necessarily be read across directly to the Concorde because of the different character of the vortex trail, it suggests that the interaction time for the Concorde on the climb-out and the approach may be considerably smaller than for existing transport aircraft.

6 SUGGESTIONS FOR FURTHER WORK

The analysis of the results in this Report could be extended, and theory could be developed, with the following aims:-

1 The development of a method of predicting when a pair of vortices will move into an asymmetric position, and what their subsequent motion will be.

2 A more detailed study of the velocities with which the vortices moved might yield information about the decay in these velocities with time.

3 Further theoretical work could include computer studies of the mechanism of the vortex mutual interaction process away from the ground (more results of the type shown in Fig.2 are available).

4 Further tests are necessary if the rate of decay of the peak velocity within vortices near the ground is to be established.

In addition, flight tests are required to study the decay process of vortices shed by a slender wing aircraft and these are now planned.

Appendix A

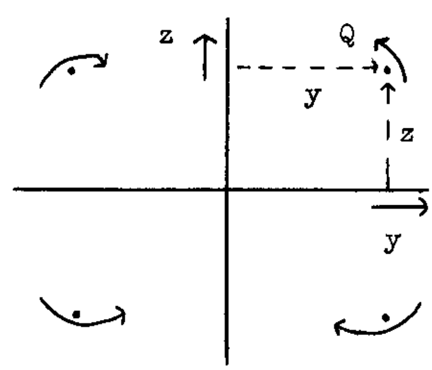
CALCULATION OF VORTEX MOTION NEAR THE GROUND IN STILL AIR

A more general treatment of vortex motions is given by Jones[4], for a multiplicity of vortex generators. A simpler approach, considering only one vortex pair, has been adopted here.

In the analysis that follows, the vortex-induced velocities are assumed not to vary with time, i.e. the effects of eddy viscosity on the circumferential velocity distribution in the vortex have been neglected, as these effects are considerable only in the region adjacent to the vortex core and the mutually-induced vortex motions are dictated by the velocities prevailing several core diameters away from the vortex centre. For typical aircraft under approach conditions the expression in Ref.1 suggests that it will be several minutes before the relevant velocities fall to 99% of their initial values.

The analysis follows the methods of classical hydrodynamics[5,1].

Let the trailing vortex pair be located a distance $2y$ apart, and at height z above the ground, then the vortex system with its image may be represented thus:-



Now the circumferential velocity v, due to a single vortex of strength K is

$$v = \frac{K}{2\pi r}$$

where r is the distance from the vortex axis.

If we consider the velocities induced at Q by the remaining vortex and the two image vortices, and resolve velocities (i) horizontally and (ii) vertically we have

$$\text{(i)} \quad \dot{y} = \frac{K}{4\pi}\left[\frac{1}{z} - \frac{z}{y^2 + z^2}\right] ,$$

and

$$\text{(ii)} \quad \dot{z} = \frac{-K}{4\pi}\left[\frac{1}{y} - \frac{y}{y^2 + z^2}\right]$$

therefore,

$$\dot{y} = \frac{K}{4\pi z}\left(\frac{y^2}{y^2 + z^2}\right) \tag{A.1}$$

and

$$\dot{z} = \frac{-K}{4\pi y}\left(\frac{z^2}{y^2 + z^2}\right) . \tag{A.2}$$

Combining (A.1) and (A.2)

$$\frac{\dot{y}}{\dot{z}} = \frac{dy}{dz} = -\frac{y^3}{z^3}$$

or,

$$\frac{dz}{z^3} = -\frac{dy}{y^3}$$

and hence,

$$\frac{1}{y^2} + \frac{1}{z^2} = A . \tag{A.3}$$

A may be found by substituting y_o and z_o for y and z respectively.

(A.1) and (A.2) may now be rewritten as

$$\dot{y} = \frac{K}{4\pi A z^3} \tag{A.4}$$

$$\dot{z} = \frac{-K}{4\pi A y^3} . \tag{A.5}$$

By eliminating z from (A.1) and rearranging:-

$$\dot{y} = \frac{K}{4\pi A}\left[\frac{Ay^2 - 1}{y^2}\right]^{3/2}$$

or,

$$\frac{4\pi}{K}\left[\frac{y^2}{Ay^2 - 1}\right]^{3/2} dy = \frac{dt}{A} \ .$$

Hence, it can be shown that,

$$\frac{4\pi}{AK}\left[\frac{Ay^2 - 2}{(Ay^2 - 1)^{1/2}}\right] = t + B$$

B may be found by substituting y_o for y at $t = 0$.

Rearranging,

$$y^2 = \frac{2\left[\frac{64\pi^2}{AK^2} + A(t + B)^2\right]}{\frac{64\pi^2}{K^2}} \left\{1 \mp \left(\frac{64\pi^2}{A^2K^2(t + B)^2} + 1\right)^{-1/2}\right\}$$

and, by symmetry, z is given by:-

$$z^2 = \frac{2\left[\frac{64\pi^2}{AK^2} + A(t + B)^2\right]}{\frac{64\pi^2}{K^2}} \left\{1 \pm \left(\frac{64\pi^2}{A^2K^2(t + B)^2} + 1\right)^{-1/2}\right\} .$$

In each of these expressions, the upper alternative sign applies when $t < B$, and the lower alternative sign applies when $t > B$.

Hence, the vertical and horizontal displacements of the vortex cores can be calculated as a function of time, to give the theoretical vortex positions shown in Figs.5-8.

Table 1

TEST DATA

Leading particulars of test aircraft (Hunter Mk.6)

Wing span	=	33.75 feet
Mean weight	=	16400 lbf
Equivalent airspeed	=	170 kn
Nominal test height	=	35 feet
Circulation K	=	$\left(\frac{4L}{\pi\rho V b}\right)$
	=	907 feet^2 sec^{-1}

Table 2

MEASURED VORTEX POSITIONS

y and z coordinates, in feet, of the points of intersection of the port (p) and starboard (s) trails with the measuring plane are tabulated against true wake age (in seconds). For each run wind direction and speed, measured by Meteorological Office recording instruments, are quoted. The Hunter's speed was 170 knots in all runs.

Run 1

Wind 0 A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	10	-26	34.6	36.0
1	3	-26	30.9	31.9
2	-2	-28	26.1	26.8
3	-6	-32	22.1	22.8
4	-8	-36	18.2	18.7
5	-10	-40	15.2	15.6
6	-14	-46	13.2	13.7
7	-14	-50	11.2	11.6
8	-14	-58	9.1	11.7
9	-12	-64	9.1	12.8
10	-10	-72	9.1	14.0
11	-6	-80	10.1	15.2
12	-6	-86		16.4
13	-4	-92		18.7
14	-4	-98		19.9
15		-102		21.1

Run 2

Wind 0 A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	20.9	-14.5	38.0	37.6
1	16.4	-12.7	35.4	34.5
2	14	-16	31.5	31.5
3	13	-18	29.6	30.6
4	10	-22	25.7	28.7
5	8	-24	21.8	26.7
6	4	-28	19.9	25.8
7	0	-32	18	23.8
8	0	-34	16	22.8
9	0	-38	14	22.9
10	0	-38	13	
11	0	-42	13	
12	0		11	
13	0		10	
14	0		10	
15	0		9	

Run 3

Wind 100°/3 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	18	-20	35.3	36.8
1.0	10	-20	31.6	32.7
2.05	8	-22	27.7	28.7
3.05	7	-24	24.8	25.7
4.1	6	-26	21.9	22.6
5.1	4	-36	17.9	19.7
6.15	4	-30	15.9	17.5
7.15	4	-36	13.9	16.6
8.2	8	-42	12.9	14.6
9.2	6	-46	10.9	14.7
10.25	12	-52		14.8

Run 4

Wind 120°/4 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	16	-17	35.4	36.6
1	16	-14	30.5	32.5
2	16	-14	26.5	27.4
3	16	-14	21.6	22.3
4	17	-18	17.7	18.3
5	20	-22	14.5	16.4
6	24	-26	11.7	15.4
7	30	-34	9.7	13.5
8	38	-42	7.7	12.5
9	50	-54	6.6	11.6
10	57	-60	6.6	

Table 2 (Contd)

Run 5

Wind 120°/14 kn   A/C track 090°

Time (secs)	y_p	y_s	z_p	z_s
0	-14	22	39.6	39.0
0.95	-24	8	36.9	36.7
1.85	-38	-8	35.4	34.3
2.8	-52	-22	32.7	31.7
3.75	-64	-36	28.9	28.1
4.65	-76	-46	26.0	25.2
5.6	-90	-56	24.1	23.3
6.55	-100	-62	24.4	22.4
7.5	-112	-70	24.7	19.4
8.4	-126	-80	21.6	17.4
9.35	-138	-94	17.2	16.5
10.3	-146	-104	16.2	15.6
11.2	-168	-110	17.7	16.8
12.15	-194	-124	16.9	15.9

Run 6

Wind 120°/14 kn   A/C track 090°

Time (secs)	y_p	y_s	z_p	z_s
0	-14	18	36.5	35.3
0.95	-26	2	35.0	35.9
1.85	-42	-12	31.4	35.5
2.8	-56	-32	29.7	32.1
3.75	-72	-46	27.0	28.4
4.65	-92	-64	25.3	24.6
5.6	-120	-76	24.9	18.4
6.55	-148	-102	23.2	17.8
7.5	-160	-120	19.9	18.0
8.4	-190	-142	19.3	18.5
9.35	-216	-158	19.7	15.2

Run 8

Wind 120°/16 kn   A/C track 090°

Time (secs)	y_p	y_s	z_p	z_s
0	-20	16	38.8	37.4
0.95	-34	-4	35.3	34.1
1.85	-54	-22	30.7	29.7
2.8	-70	-40	26.9	28.2
3.7	-94	-60	23.1	25.6
4.65	-112	-76	21.3	22.7
5.55	-136	-90	18.4	20.8
6.5	-150	-108	16.3	20.1
7.4	-174	-120	15.4	19.2
8.35	-186	-132	14.4	19.4
9.3	-206	-148	14.7	18.6
10.2	-228	-166	15.0	20.0
11.15	-248	-178	16.5	21.5

Run 11

Wind 130°/14 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	0	-32	20.0	20.7
0.95	-16	-52	18.3	19.0
1.85	-34	-72	15.6	17.2
2.8	-48	-94	13.7	17.6
3.75	-62	-118	12.8	18.0
4.65	-72	-138	10.8	18.4
5.6	-88	-162	11.0	18.8
6.55	-96	-182	11.0	18.0
7.5	-106	-202	11.1	18.3
8.4	-124	-228	13.6	18.7
9.35	-142	-250	18.5	20.3
10.3	-162		20.0	

Table 2 (Contd)

Run 12

Wind 130°/14 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	22	-18	26.4	27.5
0.95	4	-32	21.9	22.8
1.85	-10	-48	20.2	18.9
2.8	-22	-64	19.5	18.2
3.7	-32	-78	17.6	16.3
4.65	-44	-94	16.8	17.6
5.55	-56	-110	15.9	21.3
6.5	-62	-120	14.9	20.3
7.4	-73.3	-130	14.0	20.5
8.35	-80	-151.7	13.0	21.0
9.3	-93.3	-175	11.0	17.8
10.2	-97.3	-201	11.1	17.0
11.15	-100	-218	11.1	18.5
12.1	-110	-238	11.2	20.1

Run 13

Wind 120°/13 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	24	-12	35.1	35.5
1.05	18	-16	30.4	30.5
2.05	12	-22	25.7	25.6
3.05	8	-28	22.8	23.7
4.05	0	-34	20.0	19.7
5.1	-2	-42	18.0	17.8
6.1	-4	-48	16.1	15.8
7.15	-8	-56	15.1	14.9
8.15	-8	-66	14.1	13.9
9.15	-8	-76	13.1	13.0
10.2	-2	-80	13.0	14.1
11.2	-10	-94	11.1	12.1
12.2	-10	-98	10.1	11.1

Run 14

Wind 080°/7 kn A/C track 090°

Time (secs)	y_p	y_s	z_p	z_s
0	-22	14	36.9	36.4
0.95	-24	8	31.8	30.7
1.9	-28	0	26.8	26.0
2.85	-32	-2	21.7	21.0
3.85	-38	-8	18.7	18.2
4.8	-44	-12	14.7	15.2
5.8	-50	-16	13.7	15.3
6.7	-58	-18	12.8	16.3
7.65	-66	-20	13.9	14.3
8.65	-76	-22	13.0	15.4
9.6	-82	-22	12.0	16.4
10.55	-92	-20	11.0	17.4
11.5	-104	-16	11.1	20.3
12.5	-	-14	-	21.3
13.45	-120	-12	11.3	

Run 15

Wind 090°/6 kn A/C track 090°

Time (secs)	y_p	y_s	z_p	z_s
0	-26	12	40.1	38.5
0.95	-28	6	35.0	34.8
1.9	-28	4	30.9	30.9
2.85	-32	0	26.9	26.0
3.85	-34	0	23.9	23.0
4.8	-36	0	19.7	19.0
5.8	-40	0	17.7	17.0
6.7	-42	0	15.7	15.0
7.65	-46	4	15.8	13.9
8.65	-48	8	14.7	12.9
9.6	-52	10	12.7	11.9
10.5	-58	14	12.8	11.8
11.5	-62	20	13.9	12.7
12.45	-70	20	15.1	13.7
13.45	-70	26	16.1	14.6
14.4	-74	28	17.3	15.5
15.35	-78	28	18.4	16.5
16.3	-80	28	20.6	18.4
17.25	-80	30	21.7	19.4

Table 2 (Contd)

Run 18

Wind 130°/10 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	28	-10	33.0	34.4
1.05	8	-24	28.7	29.8
2.1	-8	-40	25.2	26.1
3.1	-22	-56	22.5	23.3
4.2	-34	-72	20.7	21.6
5.25	-46	-88	16.8	20.8
6.3	-60	-108	13.8	19.0
7.35	-68	-126	11.8	17.0
8.4	-64	-146	9.6	17.4

Run 19

Wind 130°/12 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	24	-14	34.1	35.5
1.05	10	-24	30.7	31.8
2.1	-4	-36	27.1	28.1
3.1	-16	-50	23.4	25.3
4.2	-28	-62	19.6	23.5
5.25	-38	-74	16.6	22.7
6.3	-48	-88	15.8	21.9
7.35	-54	-102	13.8	21.1
8.4	-52	-112	12.7	21.3
9.4	-58	-128	11.7	21.6
10.45	-60	-140	10.6	18.4

Run 20

Wind 130°/12 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	16	-24	31.5	32.8
1.05	-2	-36	29.1	31.2
2.1	-18	-50	24.5	25.8
3.1	-32	-68	19.7	22.6
4.15	-44	-84	17.8	18.5
5.2	-58	-104	15.9	16.7
6.25	-60	-118	13.8	19.2
7.3	-72	-138	11.9	18.4
8.35	-84	-160	13.1	19.9
9.4	-94	-178	13.2	20.3

Run 21

Wind 130°/10 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	20	-18	32.3	33.6
1.05	10	-24	30.7	31.8
2.15	0	-32	29.0	29.0
3.2	-8	-42	27.2	26.1
4.3	-14	-50	25.4	23.2
5.35	-22	-62	21.5	20.3
6.45	-30	-72	18.6	19.4
7.5	-38	-84	15.6	16.4
8.55	-42	-94	14.6	15.4
9.65	-50	-108	13.7	14.5
10.7	-54	-126	12.7	15.9
11.8	-58	-138	10.6	17.2
12.85	-62	-150	10.7	18.6
13.9	-66	-162	10.7	18.8
14.95	-68	-174	11.8	20.2

Table 2 (Contd)

Run 23

Wind 220°/12 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	28	-12	30.1	30.4
0.95	8	-24	24.8	24.6
1.9	-8	-40	21.2	21.9
2.85	-20	-56	18.4	21.2
3.8	-34	-72	16.6	19.4
4.75	-46	-86	14.7	17.5
5.7	-52	-100	11.6	16.6
6.65	-54	-114	9.5	15.7
7.6	-56	-128	10.6	14.8
8.55	-60	-148	16.0	13.9
9.5	-66	-156	15.0	15.2
10.45	-	-162		21.2

Run 24

Wind 230°/11 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	14	-22	35.5	36.9
0.95	0	-32	30.0	31.1
1.9	-10	-44	26.3	28.3
2.86	-20	-54	20.4	24.3
3.8	-36	-68	16.6	17.2
4.75	-54	-82	15.9	14.3
5.7	-70	-	15.1	-
6.65	-80	-114	14.1	14.6
7.6	-84	-128	9.8	17.1
8.55	-82	-144	6.5	16.2

Run 25

Wind 220°/11 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	6	-32	38.8	34.2
0.95	-6	-40	29.2	29.2
1.9	-18	-52	25.5	26.4
2.85	-30	-62	21.7	24.5
3.8	-42	-74	16.7	21.6
4.75	-54	-86	14.8	18.6
5.7	-64	-98	11.8	16.6
6.65	-72	-112	10.8	14.6
7.6	-80	-126	9.8	14.8
8.55	-80	-138	9.8	14.9
9.5	-82	-150	8.7	16.3
10.5	-80	-160	7.6	19.9
11.4	-80	-170	7.6	23.7
12.40		-178		26.2
13.35		-188		30.1

Run 26

Wind 330°/5 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	18	-20	59.8	59.3
1.0	24	-8	55.5	56.5
2.0	32	0	51.1	50.0
3.0	40	4	47.9	45.8
4.0	48	16	42.7	42.3
5.05	54	20	39.6	39.1
6.05	60	30	35.5	34.8
7.05	66		32.5	

Table 2 (Contd)

Run 27

Wind 340°/6 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	14	-24	47.3	48.2
0.95	22	-10	43.9	45.5
1.95	32	0	41.5	42.0
2.95	42	10	39.2	38.6
3.95	48	20	36.0	35.2
4.9	52	26	33.0	33.0
5.9	58	32	29.0	28.0
6.9	62	36	27.1	24.0
7.85	66	40		19.1
8.85		46		16.2
9.8		48		13.3
10.8		50		11.4
11.8		54		7.5
12.75		56		6.6
13.75		56		7.5
14.75		54		8.5
15.7		54		10.4
16.7		52		12.3
17.7		52		13.2

Run 28

Wind 180°/12 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	12	-24	27.6	27.7
1	-4	-34	24.1	24.9
2	-18	-50	20.4	22.1
3	-30	-66	17.5	21.4
4	-40	-80	15.6	19.5
5	-50	-98	13.7	19.9
6	-58	-112	11.7	19.1
7	-68	-130	9.7	19.4
8	-74	-140	8.6	19.6

Run 29

Wind 180°/12 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	18		29.4	
0.95	-4		25.1	
1.95	-18	-48	21.4	18.9
2.95	-30	-60	18.6	16.0
3.95	-44	-78	16.8	14.1
4.95	-54	-96	15.9	12.1
5.95	-66	-116	16.1	13.5
6.9	-76		15.1	

Run 30

Wind 180°/12 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	12	-24	30.6	31.8
0.95	-18	-48	25.5	29.5
1.95	-44	-76	21.0	27.1
2.95	-66	-102	16.1	24.4
3.95	-84	-124	14.2	22.7
4.95	-108	-152	12.3	19.8
5.95	-128	-178	10.2	20.3
6.9	-146	-206	8.1	20.8
7.9	-164	-234	7.1	22.6
8.9	-184	-266	4.8	24.5

Table 2 (Contd)

Run 31

Wind 190°/12 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	12	-22	24.7	23.6
0.95	-16	-48	22.4	21.0
1.95	-40	-76	19.8	19.5
2.95	-66	-104	17.1	17.8
3.9	-90	-132	15.4	17.1
4.9	-116	-164	13.5	18.8
5.85	-132	-188	12.6	20.5
6.85	-156	-216	12.9	19.7
7.8	-180		11.9	
8.75	-202		12.2	
9.75	-218		13.6	

Run 32

Wind 220°/8 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	24	-12	22.4	22.3
0.95	26	-6	17.5	18.1
1.95	30	-2	13.6	16.0
2.9	32	0	11.6	15.0
3.85	36	4	11.5	13.9
4.8	38	6	11.5	13.9
5.8	42	10	10.5	13.8
6.75	48	10	10.4	14.8
7.7	54	10	13.2	16.8
8.7	62	8	15.9	18.9
9.6	70	6	15.7	18.9
10.6	78	6	11.9	19.9
11.5		6		19.9
12.55		8		20.8
13.5		8		19.8

Run 33

Wind 220°/10 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	28	-8	27.2	27.2
0.95	26	-4	22.4	23.1
1.9	28	-4	19.4	20.1
2.85	28	-4	17.5	17.1
3.85	30	-6	15.5	14.1
4.8	34	-6	13.5	13.1
5.8	36	-8	11.5	12.1
6.7	40	-10	11.5	12.1
7.7	44	-14	10.5	13.2
8.65	46	-18	10.5	12.2
9.6	50	-24	11.4	12.3

Run 34

Wind 220°/9 kn A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	20	-14	30.3	30.5
0.95	18	-12	26.5	26.3
1.9	18	-14	24.5	22.3
2.85	20	-16	21.5	19.3
3.8	22	-18	17.6	18.3
4.8	26	-24	13.6	17.4
5.7	28	-28	10.7	17.5
6.65	32	-36	9.7	17.7
7.6	38	-42	8.6	16.7
8.6	44	-50	8.6	19.0
9.5	52	-54	8.5	20.1
10.5	54	-58	8.5	20.2
11.45	58	-66	8.4	21.4
12.4	68	-66	8.3	21.4

Table 2 (Contd)

Run 35

Wind 230°/11 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	24	-12	30.2	30.4
0.95	22	-8	28.3	28.3
1.9	24	-10	26.3	24.3
2.85	26	-10	23.3	20.2
3.8	28	-10	21.3	16.2
4.8	34	-14	20.2	15.2
5.7	40	-18	17.2	15.3
6.65	46	-20	16.2	16.4
7.6	54	-26	16.0	15.4
8.6	60	-30	15.0	13.4
9.5	64	-34	14.0	13.5
10.5	70	-36	12.9	13.5

Run 36

Wind 030°/4 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	20	-14	28.4	29.4
1.0	10	-20	26.7	27.6
2.05	6	-30	22.9	24.8
3.05	0	-40	20.0	22.9
4.05	-4	-48	18.1	20.0
5.1	-8	-62	15.1	18.1
6.1	-14	-70	14.2	17.2
7.1	-16	-82	12.2	16.3
8.15	-20	-92	11.2	15.4

Run 37

Wind 030°/4 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	12	-20	24.7	25.6
1.0	4	-24	21.9	21.5
2.05	0	-30	19.0	18.6
3.05	-4	-38	17.1	15.6
4.05	-8	-44	15.1	13.6
5.05	-10	-50	14.2	12.6
6.1	-12	-60	14.2	10.6
7.1	-14	-68	14.2	9.7
8.1	-16	-76	14.2	10.8
9.15	-16	-86	15.3	12.0
10.15	-18	-96	15.3	13.2
11.15	-16	-106	16.3	14.5

Run 38

Wind 030°/5 kn   A/C track 270°

Time (secs)	y_p	y_s	z_p	z_s
0	14	-22	25.6	26.6
1.05	8	-22	22.8	23.6
2.05	4	-24	19.9	20.5
3.1	0	-30	17.0	17.5
4.1	-4	-36	14.1	16.6
5.15	-10	-44	13.1	14.7
6.2	-12	-52	11.1	14.8
7.2	-12	-62	8.1	13.9
8.2	-12	-72		11.9
9.2	-12	-82		13.1
10.25	-14	-94		14.3
11.3		-104		14.5

SYMBOLS

A, B	constants of integration
b	aircraft wingspan, feet
C	a constant
K	circulation ($= 4L/\pi\rho Vb$) feet^2 sec^{-1}
L	aircraft lift, lbf
r	radial distance from vortex centre, feet
s	half the horizontal separation of the vortices
T	vortex interaction time, seconds
t	vortex age, seconds
u	headwind component along aircraft track, kn
V	aircraft airspeed, kn
v	circumferential vortex velocity, feet sec^{-1}
y	lateral position of vortex core in measuring plane, +ve towards ground camera, feet
$\dot{y}$	horizontal velocity of vortex core in measuring plane, +ve away from plane of symmetry, feet sec^{-1}
z	vertical position of vortex core in measuring plane, +ve upwards, feet
$\dot{z}$	vertical velocity of vortex core in measuring plane +ve upwards, feet sec^{-1}
ρ	air density, slugs ft^{-3}
ν	kinematic viscosity of air,

Suffices

P	port
S	starboard
0	value at instant of generation

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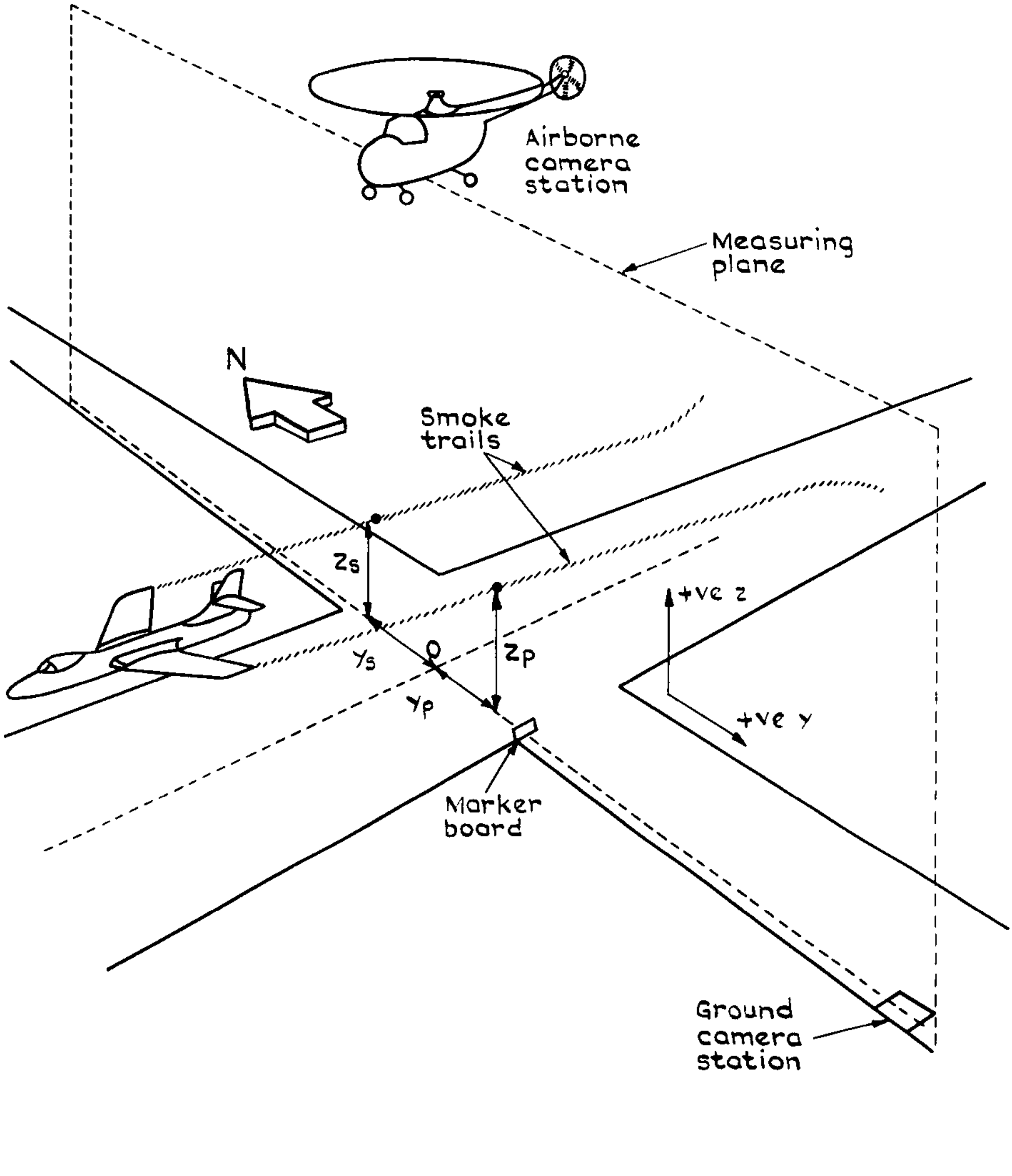


Fig. 1 Diagrammatic view of test area

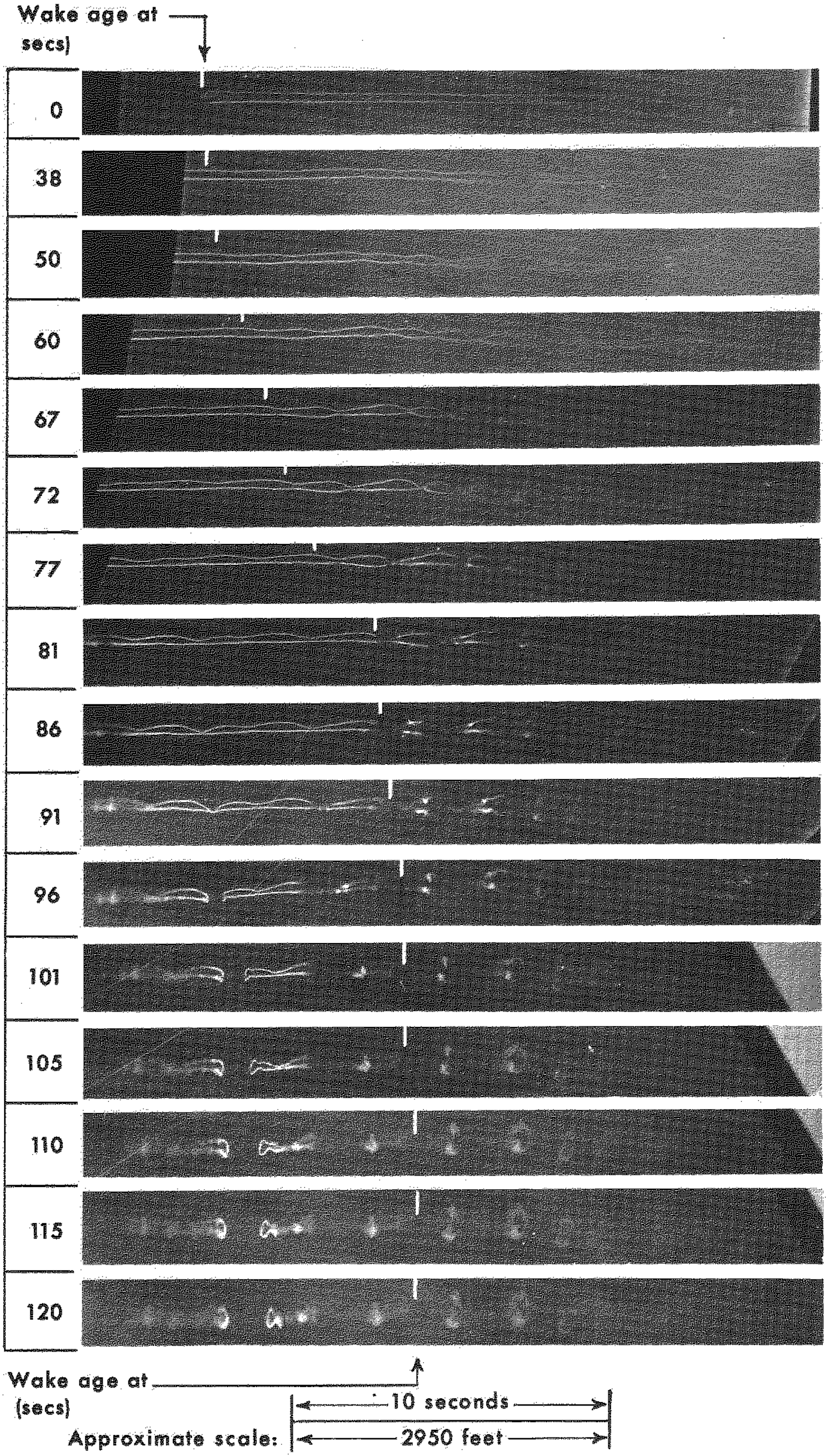


Fig.2. Trailing vortex mutual interaction away from the ground.
Comet aircraft. 150kn. E.A.S. at 10,000 ft.

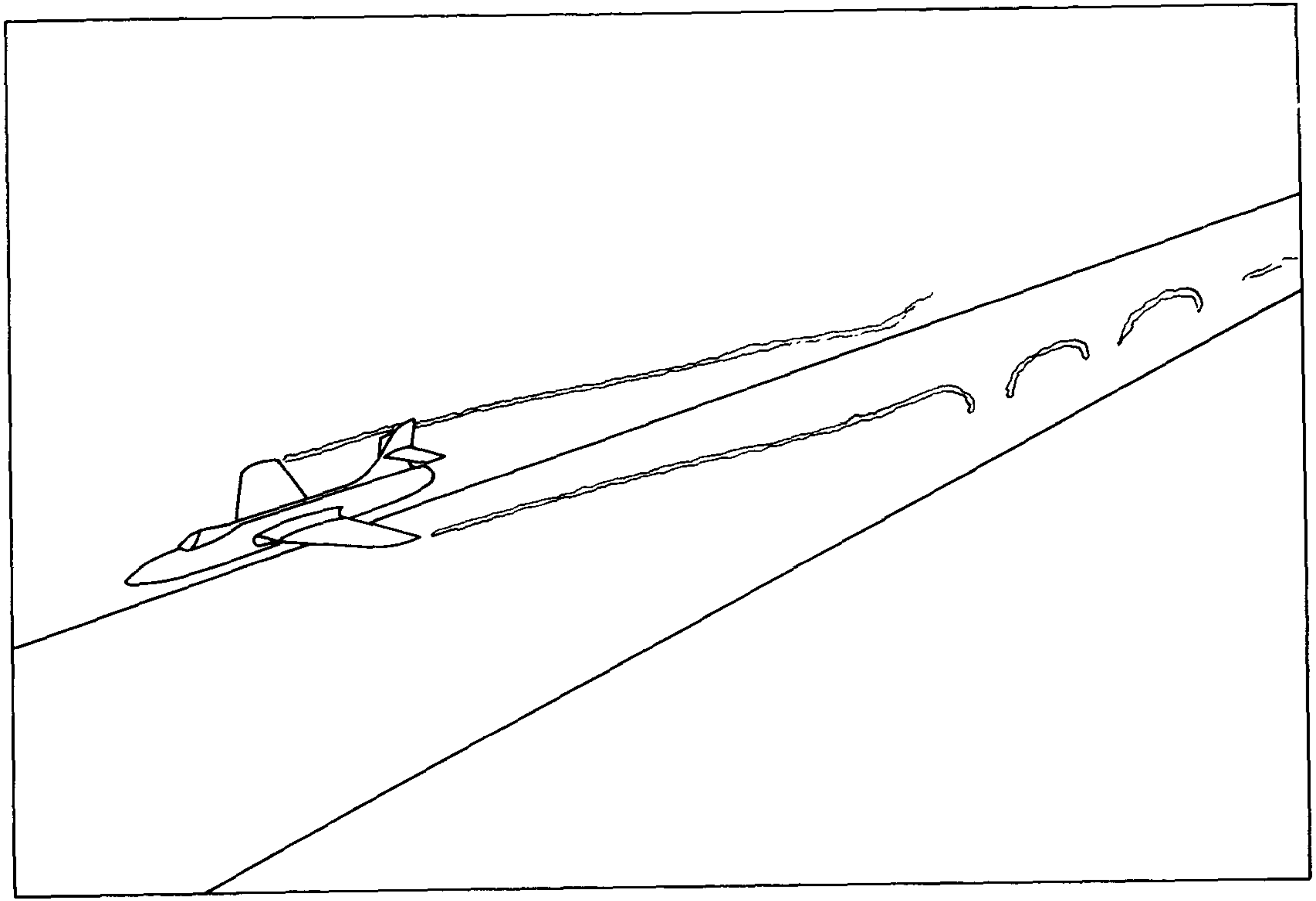


Fig 3 Sketch showing interaction sometimes observed between individual vortices and the ground

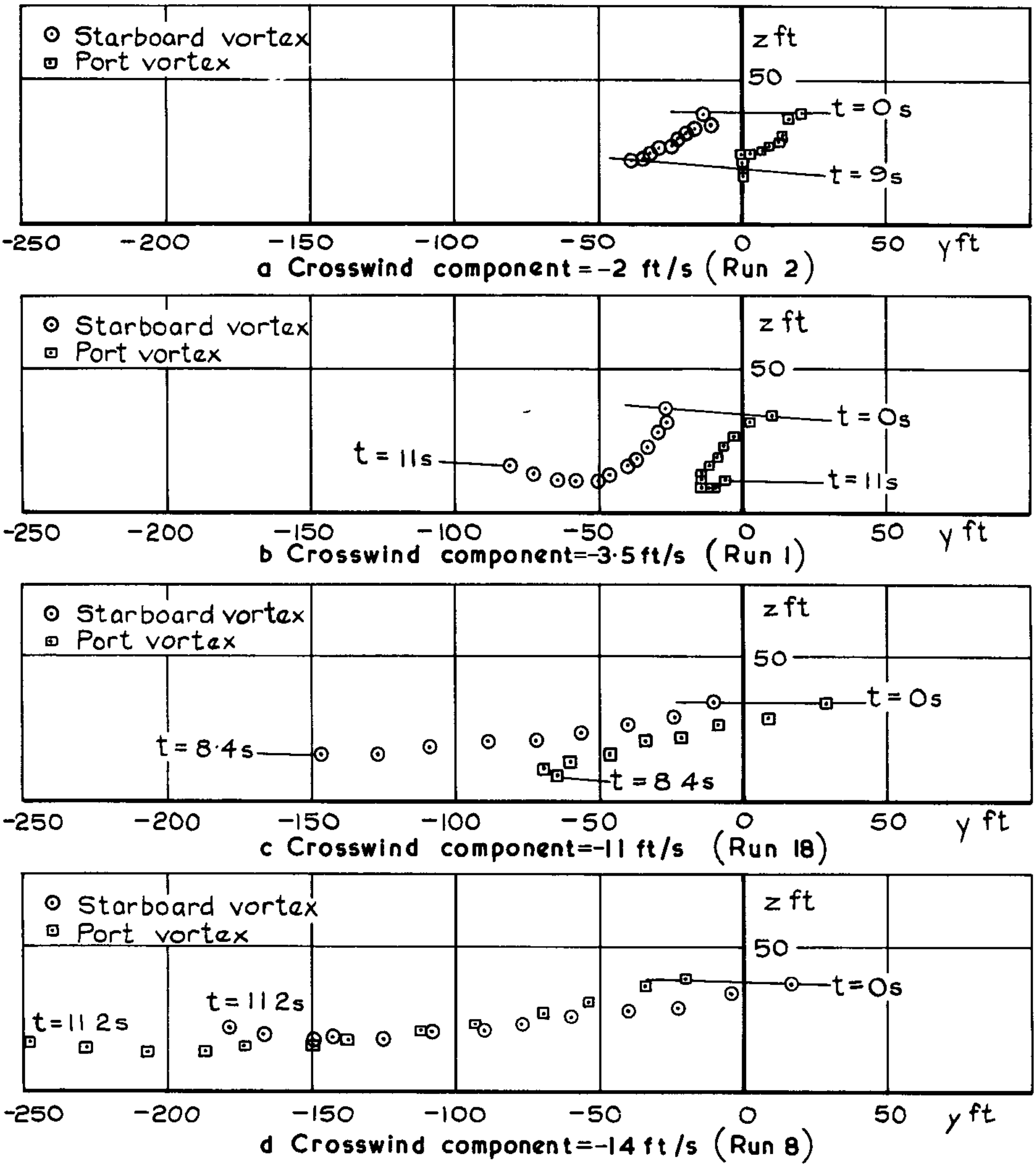


Fig 4a-d Measured vortex positions with different crosswind components

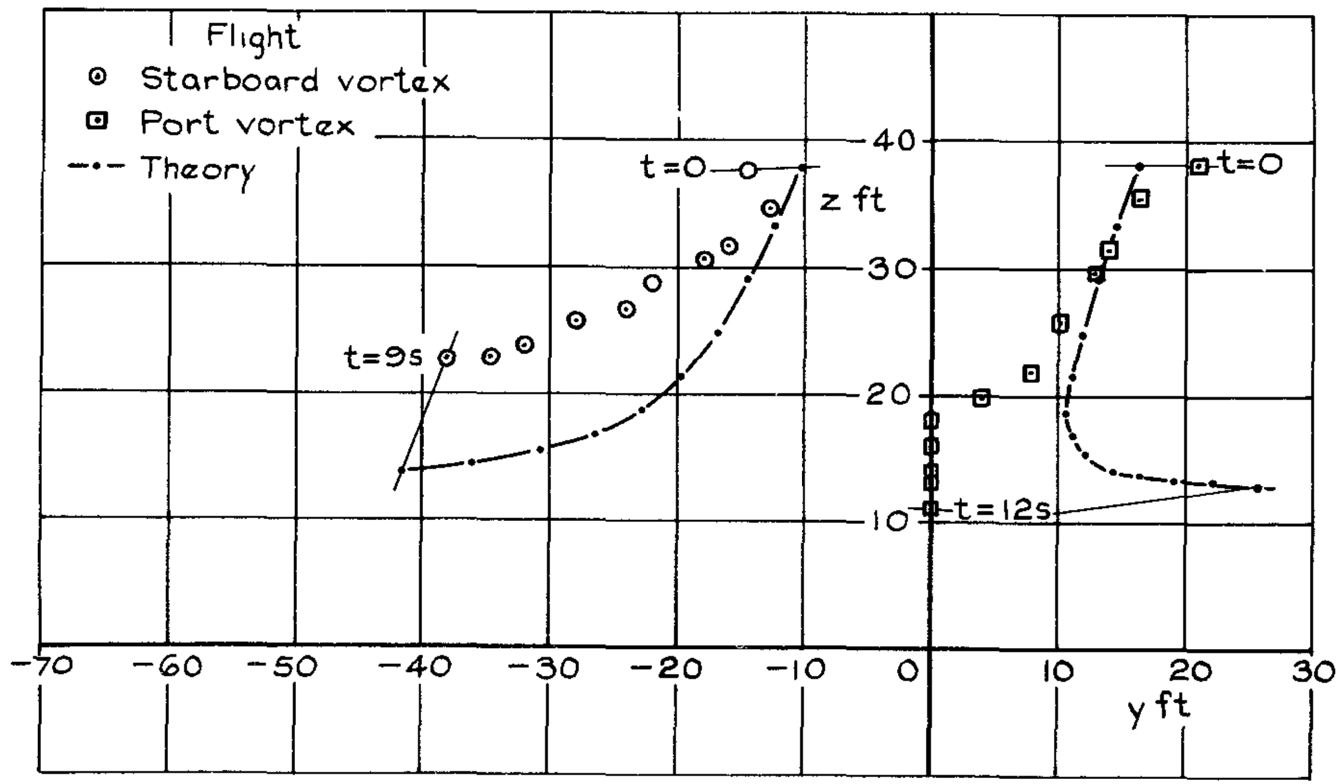


Fig 5 Comparison of measured and theoretical vortex positions
Crosswind = -2 ft/s (Run 2)

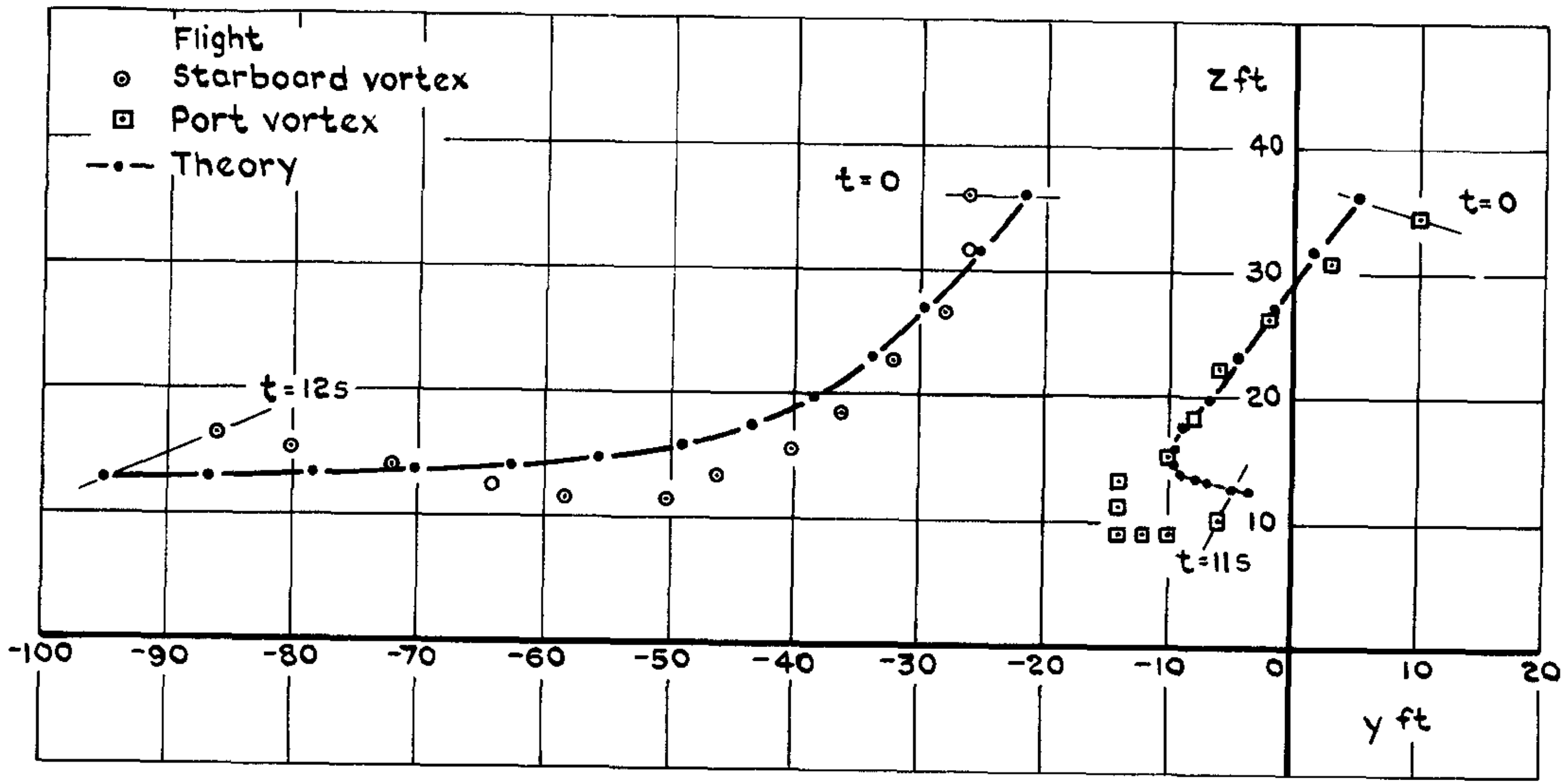


Fig. 6 Comparison of measured and theoretical vortex positions
Crosswind = $-3 \cdot 5$ ft/s (run 1)

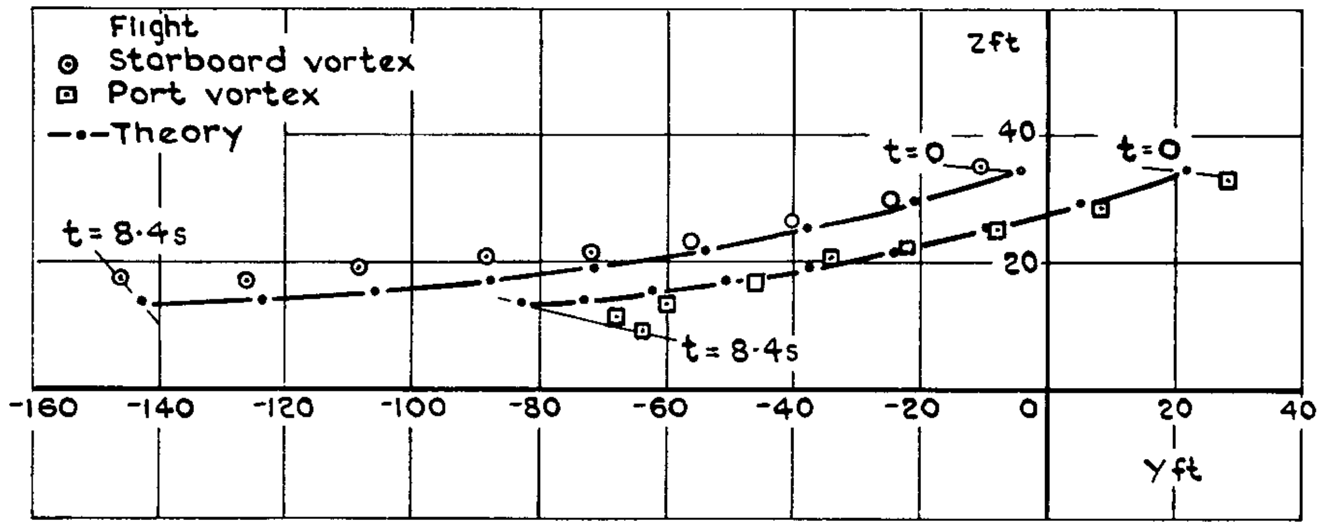


Fig. 7 Comparison of measured and theoretical vortex positions
Crosswind = −11 ft/s (run 18)

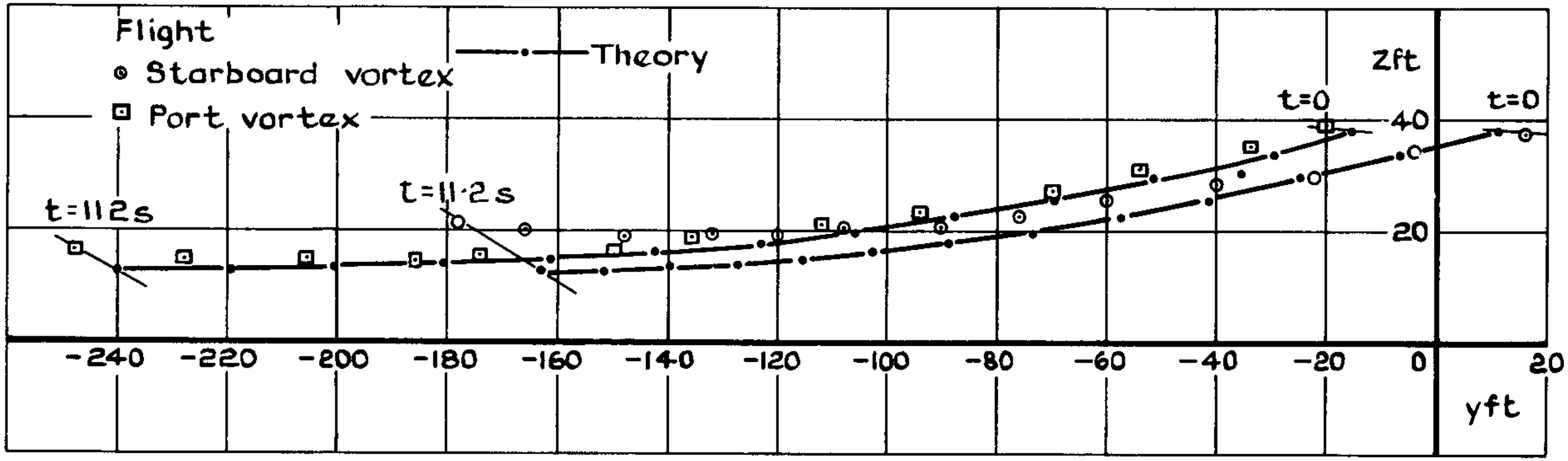
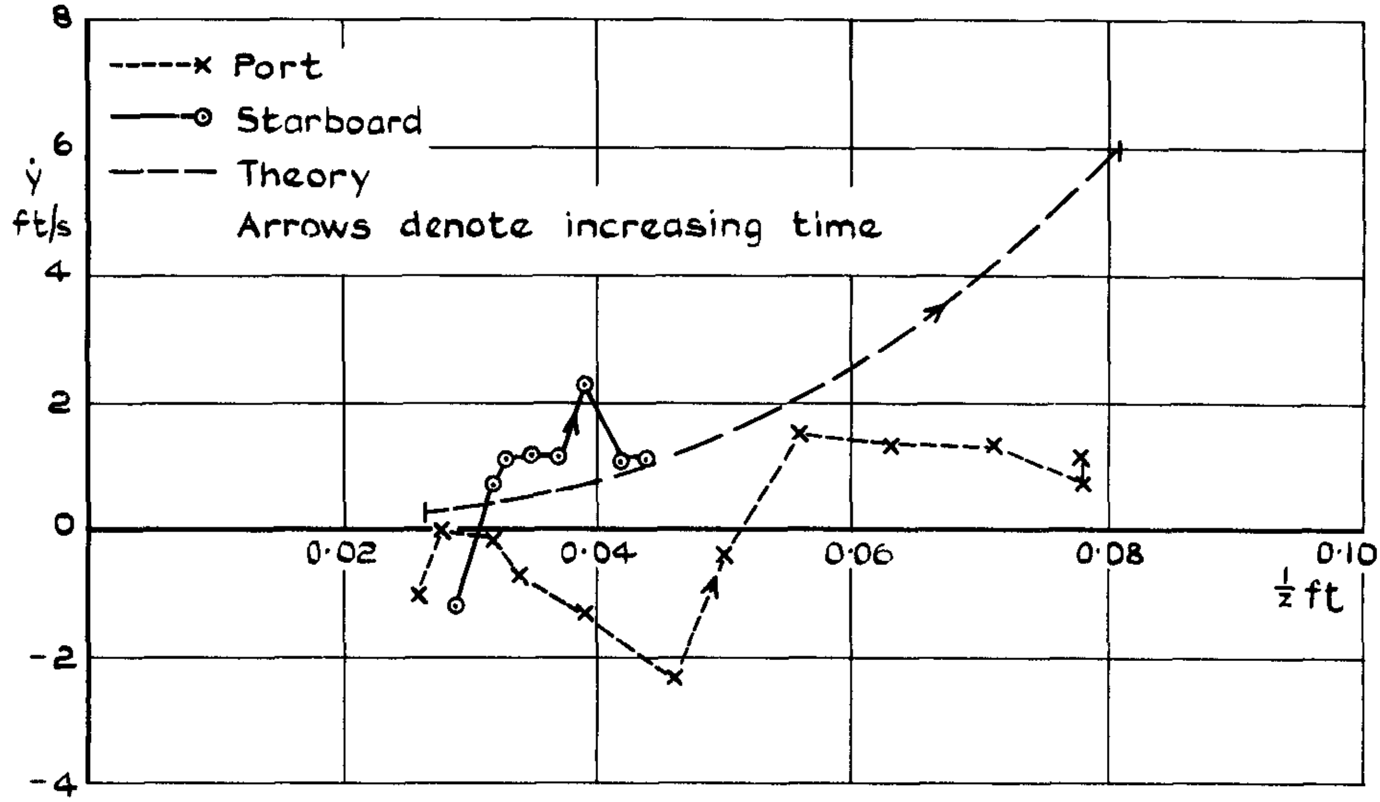
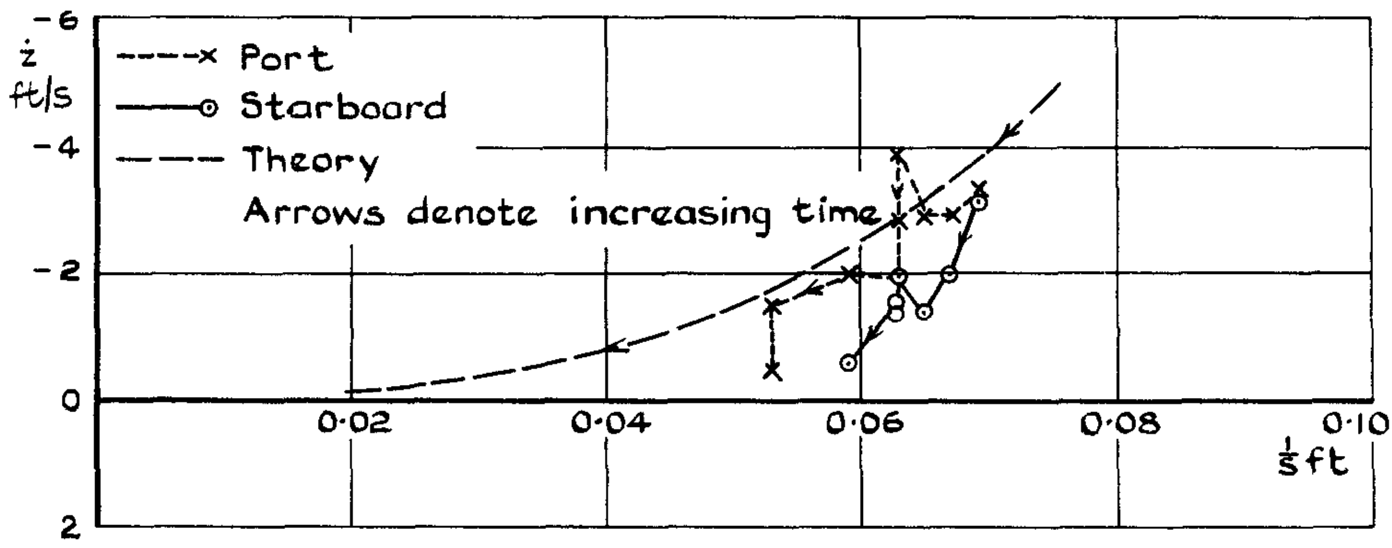


Fig. 8 Comparison of measured and theoretical vortex positions
Crosswind = -14 ft/s (run 8)

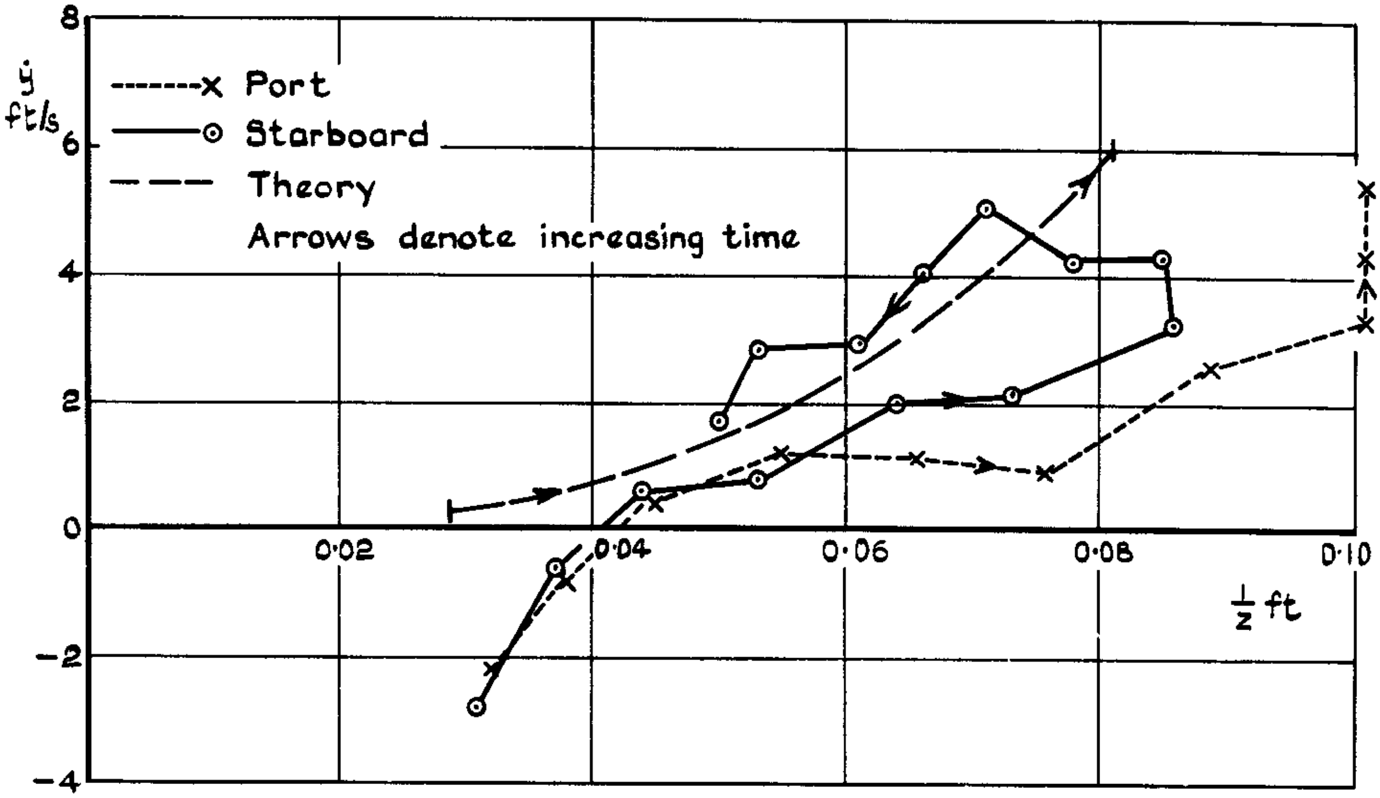


a Horizontal velocity vs (height)$^{-1}$

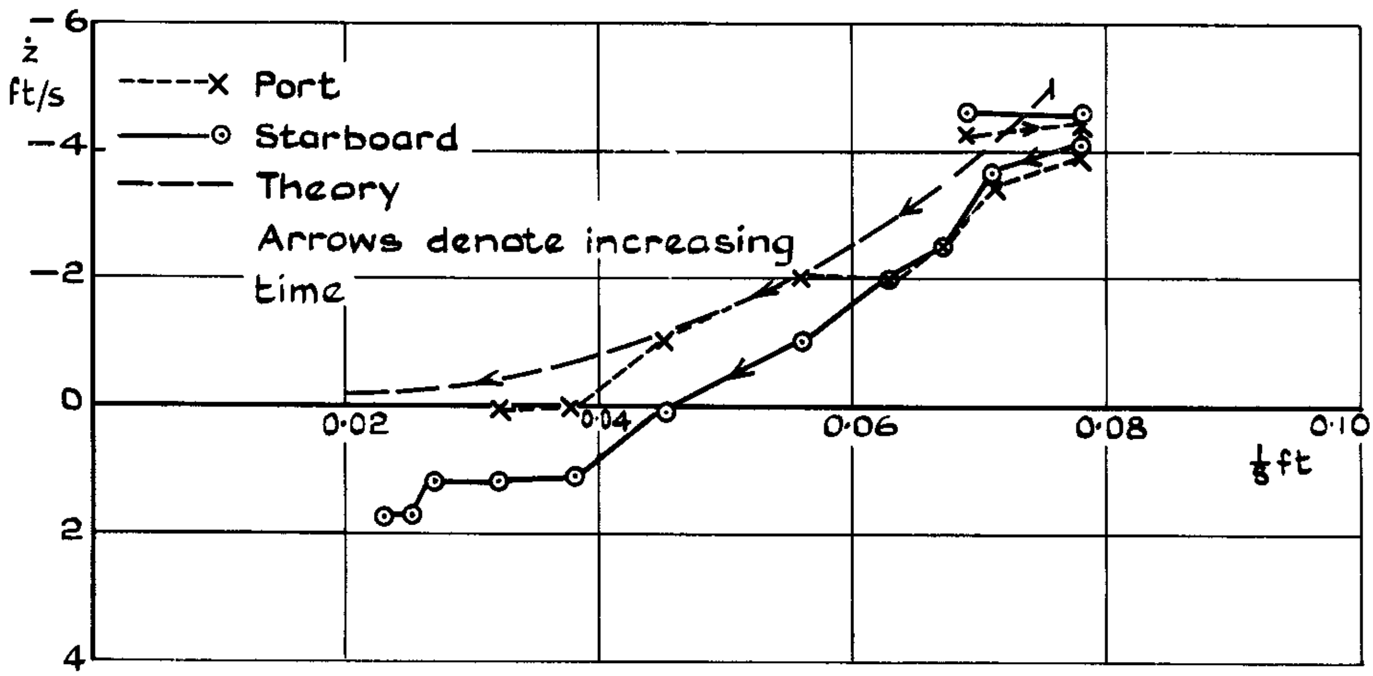


b Vertical velocity vs (half-separation)$^{-1}$

Fig. 9a & b Vortex horizontal and vertical velocity components (run 2) Crosswind − 2 ft/s



a Horizontal velocity vs $(\text{height})^{-1}$



b Vertical velocity vs $(\text{half-separation})^{-1}$

Fig.10a&b Vortex horizontal and vertical velocity components (run 1)
Crosswind −3·5 ft/s

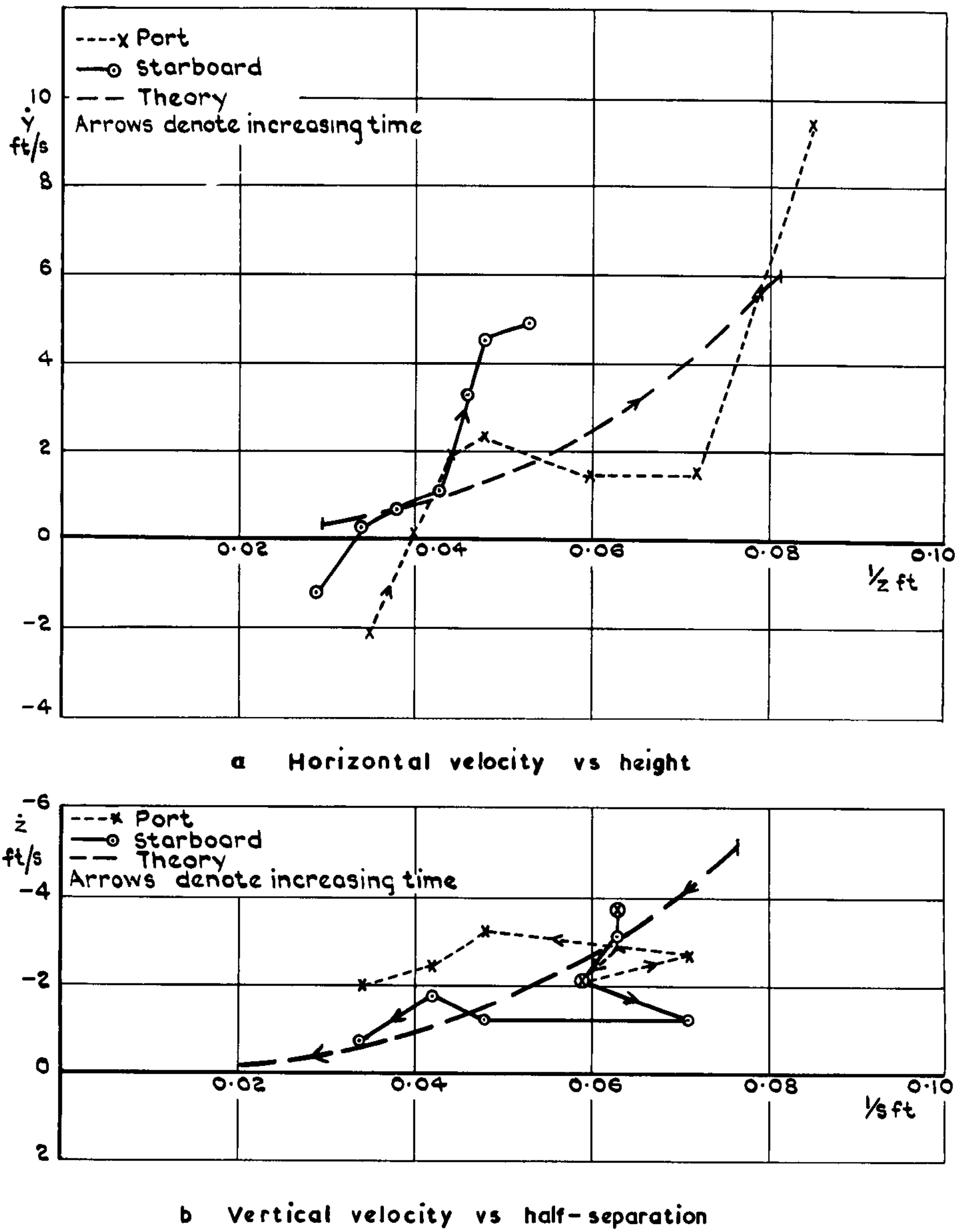
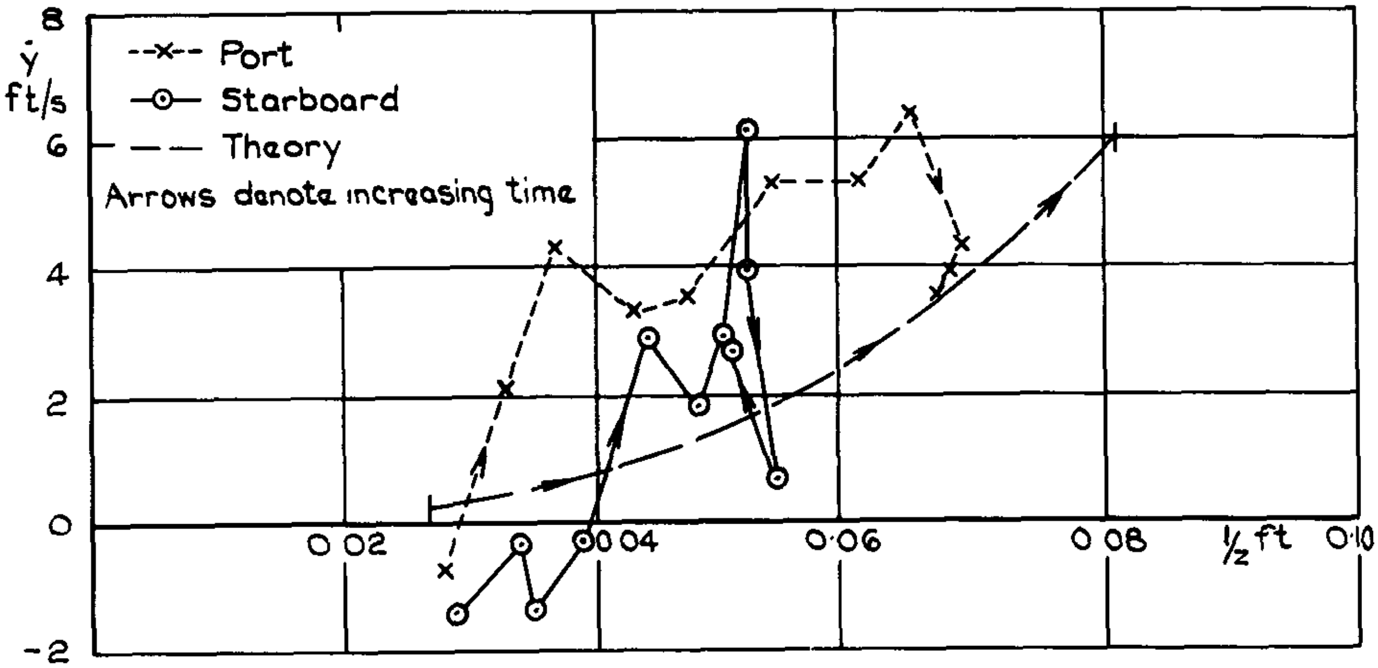
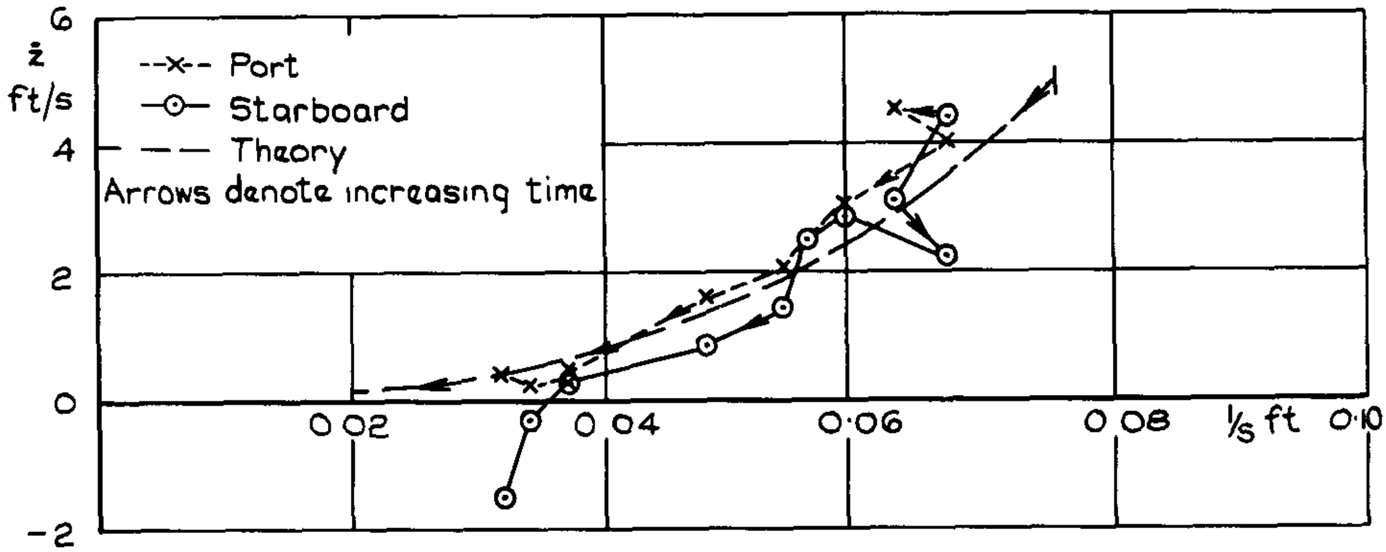


Fig.11 a & b Vortex horizontal & vertical velocity components (run 18)
Crosswind −11 ft/s



a Horizontal velocity vs height



b Vertical velocity vs half-separation

Fig.12a & b Vortex horizontal & vertical components (Run 8)
Crosswind – 14 ft/s

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