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Discrete Element Analysis of the Lateral Vibration of Rectangular Plates in the Presence of Membrane Stresses

by

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DISCRETE ELEMENT ANALYSIS OF THE LATERAL VIBRATION OF RECTANGULAR PLATES IN THE PRESENCE OF MEMBRANE STRESSES

by
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SUMMARY

The discrete element displacement method is applied to the prediction of the natural frequencies of lateral vibration of rectangular plates which may be subjected to arbitrary systems of in-plane loading. Numerical results are given for several problems in which the applied stress system is uniform and these results are shown to be in good agreement with available solutions.

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CONTENTS

1 INTRODUCTION

The prediction of the natural frequencies of transverse vibration of thin rectangular plates and the estimation of their buckling loads under systems of middle-surface forces has received considerable attention. The combined problem of transverse vibration in the presence of in-plane loading has, however, been studied by relatively few authors[1-6]. It is well-known that the frequency of a particular mode of vibration tends to zero as the corresponding critical buckling load is approached, and that the membrane stress level is linearly related to the square of frequency in cases in which the vibrational mode shape is exactly the same as the buckled shape.

In general, however, no such relationship can be deduced and the combined problem must be analysed at various stress levels. An application of the discrete element displacement method to rectangular plates under these conditions is described in this paper, which uses a non-conforming type of rectangular element employed previously by the author in investigations of the limiting cases mentioned above[7,8]. A computer program has been prepared which is applicable with any system of membrane loads but, because of the lack of comparative results for more complex stress distributions, the examples given in this paper are restricted to problems in which the in-plane stresses are uniform. The displacement method results are shown to be in close agreement with exact or accurate approximate values.

2 ANALYSIS PROCEDURE

The plate is divided for purposes of analysis into a number of rectangular regions or elements (shown in Fig.1) within each of which a polynomial expression is assumed for the lateral deflection such that the latter can be defined explicitly by the generalised displacements at the four corner nodal points of the element. The elements are deformed under the combined action of a distributed lateral inertia loading, an applied membrane stress distribution and shear forces and moments transmitted by adjacent elements. If a typical element is vibrating harmonically with a circular frequency p and an amplitude $w(x,y)$, the maximum element flexural potential energy, V, is given by

$$V = U_b - W_g - W_1, \tag{1}$$

where
$$U_b = \frac{D}{2}\int_{-b}^{b}\int_{-a}^{a}\left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 + 2(1 - \nu)\left\{\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2}\right\}\right] dx\, dy, \quad \ldots\ (2)$$

$$W_g = \frac{h}{2}\int_{-b}^{b}\int_{-a}^{a}\left[\sigma_{xx}\left(\frac{\partial w}{\partial x}\right)^2 + \sigma_{yy}\left(\frac{\partial w}{\partial y}\right)^2 + 2\,\sigma_{xy}\frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y}\right] dx\, dy, \qquad (3)$$

and W_l represents the potential of shear forces and moments distributed along the element edges. The corresponding maximum kinetic energy is

$$T = \frac{1}{2}\rho p^2 \int_{-b}^{b}\int_{-a}^{a} w^2\, dx\, dy. \qquad (4)$$

Here ρ is the mass density per unit area, h is the plate thickness, E is Young's modulus, ν is Poisson's ratio and $D[= E h^3/12(1 - \nu^2)]$ is the plate flexural rigidity; σ_{xx}, σ_{yy} and σ_{xy} are the components of in-plane stress whose sign convention is such that direct stresses are positive in compression and shear stress is positive when acting towards corners 2 and 4 of the plate element illustrated in Fig.1.

In the above expression for the element potential energy, U_b is the strain energy of bending whilst W_g represents the work done by the midplane loads during bending deformation. This latter term arises from the need, in the presence of significant membrane stress, to include rotational terms in the middle-surface strain-displacement equations.

The lateral deflection of the element is a linear function of N generalised displacements, q_i, and the expressions for U_b, W_g and T are consequently quadratic functions of these displacements; the remaining term, W_l, can be expressed as the sum of the products of the generalised displacements and the corresponding generalised forces. The appropriate values of the generalised displacements are those which make the energy $(V - T)$ a minimum[11]. This minimisation procedure leads to the following linear system of equations for the element:

$$\frac{\partial}{\partial q_i}\left(\frac{1}{2}\underline{q}^t\,\underline{K}_b\,\underline{q} - \frac{\lambda}{2}\underline{q}^t\,\underline{K}_g\,\underline{q} - \frac{p^2}{2}\underline{q}^t\,\underline{M}\,\underline{q} - \underline{q}^t\,\underline{Q}\right) = 0, \qquad i = 1\ldots N, \qquad (5)$$

which may be expressed in the form

$$(\underline{K}_b - \lambda\,\underline{K}_g - p^2\,\underline{M})\,\underline{q} = \underline{Q}\,. \tag{6}$$

Here $\underline{q}$ and $\underline{Q}$ are the column vectors of the generalised displacements and forces respectively at the corners of the element; $\underline{K}_b$ is the elastic bending stiffness matrix; $\underline{K}_g$ is the geometric or initial-stress stiffness matrix which reflects the influence of membrane stress on bending behaviour and which corresponds to a unit value of the load factor λ; $\underline{M}$ is the consistent mass matrix.

The energy expressions for the complete plate are simply summations of the energies of the discrete elements, provided that the localised displacement patterns are such that the overall displacement of the assembled structure is kinematically admissible. Similarly, the complete-plate matrices, $\underline{K}_b^*$, $\underline{K}_g^*$ and $\underline{M}^*$, which relate the vectors of generalised force and displacement, $\underline{Q}^*$ and $\underline{q}^*$, for the complete structure, are appropriate summations of element matrices with rows and columns deleted which correspond to displacements that are prescribed zero. In the absence of static normal forces the vector of those complete-plate generalised forces corresponding to the unprescribed displacements is null and equation (6) can then be rewritten for the complete plate in the standard eigenvalue equation form,

$$\left(\frac{1}{p^2}\right)\underline{q}^* = \left(\underline{K}_b^* - \lambda\,\underline{K}_g^*\right)^{-1}\underline{M}^*\,\underline{q}^*\,, \tag{7}$$

from which the natural frequencies of vibration may be found for a given value of λ.

The accuracy of the displacement method results is dependent upon the suitability of the assumed localised displacement patterns. In the plate bending problem such patterns should maintain both lateral deflection and slope continuity with corresponding patterns in adjacent elements if a bound on the total energy is to be obtained. A conforming expression of this type may be generated[9] for the rectangular element based on the product of cubic expressions in x and in y and involving four generalised displacements at each node. However, numerical convergence with decreasing grid size may be achieved using elements which permit small normal slope discontinuities at element boundaries and whose assumed deflected forms may be expressed in terms of only three generalised displacements at each node. Although the energy of the complete plate composed of such non-conforming elements is not given strictly by the sum of the discrete element energies, convergence to true energy levels will occur if the assumed displacement pattern is capable of representing arbitrary states of uniform strain[10]; such idealisations do not, however, give a bound on the total energy.

A non-conforming element of this type which has demonstrated excellent convergence characteristics in previous applications[7,8] has been adopted in the present work. The nodal generalised displacements are the lateral deflection, w, and the two slopes $\phi = \partial w/\partial Y$ and $\theta = \partial w/\partial X$, and the deflection pattern is given by,

$$w = \frac{1}{8}\sum_{i=1}^{4}\Big([(1+X_i X)(1+Y_i Y)\{2+X_i X(1-X_i X)+Y_i Y(1-Y_i Y)\}+2\mu X_i Y_i XY]\, w_i$$
$$+ [(-Y_i)\{(1+X_i X)(1+Y_i Y)^2 (1-Y_i Y)+\mu \cdot X_i X(1+Y_i Y)\}]\, \phi_i$$
$$+ [(-X_i)\{(1+X_i X)^2 (1-X_i X)(1+Y_i Y)+\mu Y_i Y(1+X_i X)\}]\, \theta_i\Big) . \quad (8)$$

Here $\mu = C(1 + X)(1 - X)(1+Y)(1 - Y)$, C being a constant taken as 1/3 in all numerical examples; $X = x/a$ and $Y = y/b$, where X_i, Y_i are the non-dimensional co-ordinates of a typical nodal point i; 2a and 2b are the length and width of the element as shown in Fig.1

3 NUMERICAL STUDIES

A computer program has been written in the Atlas dialect of Fortran to calculate the natural frequencies and mode shapes of uniform rectangular plates in the presence of in-plane loading. Because of the absence of comparative data the results given below concern plates subjected to uniform stress systems only. The program includes, however, operations to calculate the approximate membrane stress distribution corresponding to a prescribed system of loads and based on the assumption of linearly varying membrane displacements between nodes[8]; the solution to the plane stress problem, which is, of course, exact for uniform stress systems, is used directly in the vibration analysis.

In all applications the value of Poisson's ratio is taken to be 0·3. The results are presented in terms of a frequency parameter, α, and a stress parameter, k, where

$$\alpha = \frac{p}{\sqrt{\dfrac{\pi^4 D}{\rho B^4}}} \qquad \text{and} \qquad k = \frac{\sigma B^2 h}{\pi^2 D} .$$

Here σ is the value of a direct or shear stress component used for reference purposes and B is the length of the plate edge parallel to the y axis.

Simply supported square plate, uniform uniaxial stress

The mode shapes of free lateral vibration of a square simply-supported plate are identical to the corresponding buckled shapes when the plate is uniformly compressed in one direction. Consequently the square of frequency varies linearly with the intensity of stress in such a way that

$$\alpha^2 = \alpha_o^2 \left(1 - \frac{k}{k_{cr}}\right) ,$$

where, for a given mode shape, α is the value of the frequency parameter at a stress σ, k is the corresponding stress parameter, α_o is the frequency parameter at zero stress and k_{cr} is the stress parameter corresponding to zero frequency, i.e. k_{cr} is the buckling stress parameter.

In the application of the displacement method to this doubly symmetric problem only a quarter of the plate was considered, this quadrant being idealised as twenty five equal square elements. Frequencies of vibration were calculated for the first ten modes at zero stress and at stress levels (in the x-direction) of +0·5, +0·99 and -1·0 times the lowest buckling stress; in addition, the corresponding buckling stress parameters were calculated directly using a plate stability program based on the same assumed displacement form and plate idealisation. For each modal pattern the five points thus obtained are linearly related to an accuracy of about six significant figures. The calculated parameters α_o and k_{cr} are given in Table 1 together with the comparative exact values[11,12]; it can be seen that all calculated parameters are within half a per cent of the corresponding exact values. The mode shapes are defined in the table in the form m/n where m and n are the numbers of half-waves in the x and y directions respectively. The calculated mode shapes are sensibly constant at all stress levels and agree with the exact sinusoidal patterns to four figure accuracy.

Simply-supported square plate with central point support, uniform uniaxial stress

Nowacki[4] has considered the variation, with increase in uniform uniaxial compressive stress, of the fundamental frequency of a square plate simply-supported around the edges and point-supported at the centre in such a way that the deflection and the slopes are zero at this point. The unusual prescribed displacements present no difficulty to the displacement method which has been applied to this problem using, in turn, nine, sixteen and twenty-five square elements in the quarter plate. The results obtained are presented along with the comparative values in Table 2.

Clamped square plate, uniform biaxial stress

Using twenty-five equal elements in the quarter-plate the first ten frequencies of the clamped square plate subjected to a uniform biaxial stress system, $\sigma_{xx} = \sigma_{yy} = \sigma$, have been calculated for a range of stress intensities between $k = +5$ and $k = -5$. Fig.2 shows that the variation of the square of frequency with stress is, in this range, practically linear for all modes; correspondingly any given mode shape changes little with stress. The accuracy of the discrete element results may be judged by comparison of the calculated natural frequencies at zero stress with accurate approximate values[13] as in Table 3 where very close agreement is demonstrated. Furthermore, linear extrapolation of calculated values of α^2 at $k = 4$ and $k = 5$ shows that the fundamental frequency becomes zero at $k = 5{\cdot}301$ whereas a direct discrete element stability calculation gives the critical stress parameter as $k = 5{\cdot}298$ and an accurate comparative solution is $k = 5{\cdot}30$[12].

It is interesting to note that the discrete element results indicate that, as the membrane stress becomes increasingly tensile, the percentage difference between the frequencies of the modes m/n + n/m and m/n - n/m is reduced for both pairs of normal modes of the type m/n ± n/m which are depicted in Fig.2. For the first pair of modes (modes 3/1 ± 1/3) the difference in the squares of the calculated frequencies reduces steadily from about two per cent at $k = +5$ to half per cent at $k = -5$ and at a very high tensile stress corresponding to $k = -200$ this difference is only about one part in ten thousand.

Weinstein and Chien[5] have calculated upper and lower bounds for the fundamental frequency of the clamped square plate under a uniform biaxial tension varying from $k = -5$ to $k = -200$ and consider that their lower bound solutions are much closer to the true frequencies than the upper bounds. The bound solutions are listed in Table 4 together with corresponding displacement method results based on idealisation of the quarter-plate into 9, 16 and 25 elements in turn; all the latter values lie within the given bound limits. Although some small decline in the rate of convergence of the numerical results is evident for very large tensions, the results obtained in the present investigation compare closely with the lower bound solutions. By far the greatest difference corresponds to a stress level of k = -100; if, however, the lower bound values of the square of frequency are plotted against the stress parameter, it appears probable that this lower bound solution is inaccurate.

Rectangular plate, uniform longitudinal stress

The variation of the natural frequencies of a rectangular plate of aspect ratio three, with the long edges simply supported and short edges clamped (see Fig.3), has been studied for values of longitudinal compressive stress ranging from zero to the lowest buckling stress. An exact solution to this problem is known[6] which demonstrates that the variation in the square of frequency with intensity of stress is considerably non-linear for certain modes of vibration. This theoretical variation is shown in Fig.4 for the first five modes along with comparative displacement method results, and the highly non-linear region of this figure is shown in more detail in Fig.5. The discrete element results are based on an idealisation of the quarter-plate into 27 elements, as shown in Fig.3, and accurately predict the true behaviour of the plate.

The effect of change in the intensity of compressive stress on the modal patterns of the vibrating plate is illustrated in Fig.6 which shows, for the first two symmetric-symmetric modes, the deflected shapes predicted by the displacement method of the longitudinal centre line (C - D of Fig.3) corresponding to specified values of the stress parameter k. As expected, the patterns vary little with stress intensity in regions where the relationship between the square of frequency and stress is almost linear, but change rapidly in regions of considerable non-linearity. In the exact solution[6] the deflected shape of the first symmetric-symmetric mode is wholly positive (in the sense of Fig.6) for values of k up to k = 3·111 and in the second mode the deflection becomes wholly positive at k = 3·778. The corresponding limiting values obtained from the discrete element results are k = 3·10 (neglecting areas within a distance of 0·002 A of the clamped edges, A being the plate length) for the first symmetric-symmetric mode and k = 3·755 for the second mode.

Clamped rectangular plate, uniform shear stress

The final problem considered is the effect of uniform shear stress on the natural frequencies of a rectangular plate of aspect ratio 3/2 with all edges clamped. So far as is known by the author no comparative solutions exist for this combined problem but bounded values for the first two buckling stresses have been calculated by Budiansky and Connor[14] and accurate approximate values for the lower natural frequencies may be deduced from the work of Claassen and Thorne[13].

The variation in the square of frequency for the first two modes, as calculated by the displacement method using both 36 and 49 equal elements in the complete plate, is shown in Fig.7 for a range of shear stress from zero to the second buckling level. The closeness of the 36-element and 49-element solutions suggests that errors in the present analysis are small. This opinion is strengthened on checking the accuracy of the end points of the two curves. The first two calculated frequency-parameters at zero stress, α_o, and the comparative values based on a Fourier series solution[13] are given in the following table. The latter values have been obtained by linear interpolation of results given for plates with aspect ratios of 100/66 and 100/68.

Mode	Displacement method value, α_o		Comparative value
	36 element	49 element	
1	2·735	2·735	2·736
2	4·232	4·229	4·226

The buckling stress parameters, k_{cr}, have been calculated here by a linear interpolation of values of α^2 just to either side of the zero frequency line and are listed below with comparative bound solutions[14].

Mode	Displacement method value, k_{cr}		Comparative values	
	36 element	49 element	Lower bound	Upper bound
1	11·45	11·43	11·45	11·56
2	11·85	11·80	11·79	12·08

4 CONCLUDING REMARKS

It has been shown that the natural frequencies of rectangular plates subjected to uniform membrane stress systems can be calculated to a high degree of accuracy by the discrete element displacement method using a particular rectangular element. In the light of previous work on plate stability[8] there is little doubt that problems involving stress systems of increased complexity can be solved to an acceptable degree of accuracy using the existing computer program.

Table 1

FREQUENCY AND STRESS PARAMETERS OF THE SIMPLY-SUPPORTED SQUARE PLATE

Mode	Shape	Frequency parameter at zero stress, α_o			Buckling stress parameter, k_{cr}		
		Discrete element solution	Exact[11] solution	Error as a percentage	Discrete element solution	Exact[12] solution	Error as a percentage
1	1/1	1·999	2	-0·05	3·996	4	-0·10
2	2/1	4·996	5	-0·08	6·240	6·25	-0·16
3	1/2	4·996	5	-0·08	24·96	25	-0·16
4	2/2	7·984	8	-0·20	15·94	16	-0·37
5	3/1	9·995	10	-0·05	11·10	$11{\cdot}1\dot{1}$	-0·10
6	1/3	9·995	10	-0·05	99·90	100	-0·10
7	3/2	12·97	13	-0·23	18·69	$18{\cdot}7\dot{7}$	-0·46
8	2/3	12·97	13	-0·23	42·06	42·25	-0·45
9	4/1	17·01	17	+0·06	18·08	18·0625	+0·10
10	1/4	17·01	17	+0·06	289·2	289	+0·07

Table 2

VALUES OF THE FUNDAMENTAL FREQUENCY PARAMETER FOR THE SIMPLY-SUPPORTED SQUARE PLATE WITH CENTRAL POINT SUPPORT

Stress parameter k	Discrete element solutions			Comparative solution[4]
	6 × 6 grid	8 × 8 grid	10 × 10 grid	
0	5·404	5·369	5·354	5·33
1	5·219	5·186	5·171	5·16
4	4·538	4·509	4·497	4·48
9	2·294	2·269	2·263	2·27

Table 3

VALUES OF THE FREQUENCY PARAMETERS AT ZERO STRESS, α_o, FOR THE CLAMPED SQUARE PLATE

Mode	Discrete element solution	Series solution[13]	Percentage difference
1	3·644	3·646	-0·05
2	7·434	7·436	-0·03
3	10·954	10·965	-0·10
4	13·336	13·332	+0·03
5	13·398	13·395	+0·02
6	16·702	16·718	-0·10
7	21·370	21·330	+0·19
8	22·259	-	-
9	24·546	24·535	+0·04
10	24·637	-	-

Table 4

VALUES OF THE FUNDAMENTAL FREQUENCY PARAMETER FOR THE CLAMPED SQUARE PLATE UNDER LARGE BIAXIAL TENSION

Stress parameter k	Discrete Element Solutions			Comparative Solutions[5]	
	6 × 6 grid	8 × 8 grid	10 × 10 grid	Lower bound	Upper bound
- 5	5·026	5·024	5·024	5·024	5·051
-10	6·078	6·073	6·072	6·071	6·119
-15	6·961	6·954	6·951	6·949	7·018
-20	7·736	7·725	7·721	7·713	7·811
-30	9·077	9·057	9·051	9·045	9·185
-50	11·265	11·231	11·218	11·207	11·438
-100	15·389	15·319	15·288	15·022	15·702
-200	21·328	21·197	21·134	21·054	21·854

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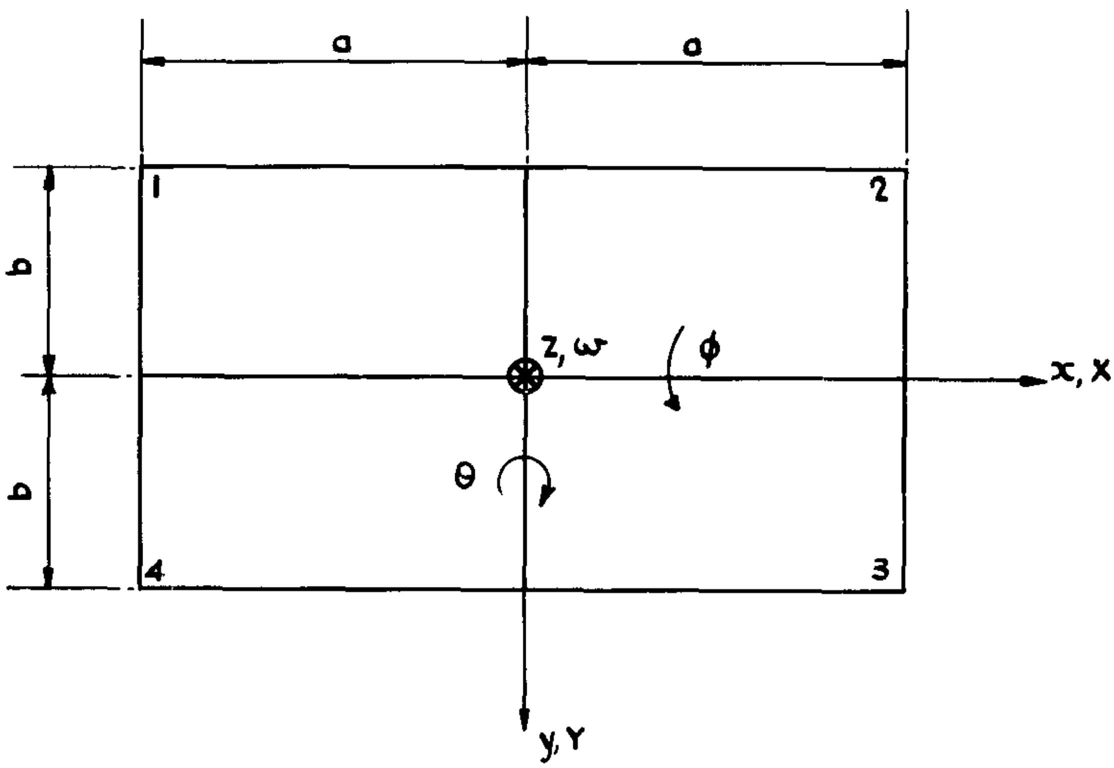

Fig. 1 Plate element details

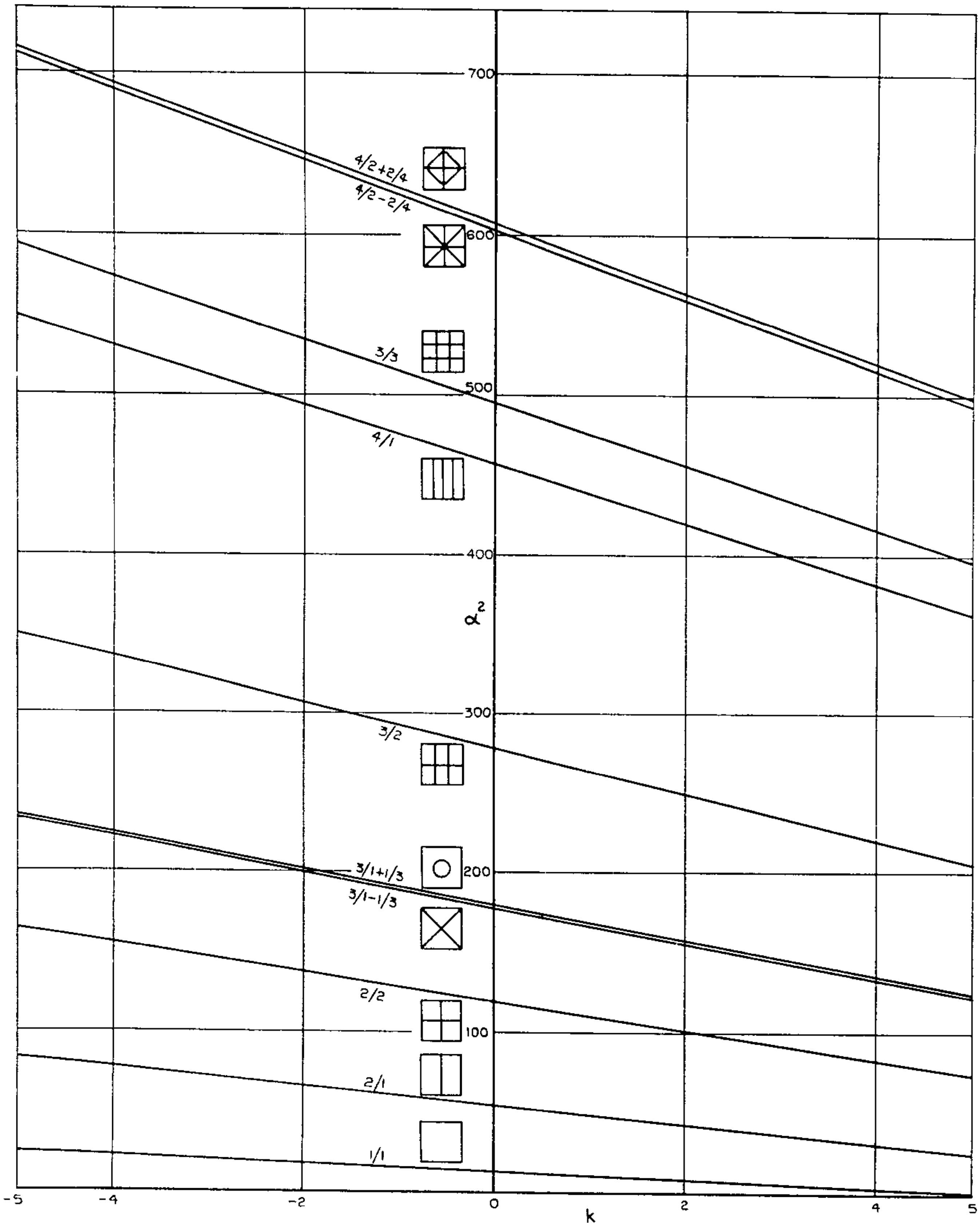

Fig. 2 Variation of frequency parameter with stress parameter for the square clamped plate with biaxial stress

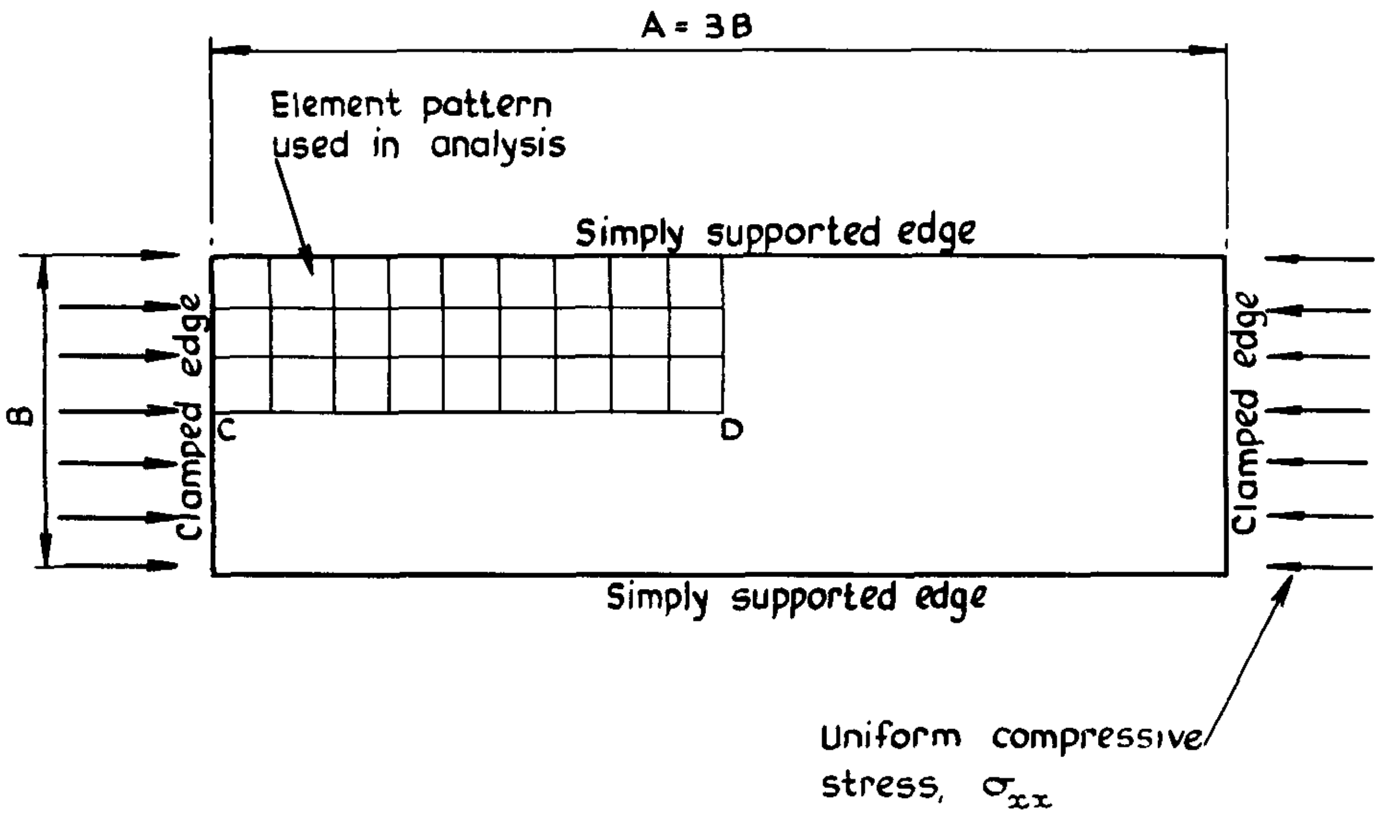

Fig.3 The rectangular plate under longitudinal compression

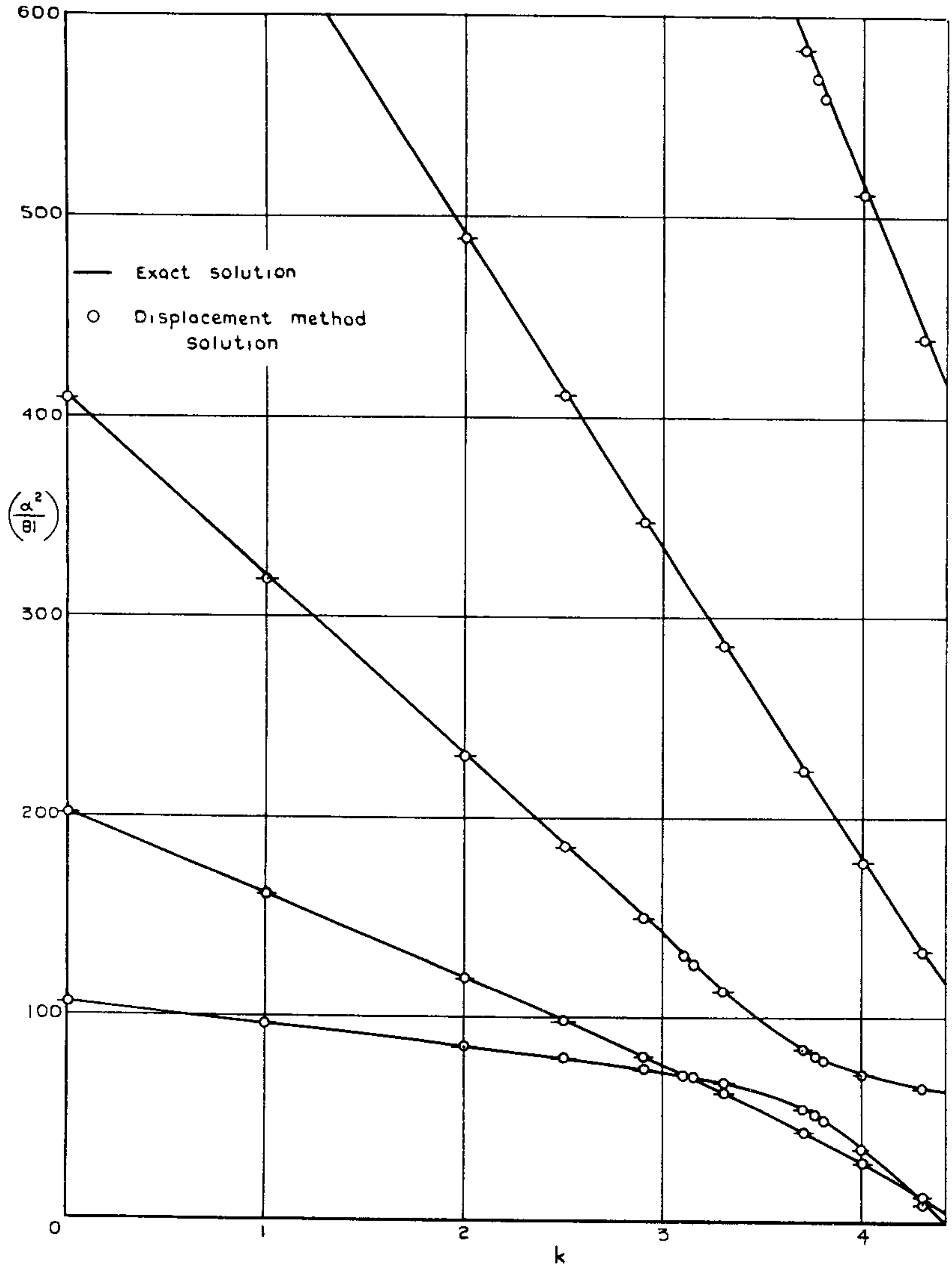

Fig. 4 Variation of frequency parameter with compressive stress for the rectangular plate

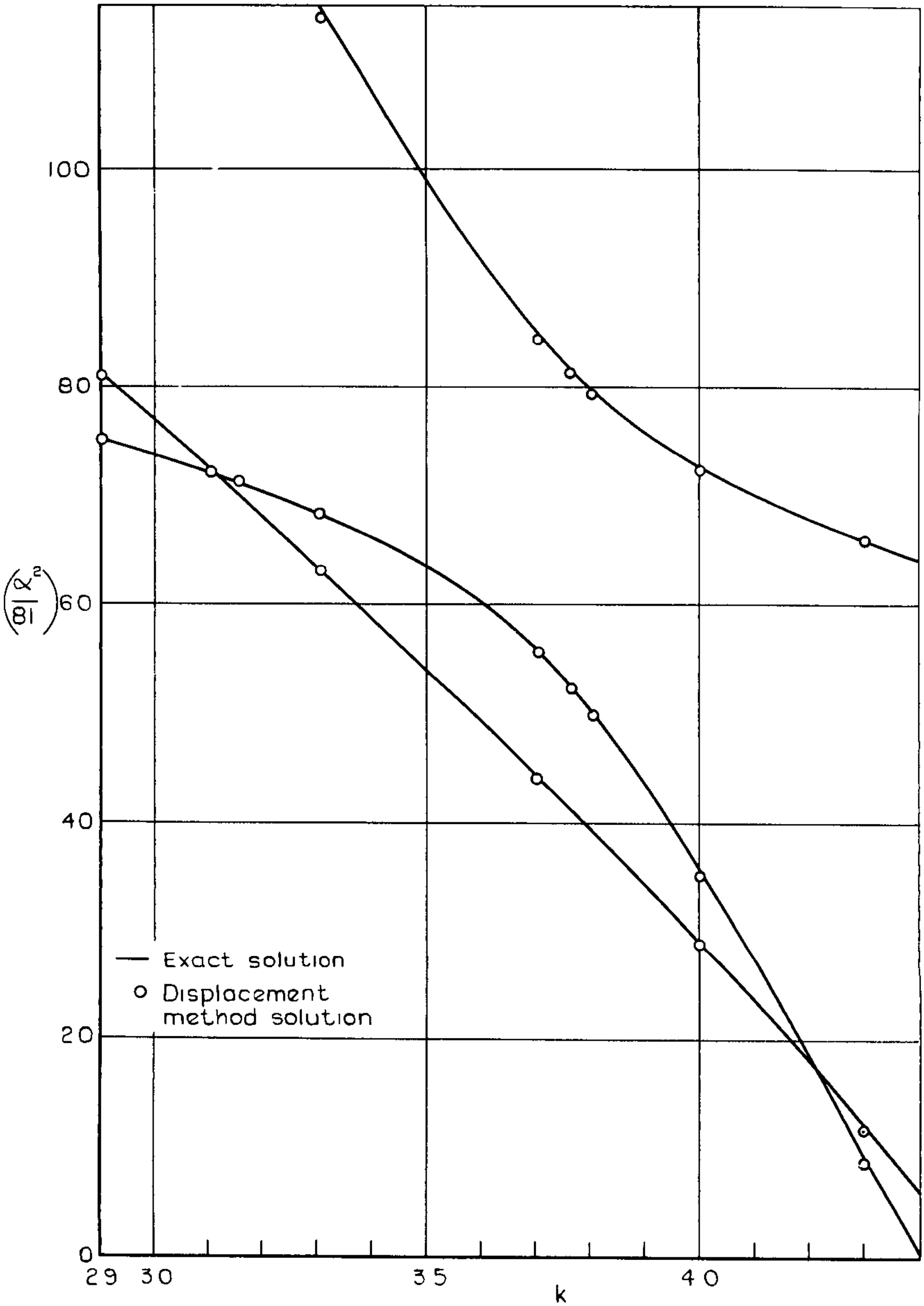

Fig. 5 Variation of frequency parameter with comprehensive stress for the rectangular plate: non-linear region

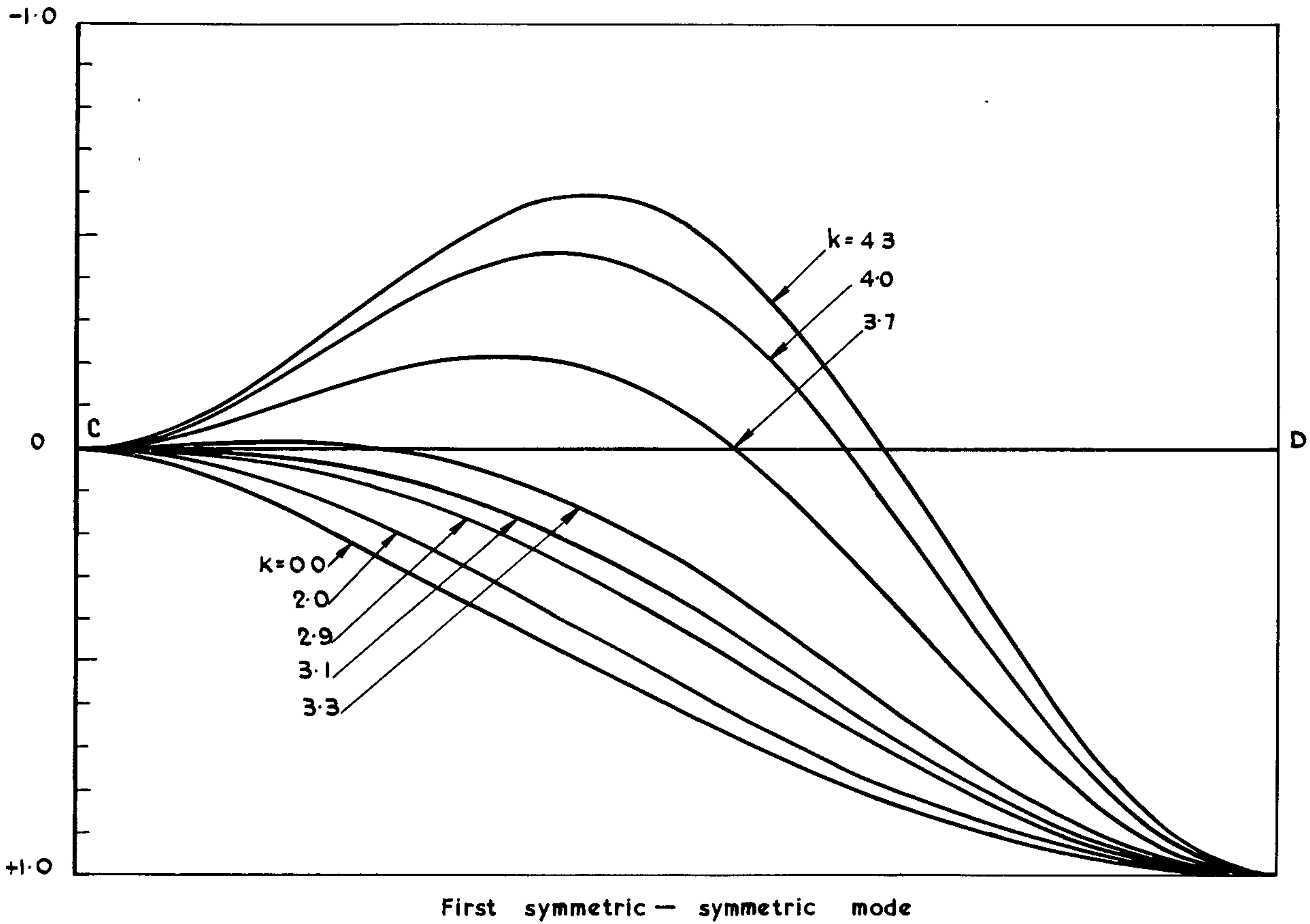

Fig. 6a Effect of compressive stress on the mode shapes of the rectangular plate

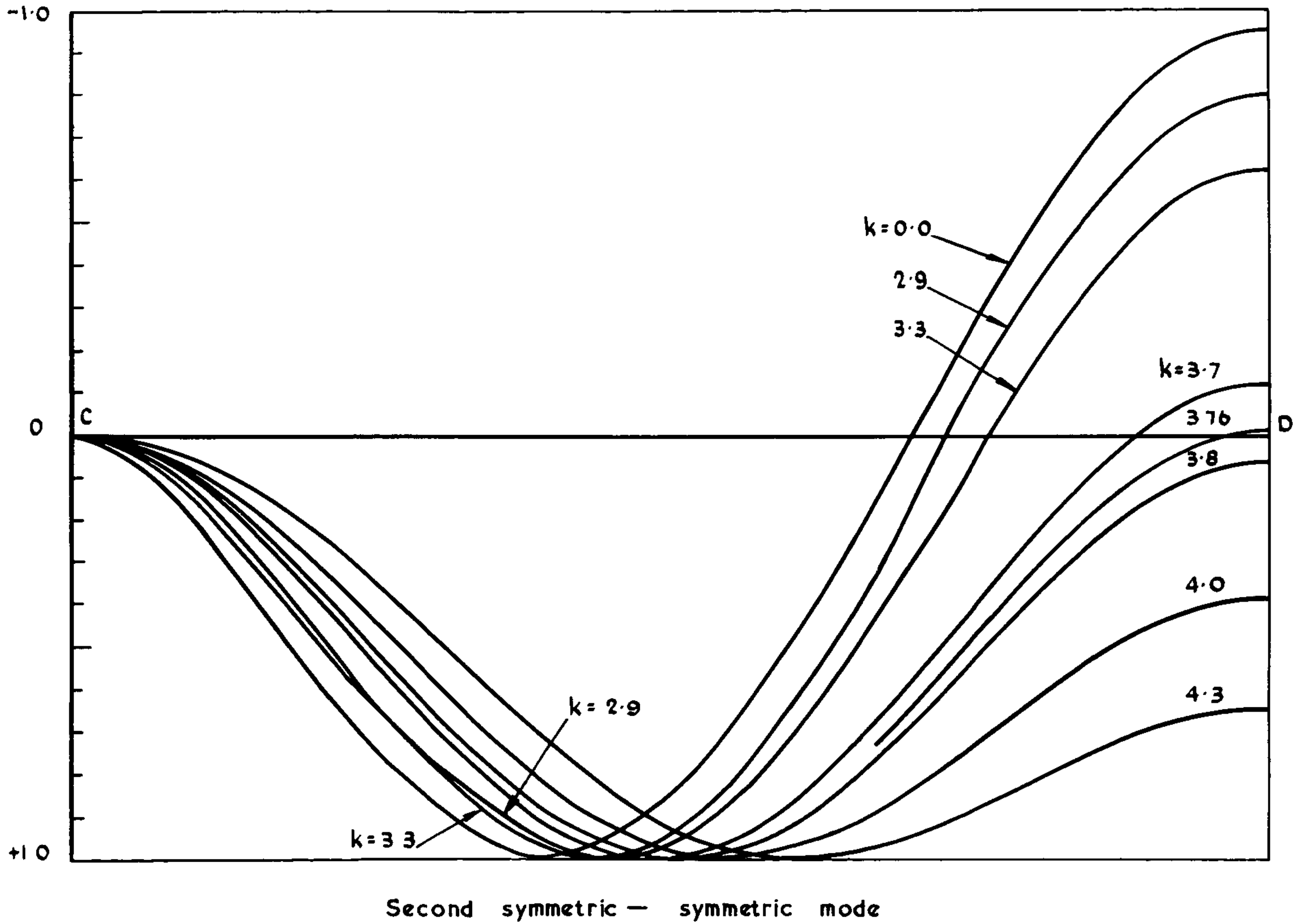

Fig 6b Effect of compressive stress on the mode shapes of the rectangular plate

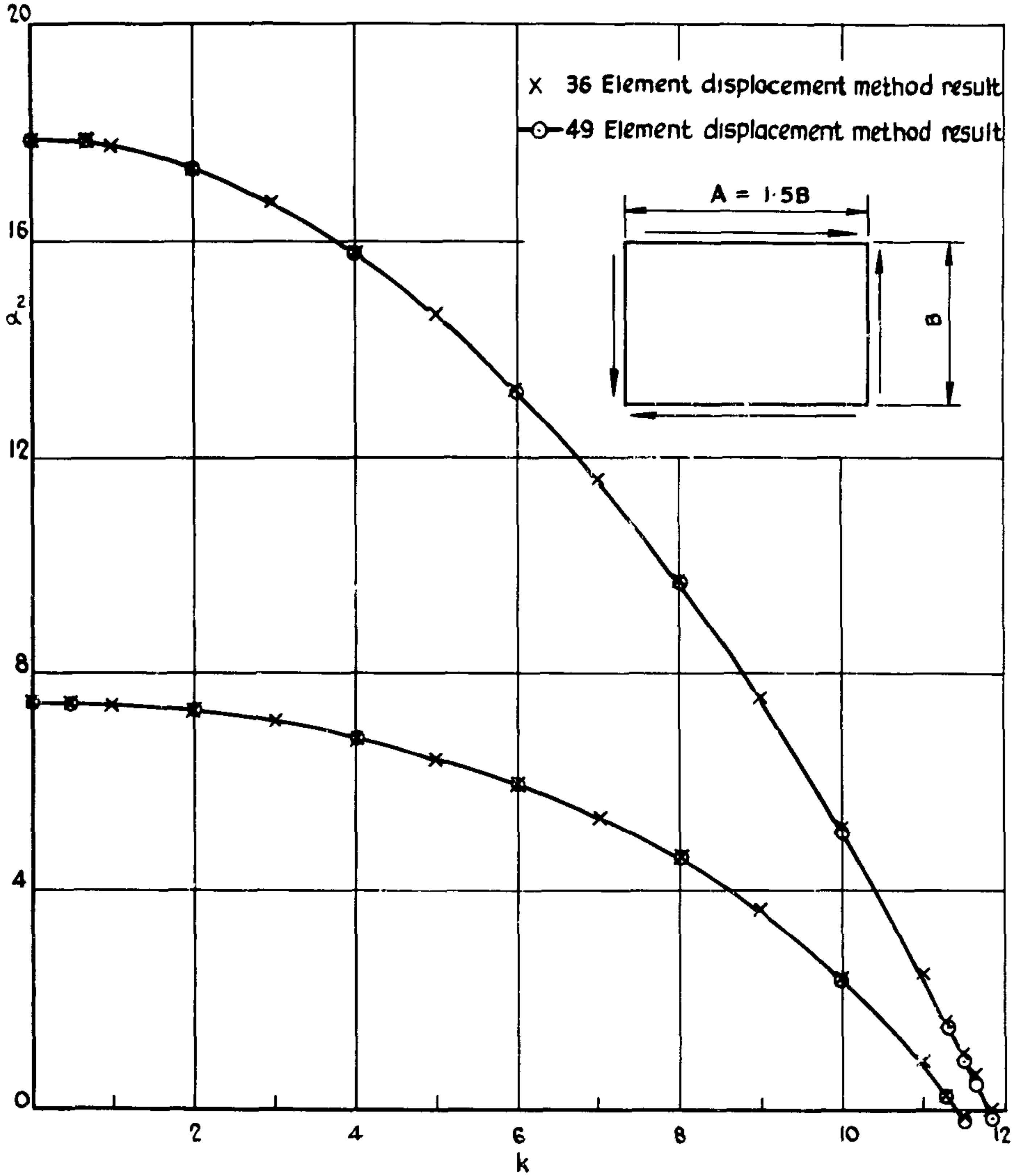

Fig.7 Variation of frequency parameter with shear stress parameter for the clamped rectangular plate

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