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FINITE DIFFERENCE SOLUTIONS FOR AN UNSTEADY INTERFERENCE PARAMETER IN SLOTTED WIND TUNNELS

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SUMMARY

Three methods of determining an unsteady interference parameter in slotted wind tunnels are described In each case the governing equation for the flow in the wind tunnel is Laplace's equation which is solved by a finite difference approximation The methods differ in the representation of the disturbance due to the wing A discussion of the merits of each method is included, results are quoted for tunnels of square section with roof and floor of varying slot parameter.

*Replaces A.R.C.30 834

1 INTRODUCTION

In a recent paper Garner, Moore and Wight[1] have presented a theory for the lift interference effects on wings in slow pitching oscillation in slotted wind tunnels at subsonic speeds This theory requires values for the steady interference parameters, δ_0 and δ_1, and a further parameter, δ_0', that arises in oscillatory flow Though information is available for the steady interference parameters, values of the oscillatory parameter are only available for limiting cases of the open and closed tunnels

This paper is concerned with the evaluation of the oscillatory parameter for slotted tunnels Three alternative methods are examined In the first a solution to the steady flow equation is obtained, the interference parameter is then evaluated from an infinite integral of the interference upwash The second method uses rearranged equations in terms of a function ϕ', δ_0' is obtained from the first derivative of ϕ' at the origin In the third method the unsteady flow equations are solved for incompressible flow over a range of frequency

These three methods are similar in that each requires a solution to the Laplace equation in three dimensions, but they have different conditions on the downstream boundary and at the wing The finite difference method[2] replaces the Laplace equation and the resultant finite difference equations solved either using an analogue computer or by the dynamic relaxation method[3] on a digital computer The accuracy of the different methods of solution illustrates both the advantages and disadvantages of the finite difference method

2 SUMMARY OF METHODS

As an aid to the comparison of the methods their salient features are presented in Table 1 Further details of the first method, in which the interference due to steady flow is considered, are to be found in Ref 2 The rearranged flow equations of Method 2 are derived in Appendix A, and the important unsteady flow equations of Method 3 are recorded in Appendix B The remainder of this section explains certain of the items of Table 1

With each method the governing equation is the Laplace equation in three dimensions Rectangular tunnels are considered which should extend from minus infinity to plus infinity However, since a finite difference method is to be used, infinity cannot be included in the solution but the field will be taken to cover the maximum streamwise distance practicable

A small wing is positioned at the centre of the tunnel, expressions are available for the perturbation velocity potential due to the presence of the small wing in unconstrained flow The relevant formulae are listed as Item 4 in Table 1

The walls of the tunnel can have various boundary conditions, in this study the only conditions considered are closed, open or slotted For a closed wall the perturbation velocity potential satisfies

$$\frac{\partial \phi}{\partial n} = 0, \tag{1}$$

where n is the direction of the outward normal, whilst for an open wall

$$\phi = 0, \tag{2}$$

on an ideal slotted wall, the homogeneous condition,

$$\phi + K\,\partial\phi/\partial n = 0, \tag{3}$$

is taken, where K is the geometric slot parameter in equation (4) of Ref 1.

As explained previously, the tunnel cannot be continued upstream to infinity but is usually terminated a distance upstream from the wing roughly four times the height of the tunnel On this plane the boundary condition is taken to be,

$$\partial\phi/\partial x = 0,$$

where x is the streamwise direction

On the downsteam boundary different conditions hold for each of the three methods From the steady flow equations the perturbation velocity potential tends to a steady value, whence the imposed condition is

$$\partial\phi/\partial x = 0$$

With the rearranged flow equations an examination of the expression for ϕ'_m, in Appendix A, shows that it steadily increases for large x, but since the streamwise ordinate, x, becomes the dominant term, the downstream condition is taken to be

$$\partial\phi'/\partial x = \text{constant}$$

For the imaginary part of the unsteady flow equation $\partial\phi_I/\partial x$ is unknown, but it is possible to find a plane on which ϕ_{mI} is zero, this plane is made the downstream boundary of the problem

Since this analysis is intended for calculating the interference parameter δ_0', accurate information is required about the interference velocity potential, defined as,

$$\phi_i = \phi - \phi_m \tag{4}$$

Since ϕ and ϕ_m satisfy the Laplace equation, so also does ϕ_i Therefore information concerning the interference potential throughout the field can be determined from a second solution to the Laplace equation, given the boundary values of ϕ and hence of ϕ_i The method of solution is described in greater detail in Ref 2 Boundary conditions for each of the three methods are listed in Table 1

The final item of Table 1 is the expression used to calculate δ_0' In the first method an integral between minus infinity and zero has to be evaluated Using the rearranged flow equation, δ_0' can be calculated directly from the slope of the interference potential at the origin With the unsteady flow equations the slope of the interference potential at the origin is used, but unless the flow is incompressible the equation is only valid as the frequency parameter ω tends to zero.

3 METHOD OF SOLUTION

From considerations of the previous section it is clear that each of the three methods reduces to a solution of the Laplace equation in three dimensions The only differences in the methods are in the boundary conditions and in the methods of evaluating the magnitude of the interference parameter

3 1 Finite difference solution

At present the most versatile method available for solving the Laplace equation is the finite difference method In the finite difference method the field is divided into a three-dimensional grid, then the Laplace equation is written in finite difference form for each node Linear simultaneous equations are obtained with one equation for each node For example, with the mesh shown in Fig.1,

$$\nabla^2\phi = \frac{\phi_1 + \phi_3 - 2\phi_0}{\Delta x^2} + \frac{\phi_2 + \phi_4 - 2\phi_0}{\Delta y^2} + \frac{\phi_5 + \phi_6 - 2\phi_0}{\Delta z^2},$$

Δx, Δy and Δz are the mesh intervals. Similar equations can be written for irregular meshes, Ref.2.

These simultaneous equations could be solved by a direct matrix inversion method, but since a variety of specific problems are to be solved it is preferable to use techniques in which the boundary conditions can be modified quickly

Two methods have been used with success The first, the electrical resistance network,[2] solves the simultaneous equations by an equivalent array of resistances Once the boundary conditions have been applied as electrical currents and potentials, the network immediately gives the potential distribution within the field

Recently the dynamic relaxation method has been used to solve the finite difference equations [3] This is an iterative method using a digital computer By introducing dynamic terms into the equations and using an explicit finite difference formulation, the equations can be solved by an iterative method which requires only a simple substitution routine on a digital computer Damping factors are chosen so that the oscillations quickly die out leading to the solution of the static equations The dynamic relaxation method has the advantage that a change in boundary conditions requires an alteration to only one statement of the computer programme.

Both the resistance network and the dynamic relaxation method have been used for each of the three methods The analogue is very useful in the development stage since any inadequacies in the technique quickly become apparent Once the technique has been developed it is advisable to use the dynamic relaxation method so that extensive results can be obtained The techniques used in the dynamic relaxation method will be described in the remainder of this report

In the finite difference method the field is divided by mesh planes, the choice of mesh spacing is governed by the need to obtain sufficient numerical values from which the required result can be calculated For methods 2 and 3 the mesh spacing is chosen to give detailed information around the origin The mesh spacing used in these two methods for a tunnel of square section is

Vertical and spanwise direction,

$$0,\ 0{\cdot}04167h,\ 0\ 0833h,\ 0\ 1667h,\ 0\ 25h,\ 0\ 375h,\ 0\ 5h$$

Streamwise spacing,

$$0,\ \pm 0\ 04167h,\ \pm 0{\cdot}0833h,\ \pm 0{\cdot}1667h,\ \pm 0{\cdot}3333h,\ \pm 0{\cdot}6667h,\ \pm 1{\cdot}1667h,\ \pm 2h,\ \pm 4h$$

The mesh spacing for a plane x = constant is drawn to scale in Fig 2 In Method 1 the integral from minus infinity to $x = 0$ has to be evaluated, thus additional mesh planes are provided for negative x, the mesh being extended as far as $-9{\cdot}375h$.

In the dynamic relaxation method the number of mesh subdivisions is limited primarily by the time taken in obtaining the solution, rather than from storage limitations

3.2 *Enforced conditions*

An examination of the mathematical expressions representing the disturbance due to the small wing (Item 4 of Table 1) shows that for $x \geqslant 0$ each expression tends to infinity as y and z

tend to zero Since an infinite potential cannot be represented in the numerical solutions, the effect of the disturbance is introduced into the finite difference solution at nodes surrounding the origin At these nodes, indicated in Fig 2, finite values of ϕ can be set.

The expressions for the respective disturbances, ϕ_m and ϕ'_m, in Methods 1 and 2 can be calculated directly, but Method 3 requires the evaluation of an infinite integral for ϕ_{mI} This infinite integral is calculated by a summation technique on the digital computer in which x' is increased in small steps until the change between two successive steps is less than 0·001%.

The boundary conditions on the walls and roof are applied through fictitious nodes Thus, for an ideal slotted roof (z = const), if the boundary passes through the nodes ϕ_1, ϕ_0, ϕ_3, the condition

$$\phi + K\,\partial\phi/\partial n = 0 \tag{3}$$

can be written in finite difference form as

$$\phi_0 + \frac{K}{2\Delta z}(\phi_5 - \phi_6) = 0,$$

hence the fictitious node ϕ_5 is given by

$$\phi_5 = \phi_6 - \frac{2\Delta z}{K}\,\phi_0 \tag{6}$$

The closed boundary is a special case of this condition with $2\Delta z/K = 0$ The open boundary condition can be enforced directly by setting the boundary point ϕ_0 equal to zero

Far upstream the condition for each boundary is that $\partial\phi/\partial x = 0$ Again fictitious nodes are used, thus

$$\phi_3 = \phi_1$$

The same condition holds on the downstream boundary for the first method For the second method where $\partial\phi'/\partial x$ = const , the equation for the fictitious node is

$$\phi'_1 = \phi'_3 + 2\text{const}\ \Delta x$$

In the third method ϕ_I and hence ϕ_{tI} is made to be zero on the appropriate plane. (7)

3 3 *Interference potential*

The method of solution and enforcement of boundary conditions for the interference potential is similar to the method for the perturbation velocity potential except that there is no singularity near the axis The different techniques used to evaluate δ'_0 will be described in the following section, but each method involves the calculation of the first differential $(\partial\phi/\partial z)_{z=0}$

Due to the antisymmetrical condition across the plane $z = 0$, $(\partial^2\phi/\partial z^2)_{z=0} = 0$, thus a finite difference formula can be constructed having an error of order Δz^4,

$$(\partial\phi/\partial z)_{z=0} = (8\phi_a - \phi_b)/6\Delta z, \quad (8)$$

where nodes a and b are at distances Δz and $2\Delta z$ above the origin

4 RESULTS

In this section the method of calculating δ_0' is described and the results for each method are presented An estimate is made of the probable accuracy

4 1 *Steady flow equation*

When calculating δ_0' from the results of a steady flow solution the following equation is used,

$$\delta_0' = \frac{-b}{USC_L} \int_{-\infty}^{0} \left(\frac{\partial\phi_t}{\partial z}\right)_x dx \quad (9)$$

Since $\partial\phi_t/\partial z$ can be calculated only at mesh points a summation method has to be used In an earlier report[2] the results were fitted to a curve of the form

$$\partial\phi_t/\partial z = (A + B\sin\theta)\cos^2\theta, \quad (10)$$

$$\text{where } \theta = \tan^{-1}(-4x/h),$$

but in the present study Simpson's Rule is used Another difficulty arises because the mesh does not extend to minus infinity However, if the function is assumed to be proportional to $1/x^2$ between the last plane and minus infinity, the contribution to the integral for this region can be calculated directly For several results both the earlier and the present method were used to evaluate δ_0' The difference was never more than 0·002, and when the curve of $(\partial\phi_t/\partial z)$ was plotted and the area under the curve determined, very good agreement was obtained with the present method.

Results for a square tunnel are included in column 2 of Table 2. The first result refers to a tunnel with all four walls open, whilst the five other results are for closed side walls but with an ideal slotted roof and floor The slot parameter,

$$(1 + F)^{-1} = (1 + 2K/h)^{-1} \quad (11)$$

varies from 0 (closed wall) to 1 (open wall) Analytical values are available only for the open and closed conditions, and agreement with these results is satisfactory. The estimated accuracy is $\pm 0{\cdot}003$, improved values could be obtained by increasing the number of mesh intervals, but this would involve an appreciable increase in time on the computer Some errors, however, do arise from the integration and also from the use of finite differences to represent a continuous system.

It is not possible to assess the magnitude of the finite difference errors but comparisons with analytical solutions indicate that they are roughly $\pm 0{\cdot}0015$. Since the errors arising from the infinite integral will be of the same order the total error will be about $\pm 0{\cdot}003$.

4 2 *Rearranged flow equations*

The method using the rearranged flow equations is apparently straightforward since the parameter δ_0' can be calculated directly from the slope of the interference potential ϕ_i at the origin Results for the completely open and the completely closed tunnel are included in column 3 of Table 2.

The reason for the inaccurate results becomes clear on examining the variation of the function ϕ' with the streamwise direction x For example, on the downstream boundary the value of ϕ' on the arc surrounding the wing is roughly 100 times the value in the plane of the wing This should be compared with the steady flow equations where the potential at the downstream boundary is only double that at the wing With such a large variation in ϕ' the finite difference errors become serious, leading to the poor results recorded in Table 2 Initially the results showed an error of up to 30% but by averaging the values given at different mesh positions the figures given in Table 2 were obtained Any method, however, requiring such averaging is not thought to be reliable Due to these errors the method of using the rearranged flow equations is not to be recommended

4 3 *Unsteady flow equations*

The third method, in which the imaginary part of the complex function, $\bar{\phi}_I$, is used, is similar to the steady flow problem but differs in the representation of the downstream boundary Downstream of the wing the velocity potential does not tend to a constant value but continues to oscillate It is possible to estimate the position of the first plane, x = constant, at which the undisturbed potential, $\bar{\phi}_{mI}$, becomes zero, this plane is then taken as the downstream boundary with zero values applied to this plane Although $\bar{\phi}_{mI}$ from equation (B8) does not vanish over the whole plane, the values at all nodal points lie close to zero

To investigate whether this assumption can lead to errors a check was made by taking the second plane on which $\bar{\phi}_{mI}$ is zero and using this as the downstream boundary The results were identical to those obtained with the first zero plane as the downstream boundary, thus demonstrating that the method of representing the downstream boundary is satisfactory

The parameter δ_0' is calculated from equation (B6) which states that at $x = 0$ and for small ω,

$$\delta_0' = -\frac{b}{\omega S \bar{C}_L} \frac{\partial \bar{\phi}_{iI}}{\partial z} \tag{12}$$

The method adopted in calculating δ_0' is to obtain solutions for three values of $\omega h/U$, 0 5, 0·1 and

0·01. By extrapolation the value at $\omega h/U = 0$ can be determined, the extrapolation is illustrated in Fig. 3 Results are calculated for the previous values of the slot parameter, and recorded in the fourth column of Table 2 The estimated accuracy for these results is ±0·0015, and they are thought to be more reliable than those given by Method 1 since the only source of errors is the finite difference approximation.

In Fig 4 the results of Table 2 are plotted against $(1 + F)^{-1}$ The curve plotted through the points indicates that, to a fair approximation, the variation with $(1 + F)^{-1}$ is linear for the particular case of a square tunnel

5. CONCLUDING REMARKS

Of the three methods, the method involving the rearranged flow equations can be discarded. At first this method appeared to be promising since the boundary conditions and the method of calculating δ_0' are straightforward However, due to the streamwise variation in the magnitude of the function the finite difference errors become serious

Although the evaluation of the infinite integral in the first method can lead to small errors it is more economical in computer time than the final method

Since it is necessary to use the computer to evaluate $\bar{\phi}_{mJ}$ in the final method a large amount of computer time is used for each solution The most accurate results, however, are given by this method which solves the unsteady equations Further, the method can be used for perforated tunnels in incompressible flow at arbitrary frequency, since the real and imaginary parts of $\bar{\phi}$ both satisfy the Laplace equation but have to be solved simultaneously.

Though the results of Table 2 are derived for incompressible flow, they can be applied to compressible flow of low frequency. The validity of this assumption is discussed in Ref 4 where it is shown that the interference can still be derived from a solution of the Laplace equation For higher frequency it is necessary to solve an equation of the form,

$$\nabla^2\bar{\phi} + k^2\bar{\phi} = 0, \tag{13}$$

where k is a function of ω and M An equation of this form can easily be solved using the dynamic relaxation method

ACKNOWLEDGEMENT

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NOTATION

b	tunnel breadth
C_L	lift coefficient
$\bar{C}_L$	complex lift coefficient, $\bar{C}_{LR} + i\bar{C}_{LI}$
F	non-dimensional slot parameter, $2K/h$
h	tunnel height
i	$(-1)^{\frac{1}{2}}$
K	geometric slot parameter
M	Mach number of undisturbed stream
n	outward normal distance from boundary
r	radial ordinate $(x^2 + y^2 + z^2)^{\frac{1}{2}}$
S	planform area of wing
U	undisturbed stream velocity
x, y, z	Cartesian coordinates
x'	increment in streamwise direction
β	$(1 - M^2)^{\frac{1}{2}}$
δ_0	steady lift interference parameter at the wing
δ_1	steady streamwise curvature parameter
δ_0'	unsteady lift interference parameter at the wing
Δ	increment
ϕ	perturbation velocity potential
ϕ'	rearranged perturbation velocity potential
$\bar{\phi}$	complex perturbation velocity potential, $\bar{\phi}_R + i\bar{\phi}_I$
ϕ_m	velocity potential in unconstrained flow
$\bar{\phi}_m$	complex velocity potential in unconstrained flow, $\bar{\phi}_{mR} + i\bar{\phi}_{mI}$
ϕ_i	interference velocity potential
$\bar{\phi}_i$	complex interference potential, $\bar{\phi}_{iR} + i\bar{\phi}_{iI}$
ω	angular frequency of oscillation.

REFERENCES

1. H C Garner, A W Moore and K C Wight. The theory of interference effects on dynamic measurements in slotted-wall tunnels at subsonic speeds and comparisons with experiment
A R C , R & M No 3500, 1968.

2 K R Rushton and Lucy M. Laing A general method of studying steady lift interference in slotted and perforated tunnels
A R C , R & M No 3567.

3 K R Rushton and Lucy M Laing. A digital computer solution of the Laplace equation using the Dynamic Relaxation method
Aeronaut. Quart., Vol. XIX, Pt.4. November, 1968. pp.375-387.

4 W.E.A Acum and H C Garner. Approximate wall conditions for an oscillating swept wing in a wing tunnel of closed circular section
A R C C P 184, January 1954

Table 1 Comparison of the three methods

	Method 1	Method 2	Method 3
1) Title	Steady Flow Equations	Rearranged Equations	Unsteady Flow Equations
2) Working function	Perturbation velocity potential, ϕ	Special function, ϕ'	Imaginary part of complex velocity potential, $\bar{\phi}_I$
3) Governing equation	$\nabla^2\phi = 0$	$\nabla^2\phi' = 0$	$\nabla^2\bar{\phi}_I = 0$
4) Disturbance	$\phi_m = \frac{USC_L z}{8\pi(y^2+z^2)}\left(1+\frac{x}{r}\right)$	$\phi'_m = -\frac{Ubz(x+r)}{8\pi(y^2+z^2)}$	$\bar{\phi}_{mI} = -\frac{US\bar{C}_L}{8\pi}\int_0^\infty \frac{z\sin(\omega x'/U)dx'}{((x-x')^2+y^2+z^2)^{3/2}}$
5) Side walls	closed or open	closed or open	closed or open
6) Roof and floor	ideal slotted	closed or open	ideal slotted
7) Condition far upstream	$\partial\phi/\partial x = 0$	$\partial\phi'/\partial x = 0$	$\partial\bar{\phi}_I/\partial x = 0$
8) Condition far downstream	$\partial\phi/\partial x = 0$	$\partial\phi'/\partial x =$ constant	$\bar{\phi}_I = 0$, on plane where $\bar{\phi}_{mI} = 0$ on tunnel axis
9) Solution in interference potential	ϕ_i set on roof and walls, $\partial\phi_i/\partial x = 0$ on upstream and downstream boundaries	ϕ'_i set on roof, walls and downstream boundary, $\partial\phi'_i/\partial x = 0$ on upstream boundary	$\bar{\phi}_{iI}$ set on roof and walls, $\partial\bar{\phi}_{iI}/\partial x = 0$ on upstream plane; $\bar{\phi}_{iI} = 0$ on downstream plane
10) Formula for δ'_0	$= -(b/USC_L)\int_{-\infty}^{0}(\partial\phi_i/\partial z)_x dx$	$= (1/U)(\partial\phi'_i/\partial z)_{x=y=z=0}$	$= (b/\omega S\bar{C}_L)(\partial\bar{\phi}_{iI}/\partial z)_{x=y=z=0}$ as $\omega \to 0$

Table 2 Values of δ_0' given by the three methods

$(1+F)^{-1}$		δ_0'			
Roof and floor	Side walls	Method 1	Method 2	Method 3	Analytical
1·0	1·0	+0·0826	+0·080	+0·0816	+0·0814
1 0	0	+0·0763		+0·0762	+0·0776
0·0769	0	+0·0532		+0 0534	
0 5	0	+0·0253		+0·0242	
0·25	0	−0·0014		−0·0050	
0	0	−0·0372	−0·033	−0·0359	−0·0361

APPENDIX A

Rearranged flow equations

The velocity potential in incompressible oscillatory flow for a small wing is given by,

$$\bar{\phi}_m = \bar{\phi}_{mR} + i\,\bar{\phi}_{mI} = \frac{US\bar{C}_L}{8\pi}\int_0^{\infty}\frac{z\,e^{-i\omega x'/U}}{[(x-x')^2+y^2+z^2]^{3/2}}\,dx', \tag{A1}$$

where ω is the angular frequency of the oscillation.

For small ω,

$$\bar{\phi}_m = \frac{US\bar{C}_L}{8\pi}\int_0^{\infty}\frac{z(1-i\omega x'/U)}{[(x-x')^2+y^2+z^2]^{3/2}}\,dx',$$

$$= \frac{US\bar{C}_L}{8\pi}\,\frac{z[(x^2+y^2+z^2)^{1/2}+x]}{(y^2+z^2)(x^2+y^2+z^2)^{1/2}}\left(1-\frac{i\omega(x^2+y^2+z^2)^{1/2}}{U}\right)$$

On substituting $r^2 = x^2+y^2+z^2$, taking the imaginary part and treating $\bar{C}_L$ as real,

$$\frac{\bar{\phi}_{mI}}{\omega} = -\frac{S\bar{C}_L}{8\pi}\,\frac{z(x+r)}{y^2+z^2} \tag{A2}$$

Now the complex interference upwash at the centre of the tunnel has been given in equation (15) of Ref. 1, which with slight rearrangement is,

$$\frac{\partial\bar{\phi}_i}{\partial z} = \frac{US\bar{C}_L}{bh}\left[\delta_0+\frac{\delta_1 x}{\beta h}+\frac{i\omega h}{U}\left(\frac{\delta_0'}{\beta}-\frac{\delta_0 x}{h}+\frac{\delta_1 x^2(2M^2-1)}{2\beta^3h^2}\right)+0\left(\frac{x}{\beta h}\right)^3\right]. \tag{A3}$$

For incompressible flow $\beta = 1$; thus at the origin where $x = 0$,

$$\frac{\partial\bar{\phi}_i}{\partial z} = \frac{US\bar{C}_L}{bh}\left[\delta_0+\frac{i\omega h}{U}\,\delta_0'\right] \tag{A4}$$

With $\bar{C}_L$ real, the imaginary part satisfies,

$$\frac{1}{\omega}\frac{\partial\bar{\phi}_{il}}{\partial z} = \frac{S\bar{C}_L}{b}\delta'_0 \tag{A5}$$

For ideal slotted tunnels $\bar{\phi}_{il}/\omega$ satisfies the boundary condition,

$$\frac{\bar{\phi}_{il}}{\omega} + K\frac{\partial(\bar{\phi}_{il}/\omega)}{\partial n} = -\frac{\bar{\phi}_{ml}}{\omega} - K\frac{\partial(\bar{\phi}_{ml}/\omega)}{\partial n} \tag{A6}$$

Moreover, in incompressible flow $\bar{\phi}_{il}$ satisfies the Laplace equation

Thus it follows from equations (A2), (A5) and (A6) that δ'_0 is equivalent to the steady interference upwash $(\partial\phi'_i/\partial z)/U$ when the unconstrained potential due to the wing is,

$$\phi'_m = -\frac{Ubz(x + r)}{8\pi(y^2 + z^2)} .$$

It should be noted that this method is not applicable to perforated or non-ideal slotted walls, when the porosity parameter enters into the boundary condition (Ref 1)

APPENDIX B

Unsteady flow equations

In oscillatory incompressible flow the complex velocity potential $\bar{\phi}$ satisfies the Laplace equation

$$\frac{\partial^2\bar{\phi}}{\partial x^2} + \frac{\partial^2\bar{\phi}}{\partial y^2} + \frac{\partial^2\bar{\phi}}{\partial z^2} = 0, \qquad \text{(B1)}$$

where $\bar{\phi} = \bar{\phi}_R + i\bar{\phi}_I$ Both the real and imaginary parts, $\bar{\phi}_R$ and $\bar{\phi}_I$, individually satisfy the Laplace equation

The velocity potential due to a small wing in unconstrained flow is,

$$\bar{\phi}_m = \bar{\phi}_{mR} + i\bar{\phi}_{mI} = \frac{US\bar{C}_L}{8\pi}\int_0^\infty \frac{z\,e^{-i\omega x'/U}}{((x-x')^2 + y^2 + z^2)^{3/2}}\,dx' \qquad \text{(B2)}$$

Boundary conditions are as follows· for a closed wall,

$$\frac{\partial\bar{\phi}_R}{\partial n} = \frac{\partial\bar{\phi}_I}{\partial n} = 0, \qquad \text{(B3)}$$

for an open boundary,

$$\bar{\phi}_R = \bar{\phi}_I = 0, \qquad \text{(B4)}$$

whilst for ideal slotted walls,

$$\bar{\phi}_R + K\partial\bar{\phi}_R/\partial n = \bar{\phi}_I + K\partial\bar{\phi}_I/\partial n = 0 \qquad \text{(B5)}$$

Since the real and imaginary parts of each of the above equations are independent it is permissible to consider the real and imaginary parts separately

The parameter δ_0' can be derived from equation (A4) of Appendix A in terms of the complex interference parameter, $\bar{\phi}_i$,

$$\delta_0' = \left(\frac{\omega h}{U}\cdot\frac{US\bar{C}_L}{bh}\right)^{-1}\frac{\partial\bar{\phi}_{iI}}{\partial z}. \qquad \text{(B6)}$$

Hence δ_0' can be calculated from the imaginary part of the complex potential The equations required to solve this problem for a small wing are as follows:

$$\nabla^2 \bar{\phi}_{il} = 0,$$

with boundary conditions,

$$\left.\begin{aligned} \partial\bar{\phi}_{il}/\partial n &= -\partial\bar{\phi}_{ml}/\partial n \\ \bar{\phi}_{il} &= -\bar{\phi}_{ml} \\ \text{or} \qquad \bar{\phi}_{il} + K\partial\bar{\phi}_{il}/\partial n &= -\bar{\phi}_{ml} - K\partial\bar{\phi}_{ml}/\partial n \end{aligned}\right\}, \tag{B7}$$

$$\text{where} \qquad \bar{\phi}_{ml} = -\frac{US\bar{C}_L}{8\pi}\int_0^\infty \frac{z\sin(\omega x'/U)}{[(x-x')^2 + y^2 + z^2)^{3/2}}dx'. \tag{B8}$$

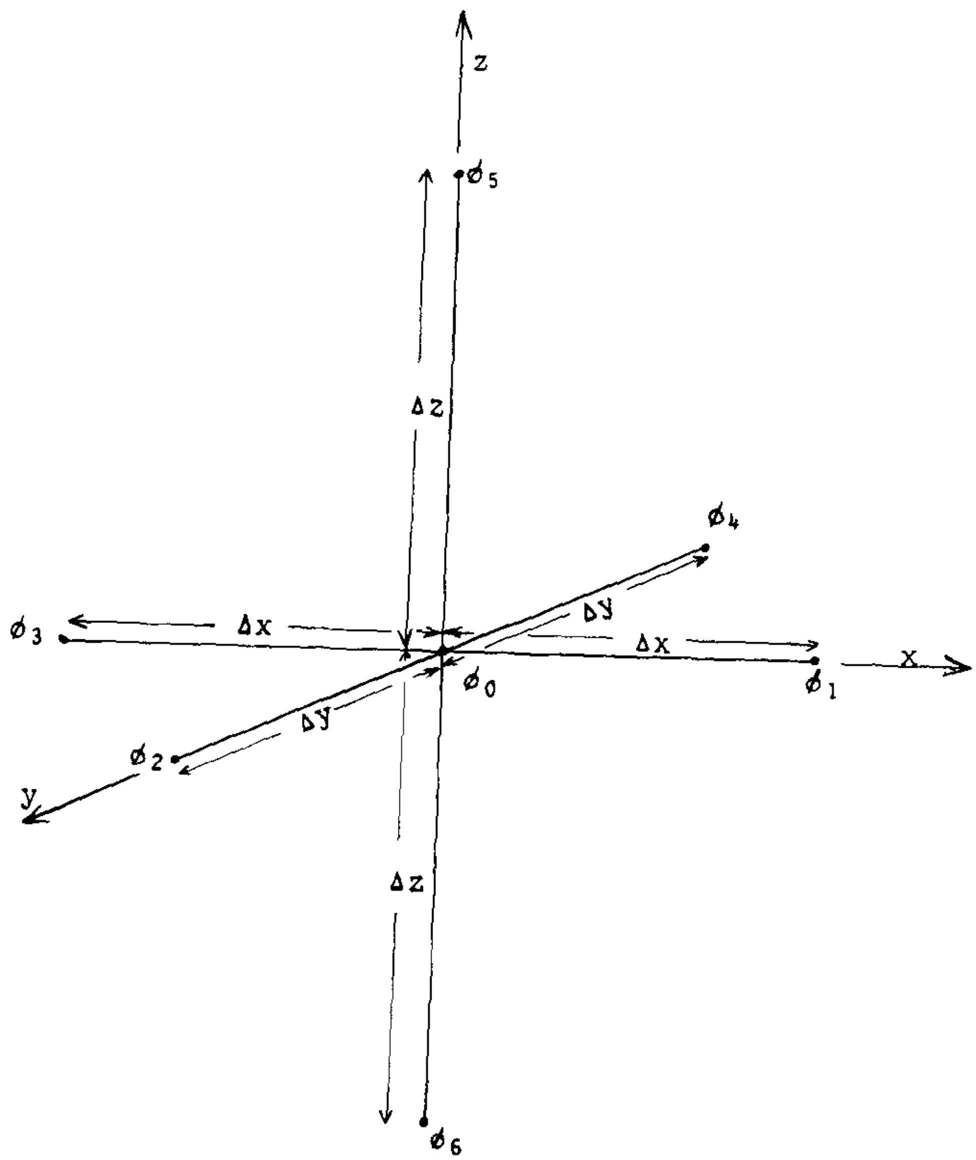

Fig. 1. Finite Difference Mesh.

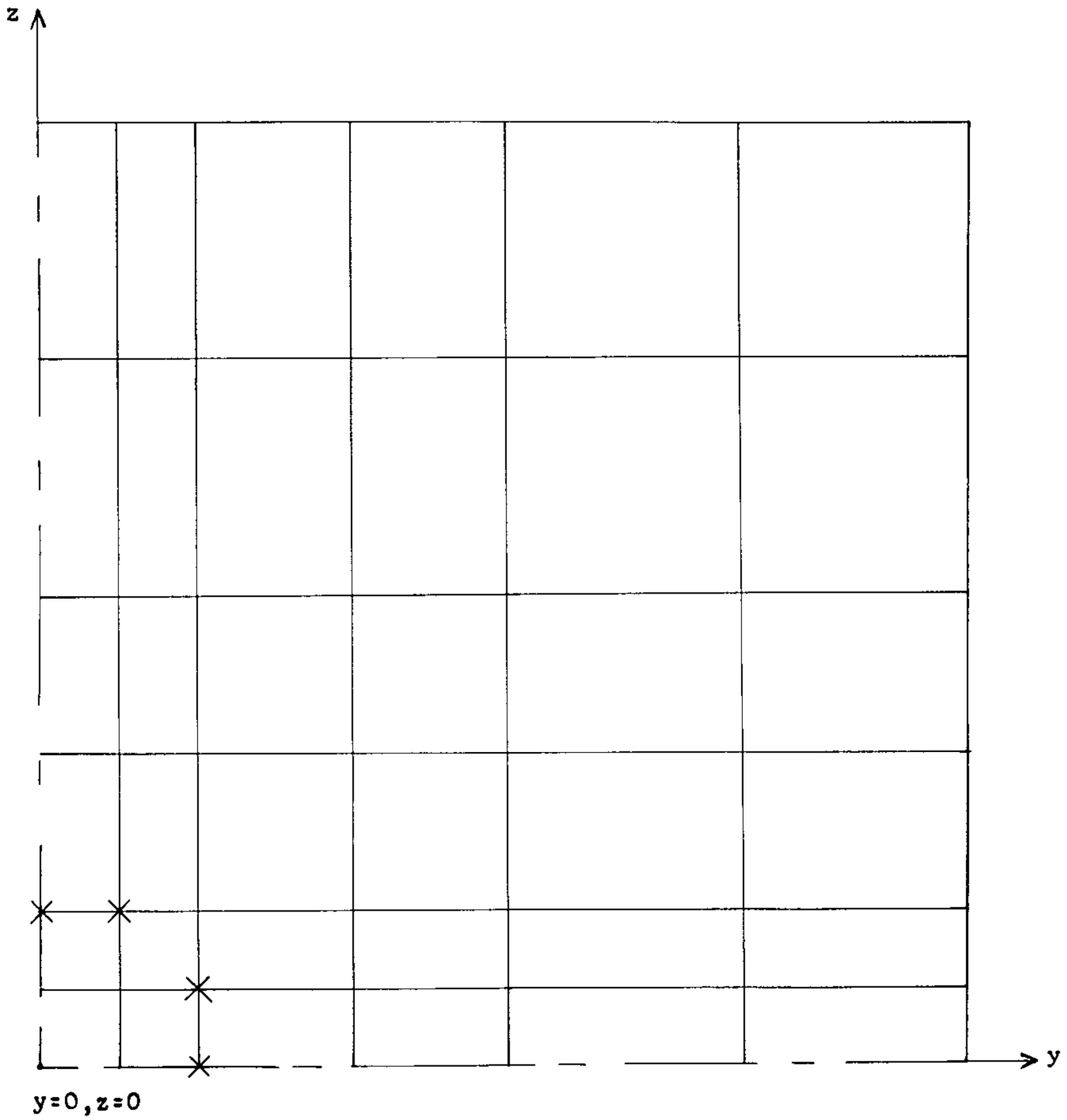

Fig. 2. Mesh spacing on a plane x = constant, showing nodes representing small wing.

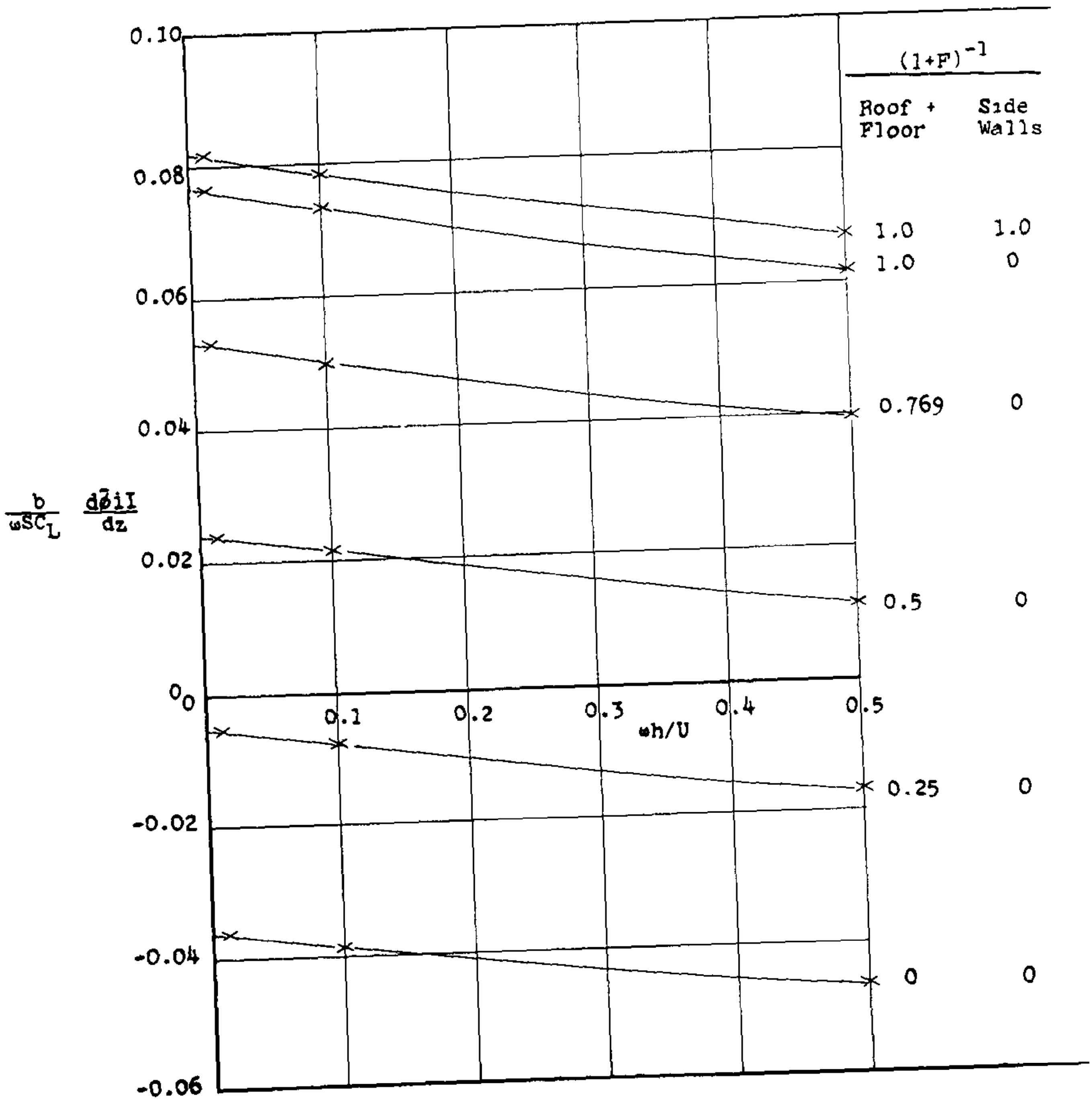

Fig. 3. Variation of unsteady interference parameter with frequency

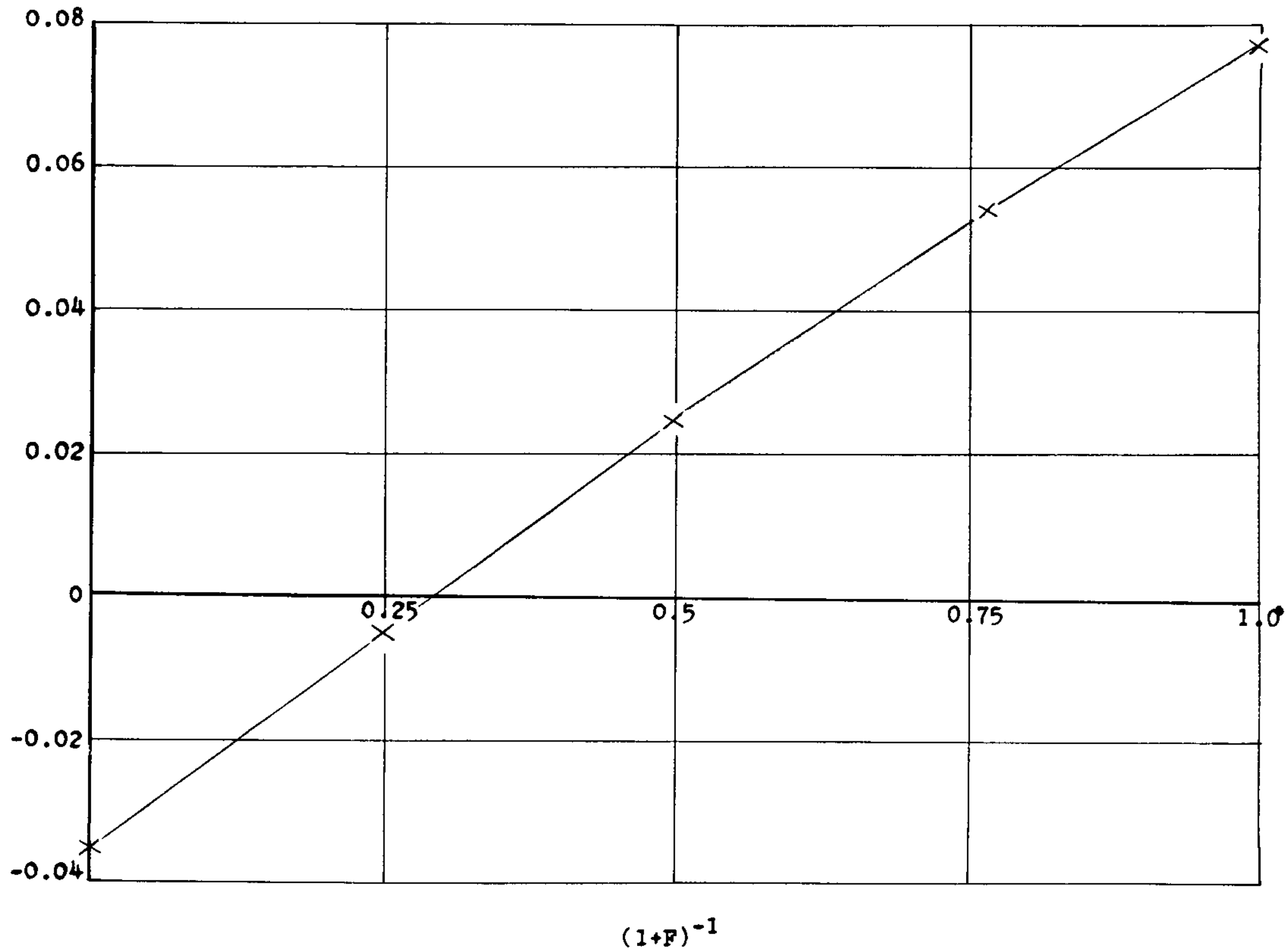

Fig 4 δ_0' for slotted tunnels (slotted roof and floor, closed side walls) from Method 3.

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