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by

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THE DRAG OF INFINITE SWEPT WINGS WITH AN ADDENDUM

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SUMMARY

The drag of an infinite swept wing is found in terms of the drag of a related unswept wing having the same relative position of transition. Results in incompressible flow are expressed in terms of a "sweep factor". Detailed calculations are made for wings of RAE 101 and 104 sections and the factor appears to have a reasonably universal character not very dependent on shape or Reynolds number if transition takes place early, but strongly dependent on thickness. Results are given as a series of curves and an empirical formula is given for the sweep factor in terms of thickness-chord ratio, angle of sweep and point of transition.

A few results are given for compressible flow over an RAE 101 section at sweep angles of 0° and 45°; these show the effect of sweep in delaying the compressibility drag rise.

* Replaces R.A.E. Technical Note Aero 2966 - A.R.C. 26302.

CONTENTS

1 INTRODUCTION

Weber and Brebner[1] attempted to make an estimate of the drag of a swept wing by relating it to the drag of a similar unswept wing, thus obtaining what has come to be known as a "sweep factor" by which to multiply the drag of an unswept wing to obtain the drag of the same wing swept. In order to do this they first calculated the sweep factor for a flat plate and gave some evidence to show that this is not much changed if the flat plate is replaced by a wing.

We must distinguish between the words "swept" and "sheared". The former means that a given wing is simply turned or "yawed" through an angle Φ called the angle of sweep. By the term "sheared" is meant that each chordwise section of the wing is moved parallel to itself downstream to such a position that the leading and trailing edges, which were normal to the mainstream to begin with, now make an angle Φ with their original directions. In each case the total area is unchanged, but the swept wing whose chord was originally c now has a streamwise chord equal to $c \sec \Phi$ whilst the sheared wing has a streamwise chord which remains equal to c.

In obtaining the factor for a flat plate Weber and Brebner[1] used the "independence principle" for both laminar and turbulent flows. This principle states that the chordwise flow may be calculated independently of the spanwise flow. By this means they found that the drag coefficient of the plate sheared to an angle Φ at a Reynolds number R is equal to $\cos \Phi$ times that of an unsheared plate at the smaller Reynolds number $R \cos^2 \Phi$. At the time the paper of Weber and Brebner was written the experimental evidence seemed to be in favour of the use of the independence principle for turbulent as well as laminar flow, but since then it has been shown both theoretically and experimentally[2] that the principle does not hold for turbulent flow; indeed it is now believed that a "line-of-flow" principle should replace it. According to the new principle the wall shearing stress is considered to be a function of the distance over which the external fluid has travelled rather than the distance perpendicular to the leading edge, which is what the independence principle leads to. Intuitively this seems to be more reasonable since one would expect the previous history to have some effect on the flow. In purely laminar incompressible flow both principles lead to the same result owing to the particular form of the boundary layer equations. It follows that if we use the line-of-flow principle, in all cases, laminar, turbulent, or mixed, <u>shearing</u> a flat plate will have no effect on the skin friction at any point, and the drag of a sheared flat plate will be always the same as that of the same plate unsheared. According to Turcotte[2], who gives a detailed account of the whole matter, and who introduced the term "line-of-flow principle", more recent experiments show results in favour of the line-of-flow principle. If this principle is to be believed all Weber and Brebner's flat plate curves should be straight lines giving a constant sweep factor of unity.

It should be noted that according to Young and Booth[3] the sweep factor for a swept flat plate in laminar flow should be equal to $\cos^{\frac{1}{2}} \phi$. This is in accordance with either the independence principle or the line-of-flow principle and would lead to a factor of unity for a sheared plate. On the other hand for fully turbulent flow they give the factor $\cos^{4/5} \phi$ for a swept plate which would give a factor $\cos^{3/5} \phi$ for the sheared plate. From the line-of-flow principle the factor for a swept plate would be $\cos^{1/5} \phi$ and would once more be unity for a sheared plate. The experiments of Young and Booth[3] are heavily in favour of the factor $\cos^{4/5} \phi$ and not $\cos^{1/5} \phi$. In this respect, however, they are at variance with Ashkenas and Riddell[4]; and Turcotte[2] is inclined to favour the experiments of the latter. It should be pointed out here that all approximate calculation methods for three-dimensional turbulent boundary layers so far given have implicitly followed the line-of-flow principle. Indeed this seems to be the only possible procedure in approximate methods for general three-dimensional turbulent boundary layers.

Even if the results of Weber and Brebner for the flat plate were correct it would still seem reasonable physically to expect that the sweep factor should vary with thickness, since the thicker the wing the more highly curved are the streamlines and the greater cross-flow in the boundary layer.

In this note a more refined procedure is attempted. We confine ourselves at first to incompressible flow, and later extend the analysis to subcritical compressible flow. First, by momentum considerations, Squire and Young's simple formula for drag is extended to apply to infinite swept wings. This extended formula was found by Young and Booth[3] by a different procedure. It involves two "components" of momentum thickness θ_{11} and θ_{21}, instead of merely θ as in the two-dimensional case. Next, approximate methods of calculating these components are devised, assuming small cross-flow in the boundary layer. Finally the result is expressed as a "sweep factor" relating the drag of the swept wing to that of a certain unswept wing. In order to obtain a set of curves for the sweep factor it is necessary to know the wing section, and RAE 101 and RAE 104 have been used for this purpose. It is not of course necessary to express the results in terms of a sweep factor but this was done in the hope that the effect of different shapes would lie mainly in the drag of the basic unswept wing and not in the factor. In other words it was hoped that the factor would not be too dependent of the particular section chosen, but would, in some degree at least, be universal. It cannot be said that this hope has been realised except when transition is early, say up to about 30% of the chord. Beyond this the drag factors for RAE 104 are considerably less than those for RAE 101, especially for the sections with the greatest thickness and the greatest sweep.

Weber and Brebner tested their theory by an experiment on a 12% thick wing and obtained good agreement. The result of the present study gives fairly good agreement with the same experiment, but would give results very different from this had the experiment been done on a thinner wing for instance, whilst their theory gave factors independent of thickness. Fair agreement is obtained with an experiment on a $4\frac{1}{2}$% thick wing by Brebner and Wyatt[10], a result which was not achieved using the Weber and Brebner factor.

Owing to the approximations used one must not expect great accuracy in the curves given here. It can only be hoped that they will give some estimate of drag at zero lift for preliminary design purposes.

In this connection we must note that our results only apply to an infinite sheared wing. They may, however, have some relevance for a finite wing, if it is designed to have straight isobars so that the external flow bears some resemblance to that over an infinite yawed wing.

2 MOMENTUM BALANCE

We surround a portion of the wing, of length ℓ, angle of sweep ϕ, by a cylinder of rectangular section but with the plane ends ABCD, A'B'C'D' swept to the angle ϕ (Fig.1). The sides BCB'C', ADA'D' are supposed fixed, but the top and bottom faces ABA'B', CDC'D' move off to infinity above and below the wing, and the ends move off to infinity upstream and downstream.

In considering the momentum balance in this control surface we note that the contributions to the mass flow and momentum in the direction of the velocity at infinity U_∞, due to fluid entering or leaving the sides BCB'C', ADA'D' cancel each other, and no fluid enters or leaves the sides ABA'B', CDC'D', and so we need only consider the ends, where the pressure has everywhere the same value. By continuity of mass we have (see Fig.2)

$$\int_{-\infty}^{\infty} \rho_\infty U_\infty \ell \cos\phi \, dz - \int_{-\infty}^{\infty} \rho u \ell \, dz = 0 \quad , \tag{1}$$

where z is measured in the direction CB.

By momentum considerations we have

$$\text{Drag} = \int_{-\infty}^{\infty} \rho_\infty U_\infty^2 \ell \cos\phi \, dz - \int_{-\infty}^{\infty} \rho u \ell (u \cos\phi + v \sin\phi) \, dz \quad . \tag{2}$$

Multiply equation (1) by U_∞ and subtract from equation (2) and we have

$$\begin{aligned} \text{Drag} &= \ell \int_{-\infty}^{\infty} \{\rho u (U_\infty \cos\phi - u) \cos\phi + \rho u (U_\infty \sin\phi - v) \sin\phi\} \, dz \\ &= \rho_\infty U_\infty^2 \ell (\theta_{11\infty} \cos\phi + \theta_{21\infty} \sin\phi) \quad . \end{aligned} \tag{3}$$

In this equation

$$\left. \begin{aligned} \rho_e \, \theta_{11} \, U_e^2 &= \int_{-\infty}^{\infty} \rho u (u_e - u) \, dz \quad , \\ \rho_e \, \theta_{21} \, U_e^2 &= \int_{\infty}^{\infty} \rho u (v_e - v) \, dz \quad , \end{aligned} \right\} \tag{4}$$

and the subscript ∞ is to mean that the values are to be evaluated at infinity downstream where

$$U_e = U_\infty, \qquad u_e = U_\infty \cos\phi, \qquad v_e = U_\infty \sin\phi \quad .$$

It will be sometimes more convenient to write θ_{11} in terms of θ'_{11} defined by

$$\rho_e \, \theta'_{11} \, u_e^2 = \int_{-\infty}^{\infty} \rho u (u_e - u) \, dz \quad , \tag{5}$$

and we note that

$$\theta_{11} = \theta'_{11}\left(\frac{u_e}{U_e}\right)^2 ,$$

and in particular at infinity

$$\theta_{11\infty} = \theta'_{11\infty} \cos^2 \phi .$$

Hence equation (3) may be written

$$\text{Drag} = \rho_\infty U_\infty^2 \ell (\theta'_{11\infty} \cos^3 \phi + \theta_{21\infty} \sin \phi) .$$

Now we wish to obtain the drag in terms of momentum thicknesses at the trailing edge of the wing, so we must find relations between θ'_{11T} and $\theta'_{11\infty}$, and between θ_{21T} and $\theta_{21\infty}$, where the subscript T denotes values at the trailing edge of the wing.

For simplicity we shall only deal with a symmetrical wing at zero incidence, though the analysis can be extended to more general cases by considering the upper and lower surfaces separately. General equations for θ_{11} and θ_{21} have been given by Cooke and Hall[5]. Here we must apply them in the wake, which has a plane "centre surface" with zero skin friction on it. These equations are obtained by integrating the equations of motion right across the wake, and will be of the same form as equations (13) and (14) of Ref.5, but with the right hand sides equal to zero. We measure x normal to the leading edge and y parallel to it. All derivatives with respect to y are to be zero, owing to the fact that the wing is a swept infinite cylinder. The equations reduce to

$$\frac{d\theta_{11}}{dx} + \frac{\theta_{11}}{\rho_e}\frac{d\rho_e}{dx} + \frac{2\theta_{11}}{U_e}\frac{dU_e}{dx} + \frac{1}{U_e}\frac{du_e}{dx}\delta_1 = 0 ,$$

$$\frac{d}{dx}(\rho_e \theta_{21} U_e^2) = 0 ,$$

where

$$\rho_e U_e \delta_1 = \int_{-\infty}^{\infty} (\rho_e u_e - \rho u)\, dz . \qquad (7)$$

The second equation shows that

$$\rho_\infty U_\infty^2 \theta_{21\infty} = \rho_{cT} \theta_{21T} U_{eT}^2 \quad .$$

The first equation may be written

$$\frac{d}{dx}\left\{\left(\frac{u_e}{U_\infty}\right)^2 \theta'_{11}\right\} + \left(\frac{u_e}{U_\infty}\right)^2 \frac{\theta'_{11}}{\rho_e}\frac{d\rho_e}{dx} + 2\left(\frac{u_e}{U_e}\right)^2 \frac{\theta'_{11}}{U_e}\frac{dU_e}{dx} + \frac{1}{U_e}\frac{du_e}{dx}\frac{u_e}{U_e}\delta'_1 = 0 \ , \qquad \ldots (8)$$

where

$$\rho_e u_e \delta'_1 = \int_{-\infty}^{\infty} (\rho_e u_e - \rho u)\, dz \quad . \qquad (9)$$

Equation (8) reduces to

$$\frac{d\theta'_{11}}{dx} + (2\theta'_{11} + \delta'_1)\frac{1}{u_e}\frac{du_e}{dx} + \frac{\theta'_{11}}{\rho_e}\frac{d\rho_e}{dx} = 0 \ , \qquad (10)$$

which is the usual momentum-integral equation for flow in a wake in two dimensions. In other words an independence principle holds in the wake, even if it is turbulent, which we shall suppose always to be the case. This result was implicitly assumed by Young and Booth[3]. Now Squire and Young[6] gave reasons for believing that in incompressible flow, to which we now confine our attention,

$$\theta'_{11\infty} = \theta'_{11T}\left(\frac{u_{eT}}{U_\infty \cos\phi}\right)^{3.5} \qquad (11)$$

approximately in a turbulent wake. (Actually their estimate for the exponent was 3.2, but, according to Thwaites[7], it is more satisfactory conceptually to use 3.5. This change in the value gives negligible difference.)

Putting in these values we find for the drag coefficient for a swept wing of streamwise chord length c, whose area is $\ell c \cos\phi$ the expression

$$C_D = 2\left\{\cos^2\phi\left(\frac{\theta'_{11T}}{c}\right)\left(\frac{u_{eT}}{U_\infty\cos\phi}\right)^{3.5} + \left(\frac{U_{eT}}{U_\infty}\right)^2 \frac{\theta_{21T}}{c}\tan\phi\right\} \quad . \qquad (12)$$

This is the extension to swept wings of the Squire and Young formula[6]. It can be shown that this is the same equation as that given by Young and Booth[3] after making allowance for the changes in notation, although the derivation is different. It is in the determination of θ'_{11} and θ_{21} that the present treatment differs from that of Young and Booth who assumed the independence principle to hold everywhere.

3 THE VALUES OF θ'_{11} AND θ_{21}

We denote values referred to streamline coordinates by the subscript s. Thus we have

$$\rho_e U_e^2 \theta_{11s} = \int_{-\infty}^{\infty} \rho \, u_s \, (u_{se} - u_s) \, dz \qquad . \qquad (13)$$

Formulae for θ_{12s}, etc, are given in Appendix 1 where it is shown that

$$\theta'_{11} = \theta_{11s} - \tan\delta' \, \theta_{21s} - \tan\delta' \, \theta_{12s} + \tan^2\delta' \, \theta_{22s} \quad ,$$

$$\theta_{21} = \sin\delta' \cos\delta' \, \theta_{11s} + \cos^2\delta' \, \theta_{21s} - \sin^2\delta' \, \theta_{12s} - \sin\delta' \cos\delta' \, \theta_{22s} \quad ,$$

where δ' is the angle between the external streamlines and the normal to the leading edge. That is

$$\delta' = \phi + \alpha \quad , \qquad \tan\delta' = \frac{v_e}{u_e} = \frac{U_\infty \sin\phi}{u_e} \quad , \qquad (14)$$

where α is the angle between streamlines and the direction of flow at infinity. For thin wings we may assume that the cross-flow is small. This will mean that θ_{21s} and θ_{12s} are of order β and θ_{22} of order β^2, where β is the angle between streamlines. We ignore terms of order β^2.

Thus we have

$$\theta'_{11} = \theta_{11s} - \tan\delta' \, (\theta_{21s} + \theta_{12s}) \quad ,$$

$$\theta_{21} = \sin\delta' \cos\delta' \, \theta_{11s} + \cos^2\delta' \, \theta_{21s} - \sin^2\delta' \, \theta_{12s} \quad .$$

It has usually been assumed[9,15,16] that in turbulent flow the streamwise and cross-flow profiles have universal forms; for example the streamwise profile has usually been supposed to follow a 1/7th power law. Relations for these forms are worked out in Appendix 2 and it is found that approximate values are

$$\theta'_{11} = \theta_{11s} (1 + 1.5 \beta \tan \delta')$$

$$\theta_{21} = \theta_{11s} (\sin \delta' \cos \delta' - 2 \beta \cos^2 \delta' - 0.5 \beta \sin^2 \delta') \quad .$$

Thus we have as an approximate formula for drag

$$C_D = 2\left(\frac{\theta_{11sT}}{c}\right)\left\{\cos^2 \phi (1 + 1.5 \beta_T \tan \delta'_T)\left(\frac{u_{eT}}{U_\infty \cos \phi}\right)^{3.5} + \right.$$
$$\left. + \tan \phi \left(\frac{U_{eT}}{U}\right)^2 (\sin \delta'_T \cos \delta'_T - 2\beta_T \cos^2 \delta'_T - 0.5 \beta_T \sin^2 \delta')\right\}$$
$$= 2\left(\frac{\theta_{11sT}}{c}\right) E \quad . \tag{15}$$

If the external flow is known everything in this equation is known except θ_{11sT} and β_T. We now proceed to determine these quantities.

4 THE VALUE OF θ_{11s}

We have (see Fig.3) for one surface of the wing (say the upper surface)

$$u_e = U_e \cos (\phi + \alpha) = U_e \cos \delta' \quad , \tag{16}$$

and u_e is in fact the velocity for flow past an unswept wing with the same thickness, but with chord $c \cos \phi$, so that its maximum thickness chord ratio is $(t/c) \sec \phi$, which we denote by $\overline{t/c}$. For this wing the velocity at infinity is $U_\infty \cos \phi$. Let $\bar{u}_e$ be the velocity of flow past this unswept wing when the velocity at infinity is increased to U_∞, so that

$$u_e = \bar{u}_e \cos \phi \quad . \tag{17}$$

We now suppose the unswept wing to be stretched so that its chord becomes c, without changing its maximum thickness-chord ratio. Bars over any quantity will refer to such an unswept wing, whose Reynolds number based on chord is the same as the Reynolds number of the swept wing, based on its streamwise chord. $\bar{u}_e$ is independent of sweep and depends only on the wing section by a plane normal to the leading edge.

To simplify notation we shall denote θ_{11s}, $\bar{u}_e$, U_e and u_e by θ, $\bar{u}$, U and u; in fact we shall drop all subscripts except T which denotes values at the trailing edge and t which denotes values at the point of boundary layer transition. For convenience we shall from now on suppose that U, u, $\bar{u}$ have been made non-dimensional by dividing by the velocity U_∞.

Now Cooke[8] has shown that for an infinite yawed wing with small cross-flow

$$U_T^3\, u_T^{1.2}\left(\frac{\theta_T}{c}\right)^{1.2} - U_t^3\, u_t^{1.2}\left(\frac{\theta_t}{c}\right)^{1.2} = \frac{0.0106}{R^{0.2}} \int_{(x_t/c)\cos\phi}^{\cos\phi} U^{3.8}\, u^{0.2}\, d\left(\frac{x}{c}\right) \quad . \qquad \ldots (18)$$

In this equation x measures distance along the surface of the wing normal to the leading edge and $R = U_\infty c/\nu$, the Reynolds number based on streamwise chord. Equation (18) is based on the results of Spence[7] for two-dimensional turbulent boundary layers, in conjunction with the line-of-flow principle.

Using equations (16) and (17) and writing $x = s \cos\phi$ we find that equation (18) becomes

$$\frac{\bar{u}_T^{4.2}}{\cos^3 \delta'_T}\left(\frac{\theta_T}{c}\right)^{1.2} - \frac{\bar{u}_t^{4.2}}{\cos^3 \delta'_t}\left(\frac{\theta_t}{c}\right)^{1.2} = \frac{0.0106 \cos^{0.8}\phi}{R^{0.2}} \int_{s_t/c}^{1} \frac{\bar{u}^4}{\cos^{3.8}\delta'}\, d\left(\frac{s}{c}\right) \qquad \ldots (19)$$

where s_t is the streamwise distance from the leading edge to the point of transition.

Now consider an unswept wing with the same chord and maximum thickness-chord ratio $(t/c)\sec\phi = \overline{t/c}$; this wing has an external velocity $\bar{u}_e$. The equation for this wing corresponding to (19) is

$$\left(\frac{\bar{\theta}_T}{c}\right)^{1.2} \bar{u}_T^{4.2} - \left(\frac{\bar{\theta}_t}{c}\right)^{1.2} \bar{u}_t^{4.2} = \frac{0.0106}{R^{0.2}} \int_{s_t/c}^{1} \bar{u}^4\, d\left(\frac{s}{c}\right) \quad , \qquad (20)$$

assuming that its transition point is at the same fraction of the chord as the swept wing. This is in fact equation (19) with $\phi = 0$.

To find $\bar{\theta}_T$ from equation (20) it is necessary to determine θ_t and $\bar{\theta}_t$. These are approximately equal if the cross-flow is small, as shown in Appendix 3.

$\bar{\theta}_t$ is found the formula of Thwaites[7], namely

$$\left(\frac{\bar{\theta}_t}{c}\right)^2 = \frac{0.45}{R\,\bar{u}_t^6} \int_0^{s_t/c} \bar{u}_e^5 \, d\left(\frac{s}{c}\right) \quad .$$

5 THE DETERMINATION OF β_T

A general method for finding β_T (which is small) seems to be too complicated to use here, and instead we give a semi-empirical value for it. β_T will depend on sweep, being zero both for 0° and 90° sweep. It is possible to give crude arguments based on Refs.8 and 9 to show that β_T has a factor $(\overline{t/c})\sin\phi\cos\phi$; the reasoning is given in Appendix 4.

We therefore write

$$\beta_T = m(\overline{t/c})\sin\phi\cos\phi \quad , \tag{21}$$

where m is a constant for a given section shape. We may then find m by experiment. Brebner and Wyatt[10] for instance, found that for a wing of RAE 101 section with t/c = 0.12, swept to 45°, (so that $\overline{t/c} = 0.17$) β_T had a value of about 8°. This leads to a value m = 1.64. From Brebner and Wyatt's second wing, with t/c = 0.045, swept to 55°, (so that $\overline{t/c} = 0.078$) this gives $\beta_T = 3.5°$ which agrees approximately with the value obtained in their experiment. In their photographs it may be noted that β_T does not seem to be greatly affected by the position of transition and so we shall use equation (21) with m = 1.64 universally. For different shapes m will of course be different but for reasonable changes of shape the change should be small and β_T is itself small in any case.

This method of finding β_T is not very satisfactory but a more rigorous determination is scarcely possible at present and might complicate the analysis to a degree not justified by the accuracy of the final results.

The drag factor is now obtained as the ratio of the drag given by equation (12) to that obtained by putting $\phi = 0$ in equation (12).

6 EXTENSION TO COMPRESSIBLE FLOW WITH ZERO HEAT TRANSFER

There is no great difficulty in carrying out the analysis for the sub-critical compressible case. In section 2 nothing is changed up to and including equation (10) which may be written

$$\frac{d\theta'_{11}}{dx} + (2 + H - M^2)\,\frac{\theta'_{11}}{u_e}\,\frac{du_e}{dx} = 0 \quad .$$

Following Thwaites[7] we replace $2 + H - M^2$ by the mean of its values at the trailing edge and at infinity. We denote this mean value by the subscript m.

Integrating the equation from the trailing edge to infinity downstream we find

$$\frac{\theta'_{11\infty}}{\theta'_{11T}} = \left(\frac{u_{eT}}{U_\infty \cos\phi}\right)^{(2+H-M^2)_m} \quad .$$

Now Spence[12] transformed the incompressible boundary layer into an incompressible one, and he found that for zero heat transfer

$$H = \frac{T_w}{T_e}(H_i + 1) - 1$$

where H_i is the corresponding incompressible value. This equation was derived only for flat plate flow, but is probably adequate for use in the momentum equation. At infinity we have $H = 1$ and following Thwaites we write $H_i = 2$ at the trailing edge. According to Spence, for zero heat transfer

$$\frac{T_{wT}}{T_{eT}} = 1 + 0.178\,M_T^2 \quad ,$$

and we write $M_T \simeq M_\infty$ which will be sufficiently accurate since $u_{eT} \simeq U_\infty \cos\phi$.

We find

$$(2 + H - M^2)_m = 3.5 - 0.733\,M_\infty^2 \quad .$$

Hence we may write in place of equation (11)

$$\theta'_{11\infty} = \theta'_{11T}\left(\frac{u_{eT}}{U_\infty \cos\phi}\right)^{3.5-0.733M_\infty^2} \quad .$$

Other formulae have been given for $\theta_{11\infty}$ for incompressible flow. For instance Young and Winterbottom[17] find that in two-dimensional flow the incompressible value should be multiplied by ρ_{eT}/ρ_∞ that is $(T_{eT}/T_\infty)^{5/2}$. Nash, Moulden and Osbourne[18] give $(T_{eT}/T_\infty)^{5/4}$. For an unswept wing, taking the power as 5/2, we can show that if u_{eT}/u_∞ does not differ greatly from unity, the value of θ_∞ is

$$\theta_T \left(\frac{u_{eT}}{U_\infty}\right)^{\frac{2+H_T}{5} - M_\infty^2}$$

and if Spence's form for H_T is used, and we put $H_i = 2$ this becomes

$$\theta_T \left(\frac{u_{eT}}{U_\infty}\right)^{3.5-0.733M_\infty^2} .$$

This will cause the same change to be made to the exponent 3.5 in equation (12). The values of θ_{21s} and θ_{12s} in equation (25) will be changed, but these are subject to some uncertainty in any case, and the change will be small in subsonic flow which is our main interest in this connection. We shall therefore leave them unchanged and so equation (15) will be unaltered except for the change in the exponent 3.5.

There will be a change in the value of θ_{11sT}. There is no point now in using $\bar{u}$, since equation (17) no longer holds. Equation (18) should be replaced by

$$U_T^3\, u_T^{1.2} \left(\frac{T_{eT}}{T}\right)^{1.665} \left(\frac{\theta_T}{c}\right)^{1.2} - U_T^3\, u_t^{1.2} \left(\frac{T_{et}}{T_\infty}\right)^{1.665} \left(\frac{\theta_t}{c}\right)^{1.2}$$

$$= \frac{0.0106 \cos \phi}{R^{0.2}} \int_{s_t/c}^{1} U^{3.8}\, u^{0.2} \left(\frac{T_e}{T_\infty}\right)^{1.343} (1 + 0.128\, M^2)^{-0.822}\, d\left(\frac{s}{c}\right) ,$$

where θ_t/c is found from

$$\left(\frac{\theta_t}{c}\right)^2 \left(\frac{T_{et}}{T_\infty}\right)^3 = \frac{0.45}{R}\left(\frac{}{u_t}\right) \int_0^{s_t/c} u_e^5 \left(\frac{T_e}{T_\infty}\right)^{1.5} d\left(\frac{s}{c}\right) \quad .?$$

In the last two equations u and U have been made non-dimensional by U_∞ as before. They come from Ref.8.

It is not so easy to see what happens to β_T when compressibility is taken into account. It would seem that equation (29) should be replaced by

$$\bar{u}_e = 1 + \frac{(t/c)\, f(x)}{\beta_1}$$

where β_1 is the Weber factor given by[13]

$$\beta_1 = [1 - M_\infty^2 (\cos^2 \phi - c_{pi})]^{\frac{1}{2}}$$

and c_{pi} is the pressure coefficient if the flow is incompressible. This means that we must write in place of (21)

$$\beta_T = \frac{m(t/c) \sin \phi \cos \phi}{\beta_1}$$

with m = 1.64 as before. Since β_T is small it will be sufficient to write

$$\beta_1 = (1 - M_\infty^2 \cos^2 \phi)^{\frac{1}{2}} \quad .$$

When u_e is known we find M and T_e/T from the equations

$$M^2 = \frac{5\, U_e^2\, M_\infty^2}{5 + M_\infty^2 (1 - U_e^2)} \quad ,$$

$$\frac{T_e}{T_\infty} = \frac{5 + M_\infty^2}{5 + M^2} \quad .$$

These equations are sufficient to determine C_D if a set of values of u_e is known. If the pressure coefficient c_p is given, for instance from experiments, then we have[8]

$$\frac{T_e}{T_\infty} = (1 + 0.7\, M_\infty^2\, c_p)^{2/7} = K \text{ (say)} \quad ,$$

$$M^2 = \frac{5 + M_\infty^2}{K} - 5 \quad ,$$

$$U_e = (5 + M_\infty^2 - 5K)^{\frac{1}{2}}/M_\infty \ ,$$

$$u_e^2 = U_e^2 - \sin^2 \phi \quad .$$

7 DRAG OF SWEPT WING OF RAE 101 AND 104 SECTIONS AT ZERO LIFT

In order to carry out the calculations it is necessary to give some form for $\bar{u}_e$. We have taken values from Ref.11. From $x/c = 0.3$ and 0.6 respectively to 0.95 the curves for $\bar{u}_e$ are straight lines, and these lines have been extended to $x/c = 1.00$. Our results for θ_T are rather sensitive to the choice of $\bar{u}_e$ at this point, as Thwaites[7] pointed out. He also showed that $\theta(\bar{u}_e)^{3.5}$ is not so sensitive to such variations. In any case some of this sensitivity is lost when one compares the drag with that of a wing whose drag is computed by the same technique.

The integrals were evaluated by Simpson's rule with intervals of s/c equal to 0.05. The computations are straightforward and the results show little dependence on Reynolds number except for the highest values of s_t/c.

Fig.4 gives the sweep factors for RAE 101 for three different values of $\overline{t/c}$. It will be seen that the effect of sweep is very much reduced if the wing is thin.

Fig.5 is a cross plot of the same curves expressed as functions of $\overline{t/c}$. It will be seen that for early transition the curves are very nearly straight lines. An empirical formula for these lines is

sweep factor $= 1 - (\overline{t/c})\{2.84 - 4.6\,(s_t/c)^2 - 0.25\,(s_t/c)^4\} \sin^2 \phi$, which gives a fair approximation up to $s_t/c = 0.6$.

Fig.6 gives the curves corresponding to those of Fig.5 for the section RAE 104. For early transition, say up to about $s_t/c = 0.3$ the curves do not differ greatly. A comparison is shown in Fig.7. It will be seen that for high values of s_t/c and for high values of ϕ the sweep factors for RAE 104 are considerably less than those for RAE 101.

Fig.8 gives the drag coefficient for a wing of RAE 101 section, having a streamwise t/c of 0.12 for varying Mach numbers at angles of sweep 0° and 45°. The drag rise due to compressibility, sometimes called "drag creep", at $\phi = 0$ is shown clearly, as is also the fact that sweep delays this rise considerably. A point of interest in this Fig.8 can be seen in the fact that at low Mach numbers the curves for $\phi = 0$, $s_t/c = 0.2$ and $\phi = 45°$, $s_t/c = 0.0$ run close together. An unsheared wing at zero incidence may well have its transition point at $s_t/c = 0.2$, but when it is sheared it is likely that transition would move to a point very close to the leading edge. In such a case there would be practically no change in drag, the reduction due to shear being cancelled by the forward movement of transition.

It would be of some interest if one could find out how this increase in drag with increasing Mach number arises. It would be useful if we could separate out the skin friction drag and the pressure drag. Unfortunately it is not possible to do this with any measure of accuracy. As pointed out by Thwaites[7], and already referred to at the beginning of this section, in the calculation of θ the value $\theta(\bar{u}_e)^{3.5}$ is insensitive to local inaccuracies in $\bar{u}_e$ near to the trailing edge whilst the pressure drag and skin friction drag are both quite sensitive to changes in $\bar{u}_e$. It is fortunate that it is just this combination that we need for the overall drag especially as the value used for $\bar{u}_e$ is very much of an estimate near to the trailing edge (theory gives $\bar{u}_e = 0$ there whilst actually it is fairly near to $U_\infty \cos \phi$). Hence we cannot separate the skin friction drag and the pressure drag with any degree of confidence.

We can perhaps estimate trends in the following way. We will confine ourselves to two-dimensional wings of 12% RAE 101 section with all-turbulent flow. It has often been observed that the skin friction drag of a wing is quite close to that of a flat plate of the same planform placed edge on to the flow. Let us assume that this is so for the sake of the discussion. Then the contributions to drag at $R = 10^7$ are given in Table 1.

Table 1

Drag coefficient of RAE 101, 12%, $R = 10^7$

M_∞	Skin friction	Pressure drag	Total
0	0.00615	0.00208	0.00823
0.4	0.00607	0.00225	0.00832
0.6	0.00596	0.00268	0.00864
0.8	0.00582	0.00469	0.01051

It will be seen that the skin friction decreases very slightly as the Mach number goes up, but there is a considerable increase in pressure drag. Now in the subcritical flow of an inviscid fluid the pressure drag is zero.

We can divide the body into two parts, the forebody (up to the point where $\partial z/\partial x = 0$) and the afterbody. The drag (or thrust) on the two halves will cancel each other in inviscid flow. Now one would expect little effect of the boundary layer in the forebody (where it is still thin) and so the large increase in pressure drag at the higher Mach numbers would seem to come mainly from the afterbody, being increased because of the considerably increased displacement thickness there, especially at the higher Mach numbers.

8 COMPARISON WITH EXPERIMENT

We shall compare our results with the experiments of Weber and Brebner[1] on a wing of RAE 101 section, swept to 45° with a streamwise thickness chord ratio of 12% so that the value of $\overline{t/c}$ is 0.17. For transition at 0.15 and 0.35 of the chord the results of this paper give factors 0.773 and 0.807 respectively, whilst Weber and Brebner found factors 0.82 and 0.84. However, they apply the factor to the drag of an unsheared wing of the same thickness chord ratio, whilst the factors given here are to be related to a wing of thickness chord ratio of 0.17. The results are shown in Fig.9 and the agreement is seen to be fair. Perfect agreement could hardly be expected unless one could assume that the end effects of the finite wing tested cancel one another.

In this connection it might be well to mention some unpublished work by Weber in connection with the drag of an unswept wing. This is briefly described in Appendix 5.

Another comparison may be made by considering the models tested by Kirby and described in his addendum to Ref.1. For wing A Kirby found the drag factor to be 0.87 whilst the present method gives 0.887, and for wing B his factor was 0.845 compared with 0.874 by the present method. The measurements were, however, made on tapered wings, and the angle of sweep used was based on that of the quarter chord line. One would expect these wings to behave even less like infinite swept wings than that described above and one would not really expect good agreement.

Another comparison may be made from Brebner and Wyatt's work[10] on a $4\frac{1}{2}\%$ thick RAE 101 section sheared to 55° at a Reynolds number of 2×10^6. Transition was at 0.79 of the chord. For this sweep and thickness we have $\overline{t/c} = 0.078$. According to Weber and Brebner[1] the sweep factor should be 0.83 whilst the measured value was very approximately 1.06. According to the present work the factor should be about 0.98. This factor, however, compares the wing with an unswept one of thickness/chord ratio of 0.078. To make the factor apply to wings of the same streamwise section we must allow for this change of thickness. This can be done by the use of Fig.11. We find that the factor becomes 1.01. Thus the error of 20% in the factor is reduced to 5%.

The factor is always nearer to unity when this type of comparison is made, that is between sheared and unsheared wings, both having the same streamwise thickness chord ratio. Indeed in such a comparison it may become greater than unity in rather extreme conditions, namely late transition and low Reynolds numbers, as in the example above.

9 DRAG OF RAE 101 AND 104 SECTIONS UNSWEPT. INCOMPRESSIBLE FLOW

As the method involves finding the drag of these sections when unswept, and as the drags do not appear to have been previously determined, it was considered that these might be worth recording. In Ref.14 the method of displaying the results is first to give curves for the drag of a flat plate, and then to give curves for a "form factor" λ by which the flat plate drag is to be multiplied to give that of the wing. We have recomputed the flat plate drag and the results are shown in Fig.9. The curves are the same as those in Fig.V 4 in Ref.7 and are obtained from equation (20) with all velocities constant and equal to the free stream values. The factor λ for the two sections is given in Figs.11 and 12. The curves given in Ref.14 show no dependence of λ on Reynolds number, but it will be seen that there is in fact quite a strong dependence in the case of the wings studied here.

10 CONCLUDING REMARKS

It is to be hoped that the curves given in this Note may be of some use in the determination of the drag of swept wings with sections not differing too much from the basic sections used.

The results strictly only apply for an infinite swept wing at zero lift. A finite swept wing of constant section has different pressure profiles at different points in its span, and for such a wing it is not true that derivatives with respect to y are zero. Such wings are, however, often designed so as to have straight isobars and for these the method would work if the reference unswept wing were properly chosen.

The method will not apply if the transition front is not parallel to the leading edge but it might be possible to use it to give approximate values even in this case, either by taking a mean position for transition or dividing the surfaces into spanwise strips and applying the appropriate factor to each.

If the drag coefficient of any other section is required it is not of course necessary to use these drag factors since the programme is available to calculate directly the drag coefficient for any Reynolds number, sweep, and position of transition. All that is necessary to be known are the values of the chordwise component of velocity or the pressure coefficient at 21 or 41 points on the chord. It is also necessary to know β_T, but if the section does not differ very much from those considered here, the change in β_T (which is itself small) will probably make little difference to the results, and so the value given in equation (21) may suffice.

Few experiments seem to be available to test the theory. It gives fair results for the wings tested by Weber and Brebner[7], but a wider series of tests would be necessary before its general usefulness could be assessed.

It is of course to be understood that the results would not be valid if separation were to occur.

The method can in principle be used for a lifting wing, but each surface would need to be considered separately, with the appropriate values of velocity for each of the two surfaces. The difficulty here would be the determination of β_T, which is indeed open to criticism even in the case of a symmetrical wing at zero lift. It would not, however, be feasible to attempt to find drag factors to be universally used in such cases. In the case of symmetrical sections at zero lift in incompressible flow, the factors have a universal quality if transition takes place early, as is usually the case. This is probably not so, however, at Mach numbers approaching the critical.

Appendix 1

VALUES OF θ'_{11} AND θ_{21}

We see from Fig.3 that

$$u = u_s \cos\delta' - v_s \sin\delta', \qquad u_e = u_{se}\cos\delta' = U_e\cos\delta',$$

$$v = u_s \sin\delta' + v_s \sin\delta', \qquad v_e = u_{se}\sin\delta' = U_e\sin\delta',$$

noting that $v_{se} = 0$, $u_{se} = U_e$ since the suffix s refers to streamline coordinates.

Now, omitting the limits, which in the wake and at the trailing edge are from $-\infty$ to $+\infty$, or from 0 to ∞ if the upper and lower surfaces are treated separately, we have

$$\rho_e u_e^2 \theta'_{11} = \int \rho\, u(u_e - u)\, dz \quad ,$$

and hence

$$\rho e U_e^2 \theta'_{11} = \frac{1}{\cos^2\delta'}\int \rho(u_s\cos\delta' - v_s\sin\delta')(U_e\cos\delta' - u_s\cos\delta' + v_s\sin\delta')\, dz$$

$$= \int \rho\, u_s(U_e - u_s)\, dz + \tan\delta' \int \rho\, u_s v_s\, dz$$

$$- \tan\delta' \int \rho\, v_s(U_e - u_s)\, dz - \tan^2\delta' \int \rho\, v_s^2\, dz \; .$$

Hence we have

$$\theta'_{11} = \theta_{11s} - \tan\delta' (\theta_{21s} + \theta_{12s}) + \tan^2\delta'\, \theta_{22s} \quad , \tag{22}$$

where

$$\left.\begin{array}{ll} \rho_e U_e^2 \theta_{11s} = \int \rho\, u_s(U_e - u_s)\, dz , & \rho_e U_e^2 \theta_{12s} = \int \rho\, v_s(U_e - u_s)\, dz , \\ \rho_e U_e^2 \theta_{21s} = -\int \rho\, u_s v_s\, dz , & \rho_e U_e^2 \theta_{22s} = -\int \rho\, v_s^2\, d\xi \end{array}\right\} \quad \ldots (23)$$

In a similar way from the equation

$$\rho_e U_e^2 \theta_{21} = \int \rho \, u(v_e - v) \, dz \quad ,$$

we may deduce

$$\theta_{21} = \sin \delta' \cos \delta' (\theta_{11s} - \theta_{22s}) + \cos^2 \delta' \, \theta_{21s} - \sin^2 \delta' \, \theta_{12s} \quad . \quad (24)$$

Appendix 2

VALUES OF θ_{12s} AND θ_{21s}

We assume, following Cooke[9], that

$$\frac{u_s}{U_e} = \left(\frac{z}{\delta}\right)^{1/7} , \qquad \frac{v_s}{U_e} = \left(1 - \frac{z}{\delta}\right)^2 \frac{u_s}{U_e} \beta ,$$

where β is the angle between streamlines and limiting streamlines. After substituting the values in equations (24) and integrating between the limits 0 and δ we find for one surface, say the upper surface,

$$\theta_{11s} = 0.0972\ \delta ,$$

$$\theta_{21s} = -0.2071\ \beta\delta ,$$

$$\theta_{12s} = 0.0527\ \beta\delta .$$

Hence we have

$$\frac{\theta_{21s}}{\theta_{11s}} = -2.13\ \beta , \qquad \frac{\theta_{12s}}{\theta_{11s}} = 0.542\ \beta .$$

If we use Becker's forms[15], which are

$$\frac{u_s}{U_e} = \left(\frac{z}{\delta}\right)^{1/7} , \qquad \frac{v_s}{U_e} = \frac{4}{3}\beta \frac{u_s}{U_e} \left\{1 - \left(\frac{u_s}{U_e}\right)^2\right\} ,$$

we find

$$\frac{\theta_{21s}}{\theta_{11s}} = -1.94\ \beta , \qquad \frac{\theta_{12s}}{\theta_{11s}} = 0.46\ \beta .$$

Since β is small we may use a simple approximation for these values without serious error, and we have in fact chosen

$$\theta_{21s} = -2\ \beta\ \theta_{11s} , \qquad \theta_{12s} = 0.5\ \beta\ \theta_{11s} . \qquad (25)$$

The same values apply for the lower surface, using the appropriate value for θ_{11s}, which will be different unless the wing is symmetrical and at zero lift.

Appendix 3

PROOF THAT $\theta_t = \bar{\theta}_t$

In the full notation, we require to prove that in the laminar flow up to transition

$$\theta_{11s} = \bar{\theta} \quad .$$

We first note that, according to equation (22)

$$\theta_{11s} = \theta'_{11} + O(\beta) \quad ,$$

and we shall prove that $\theta'_{11} = \bar{\theta}$.

Here θ'_{11} is equal to the momentum thickness for a two-dimensional wing whose chord is $c \cos \Phi$, thickness chord ratio $(\overline{t/c})$ and velocity at infinity $U_\infty \cos \Phi$. This follows from equation (5) and the independence principle in laminar flow. At the same time $\bar{\theta}$ is the momentum thickness for a wing, geometrically similar to the other one, whose chord is c and velocity at infinity U_∞. That these are equal follows from the general two-dimensional laminar boundary layer equations, which are unchanged if u, u_e and x are each multiplied by a factor $\cos \Phi$, with w and z unchanged.

Hence it is true that $\theta_{11s} = \bar{\theta}$ with an error of order β and in particular $\theta_{11st} = \bar{\theta}_t$, which is the desired result.

Appendix 4

ESTIMATE FOR β_T

In Ref.9 is given the equation

$$\frac{\partial\eta}{\partial\varphi} + \frac{a\eta}{U_e\,\Theta} = \frac{e}{U_e^{3.5}}\frac{\partial U_e}{\partial\psi} , \qquad (26)$$

where the line element in streamline coordinates φ and ψ is given by

$$ds^2 = \frac{d\varphi^2}{U_e^2} + \frac{d\psi^2}{\bar{\rho}\,U_e^2} \qquad (27)$$

and

$$\eta = \frac{\beta}{\bar{\rho}^{0.5}\,U_e^{2.5}} , \qquad \Theta = \theta_{11s}\left(\frac{U_e\,\theta_{11s}}{\nu}\right)^{0.2} . \qquad (28)$$

a and e are constant and φ is the velocity potential of the external flow. It can be shown[16] that for an infinite yawed wing

$$\bar{\rho} = \frac{A^2}{u_e^2} , \qquad \frac{\partial}{\partial\varphi} = \frac{u_e}{U_e^2}\frac{d}{dx} , \qquad \frac{\partial}{\partial\psi} = \frac{\sin\Phi}{\bar{\rho}^{0.5}\,U_e}\frac{d}{dx} .$$

Suppose we take a given wing and vary its sweep, without changing anything else. Then x is not changed; although u_e varies with sweep $\bar{u}_e = u_e/\cos\Phi$ is not changed.

In linear thin wing theory

$$\bar{u}_e = 1 + (\overline{t/c})\, f(x) , \qquad (29)$$

where, as usual, $\overline{t/c}$ is the maximum thickness-chord ratio normal to the leading edge. $\overline{t/c}$ is unchanged by sweep. $f(x)$ is also unchanged by sweep or by changes in thickness-chord ratio.

Now, since from equation (31) below $\alpha\tan\Phi$ is small,

$$U_e = \frac{u_e}{\cos(\alpha+\Phi)} = \frac{\bar{u}_e\cos\Phi}{\cos(\Phi+\alpha)} \simeq \bar{u}_e\,(1+\alpha\tan\Phi) . \qquad (30)$$

We can show from equation (14) that

$$\tan\alpha = \frac{(1-\bar{u}_e)\sin\phi\cos\phi}{1-(1-\bar{u}_e)\cos^2\phi}$$

and so to the first order in $\overline{t/c}$ we have

$$\alpha = -(\overline{t/c})\,f(x)\sin\phi\cos\phi \quad . \tag{31}$$

Substituting in equation (26) we have, keeping the first order in powers and products of α,β and $\overline{t/c}$ and noting that dU_e/dx is of order $\overline{t/c}$,

$$\cos^2\phi\,\frac{d\beta}{dx} + \frac{\alpha\beta\cos\phi}{\theta} = e\sin\phi\cos\phi\,\frac{dU_e}{dx} \quad . \tag{32}$$

For θ we have the equation[9]

$$\frac{\partial}{\partial\varphi}\left(\frac{\theta\,U_e^{2.8}}{\bar{\rho}^{0.6}}\right) = 0.0106\,\frac{U_e^{1.8}}{\bar{\rho}^{0.6}}$$

in turbulent flow, and we require the value of θ to zero order, for which we may write $U_e = \bar{u}_e$, and so

$$\theta = \frac{0.0106}{\bar{u}_e^4\cos\phi}\int_{x_t}^{x}\bar{u}_e^4\,dx + \theta_t\left(\frac{\bar{u}_{et}}{\bar{u}_e}\right)^4$$

$$= \frac{\bar{\theta}}{\cos\phi} + \theta_t\left(\frac{\bar{u}_{et}}{\bar{u}_e}\right)^4$$

where $\bar{\theta}$ is independent of sweep. To zero order we could in fact have written $\bar{u}_e = 1$ but this might have confused the argument. However, to zero order $\bar{\theta}$ is independent of $\overline{t/c}$ as well as of sweep. θ_t depends on θ_{11t} which is supposed to be small and will be ignored.

Finally we have on using equations (30) and (31)

$$U_e = \bar{u}_e\,(1+\alpha\tan\phi)$$

$$= \{1+(\overline{t/c})\,f(x)\}\{1-(\overline{t/c})\,f(x)\sin^2\phi\}$$

and so to the first order in $\overline{t/c}$

$$\frac{dU_e}{dx} = (\overline{t/c})\ f'(x) \cos^2 \phi \quad .$$

Hence equation (32) may be written to the first order

$$\frac{d\beta}{dx} + \frac{\alpha\beta}{\bar{\Theta}} = e(\overline{t/c})\ f'(x) \sin\phi \cos\phi \quad .$$

This is a linear equation for β and the right hand side is the only place where the effect of $\overline{t/c}$ and ϕ is to be found. Hence β_T must have the factor $(\overline{t/c}) \sin\phi \cos\phi$.

Appendix 5

WEBER'S CALCULATIONS OF THE DRAG OF AN UNSWEPT WING

This unpublished work was done in connection with the experiments of Brebner and Bagley[10] on an unswept 10% RAE 101 section wing at a Reynolds number of 1.6×10^6. Boundary layer measurements were made just behind the trailing edge and the drag was estimated by the methods of Betz[19] and of Jones[20], and also by the method of Squire and Young[6], using measured values at the trailing edge. The following results were found:-

Incidence	Surface	x_t/c	Betz	Jones	Squire - Young
0°	Upper Lower	0.62	0.0030	0.0030	0.0037
4.1°	Upper	0.11	0.0068	0.0070	0.0077
	Lower	0.85	0.0015	0.0015	0.0019
8.2°	Upper	0.01	0.0118	0.0123	0.0133
	Lower	1.0	0.0008	0.0009	0.0009

It will be seen that there are differences between the methods of Betz and Jones, but they are nowhere more than 5% (except in the last case where the drag is very small). For zero lift the method of the present paper gives 0.00275 for each surface. We see that the Squire and Young method (on which the present paper is based) can be considerably in error (if the results of Jones and Betz are accepted as correct). It gives results too high if based on measured values and too low if based entirely on calculations.

The error in the Squire - Young method based on measured values is about 10% in the cases of early transition and can be over 25% when transition is late. It should be noted, however, that the method assumes that the flow in the wake is fully turbulent. If transition is late this does not seem to be realised in Brebner and Bagley's work, and this may account for the discrepancy. One must also bear in mind that the methods of Betz and Jones both amount to applying a correction term to allow for the fact that the wake traverse is not made at infinity downstream as in theory it ought to be. It has been suggested[21] that the distance downstream ought to be at least 5% of the chord. Here the measurements were made at the trailing edge itself, where it is possible that the Jones and Betz methods may not be so satisfactory; however, the fact that they agree so well is a strong point in their favour.

ADDENDUM

September 1967

There are errors in equations (1) and (2) of the main text which should be corrected though they do not affect equation (3) or any of the subsequent work. It is not correct to say that no fluid enters or leaves the sides ABA'B' and CDC'D' of the control surface of Fig.1. If the fluid has an outflow component w normal to these surfaces there must be an additional term

$$-\int \rho w \, dS$$

in equation (1), where the integral is taken over the two surfaces. There must also be an additional term in equation (2) equal to

$$-\int \rho w U_\infty \, dS \qquad ,$$

namely, the loss in streamwise momentum due to the outflow w. (We may take the streamwise velocity to be equal to U_∞ on the planes ABA'B' and CDC'D'.)

These additional terms cancel in equation (3). I am indebted to Dr. Pankhurst for pointing out this error.

It has been suggested that Figs.5 and 6 are not the best way to display the sweep factor F, and that it would be better to give it based on the unsheared wing as defined in section 1, paragraph 2. The most important case is when the flow is all turbulent, i.e., $s_t/c = 0$ and we shall only consider this case.

The curves in Figs.11 and 12 for $s_t/c = 0$ can be given to a good approximation as

$$\lambda = 1 + 2.25\,(t/c) + 5(t/c)^2$$

for a large range of Reynolds numbers, and those in Figs.5 and 6 for $s_t/c = 0$ are given by

$$F = 1 - 2.85\,(\overline{t/c})\,\sin^2\phi$$

Hence the factor on the new basis is given by

$$F = \left[1 - 2.85\,(t/c)\sec\phi\,\sin^2\phi\right] \frac{1 + 2.5\,(t/c)\sec\phi + 5\,(t/c)^2\sec^2\phi}{1 + 2.5\,(t/c) + 5\,(t/c)^2} \quad .$$

Where? These values of F are shown in Fig.13 and will apply for sections RAE 101 and 104 and probably therefore for intermediate sections as well.

SYMBOLS

a	constant in equation (26)
c	streamwise chord
c_p	pressure coefficient
C_D	drag coefficient
e	constant in equation (26)
E	defined in equation (15)
K	$(1 + 0.7\, M_\infty^2\, c_p)^{2/7}$
ℓ	length of the leading edge
m	constant in equation (21)
M	Mach number
s	distance in direction of velocity at infinity
T	temperature
t	maximum thickness
$\overline{t/c}$	maximum thickness-chord ratio in direction normal to leading edge
U	resultant velocity
u, v	velocity components normal to and parallel to the leading edge
u_s, v_s	velocity components along and normal to the external streamlines
x, y, z	Cartesian coordinates as shown in Figs.1 and 2
α	angle between external streamlines and direction of flow at infinity
β_1	$(1 - M_\infty^2 \cos^2 \phi)^{\frac{1}{2}}$
β	angle between streamlines and limiting streamlines
δ	boundary layer thickness
δ'	$\phi + \alpha$
δ_1, δ_1'	defined by equation (7) and (9)
θ_{11}, θ_{21}	defined by equation (4)
$\theta_{11s}, \theta_{12s}, \theta_{21s}, \theta_{22s}$	defined by equation (23)
Θ	defined by equation (28)
ϕ	angle of sweep
ρ	density
$\bar{\rho}$	defined in equation (27)
φ, ψ	streamline coordinates defined by equation (27)

Subscripts

t	refers to values at the point of transition
T	refers to values at the trailing edge
s	refers to values in a streamline coordinate system
e	refers to values just outside the boundary layer
∞	refers to values at infinity
w	refers to values at the wall

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19	A. Betz	Ein Verfahren zur direkten Ermittlung des Profilwiderstandes, ZFM, Vol.16, p.42 (1925)
20	B.M. Jones	The measurement of profile drag by the pitot traverse method. A.R.C. R. & M. 1688 (1936)
21	H. Doetsch	Profilwiderstandsmessungen in großen Windkanal der DVL. Luftfahrtforschung, Vol.1, p.111 (1941)

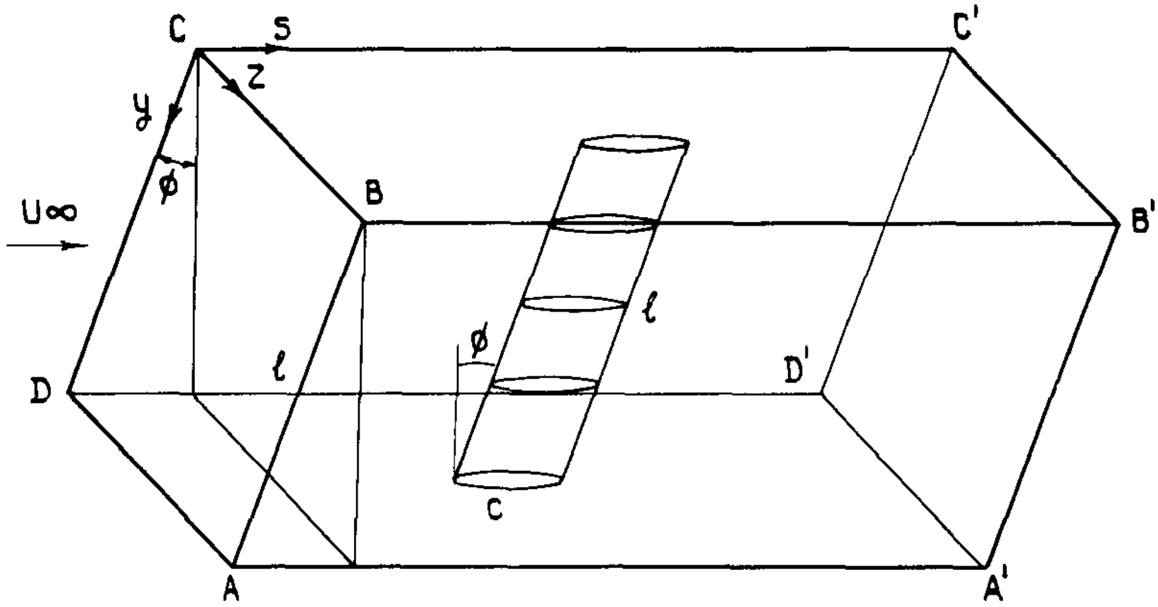


FIG I THE CONTROL SURFACE

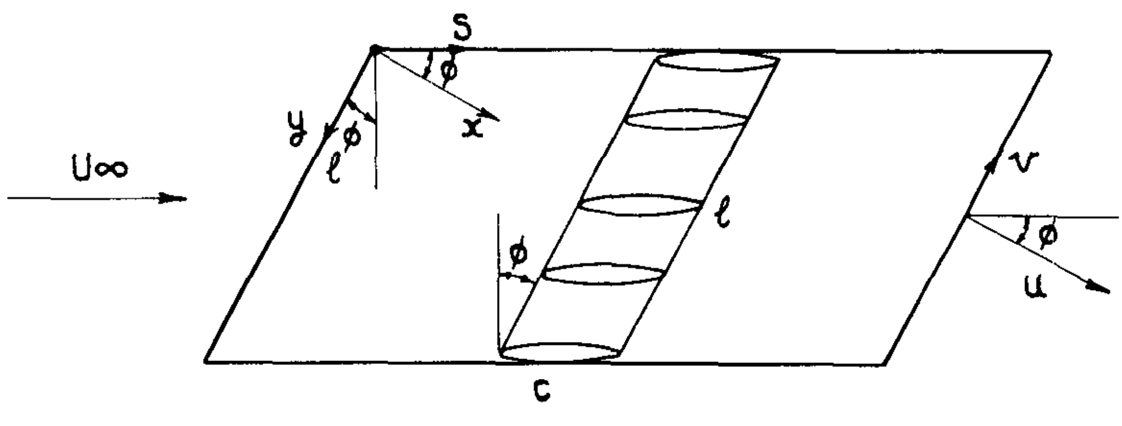


FIG. 2. VELOCITY COMPONENTS AND COORDINATE SYSTEM.

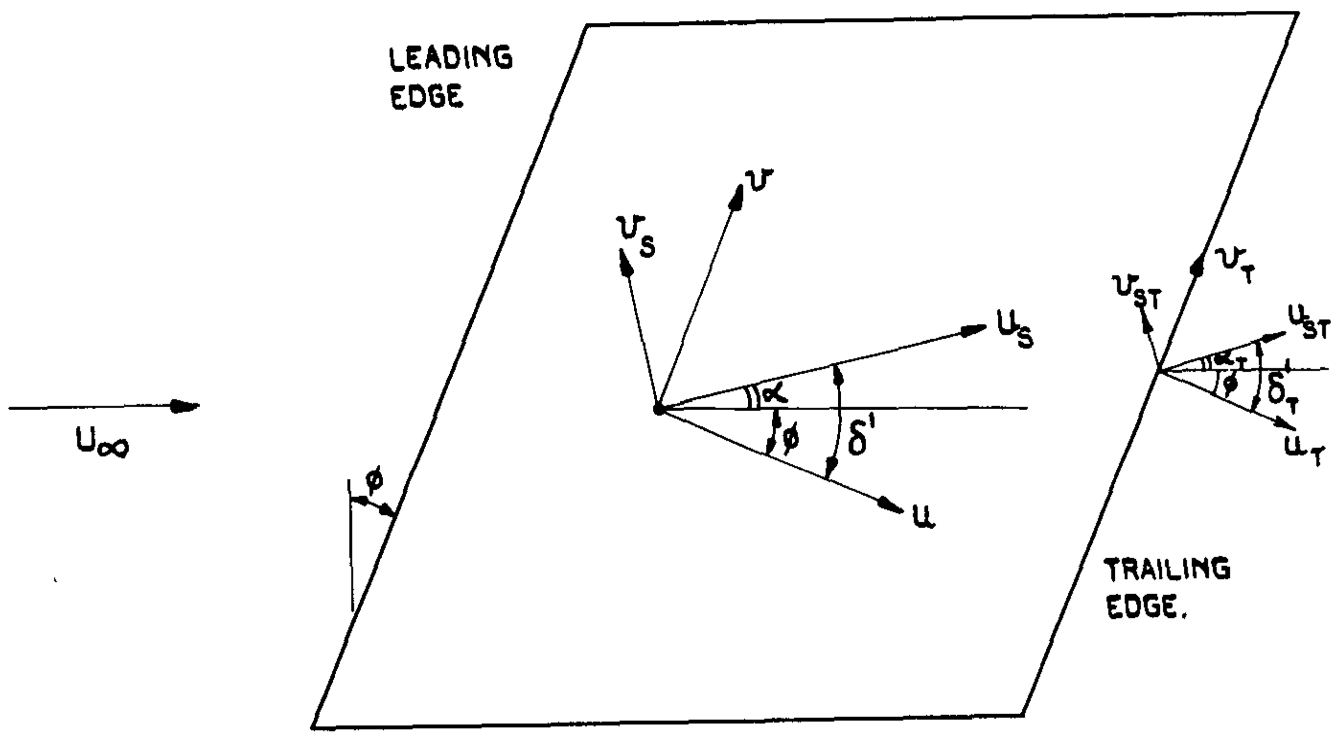


FIG.3. VELOCITY COMPONENTS ON THE WING SURFACE.

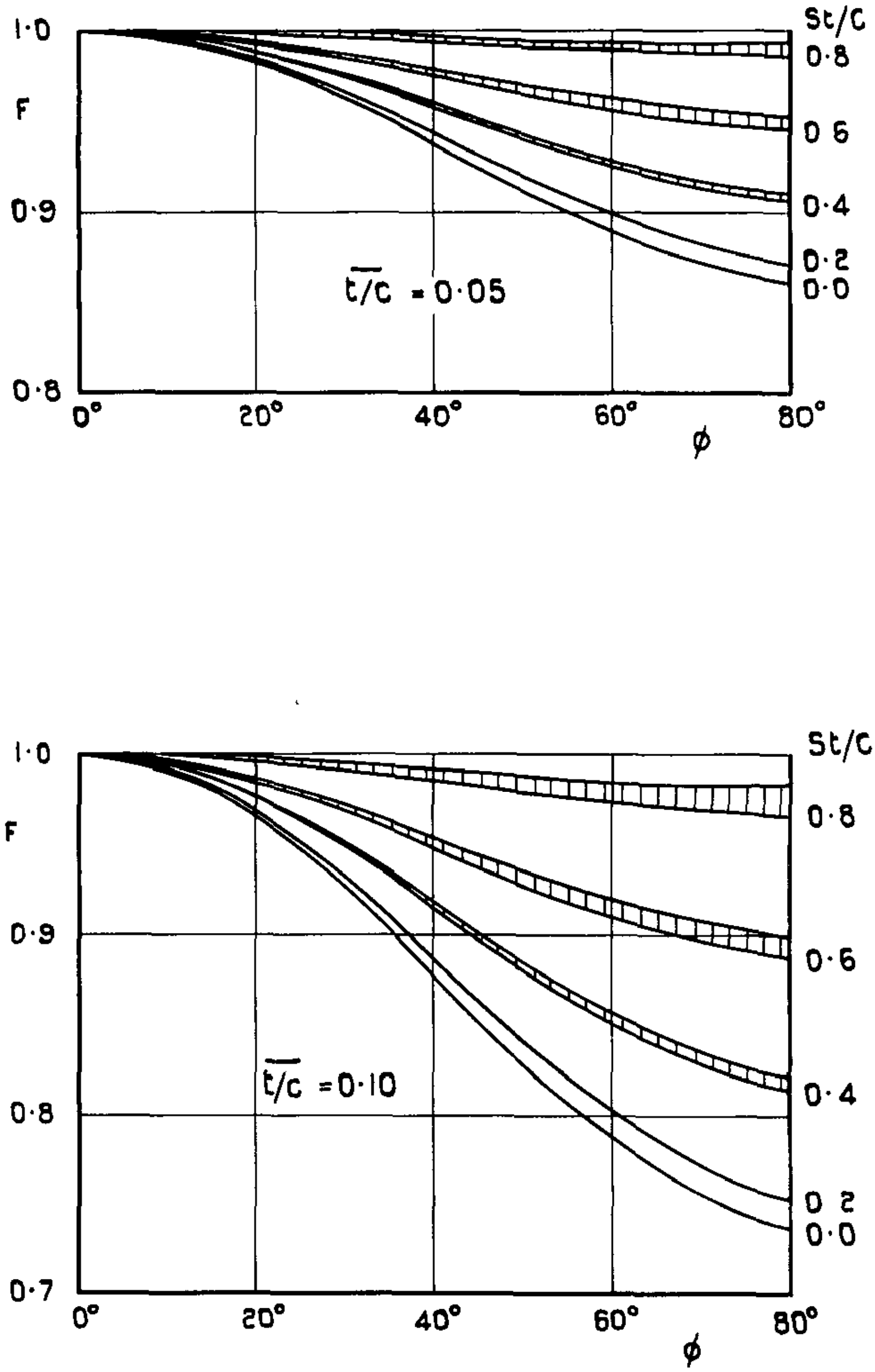


FIG. 4(a). FACTOR FOR R.A.E. 101. IN CURVES JOINED BY SHADING THE UPPER IS FOR $R = 10^8$ AND THE LOWER FOR $R = 10^7$ SINGLE CURVES ARE FOR BOTH 10^7 AND 10^8.

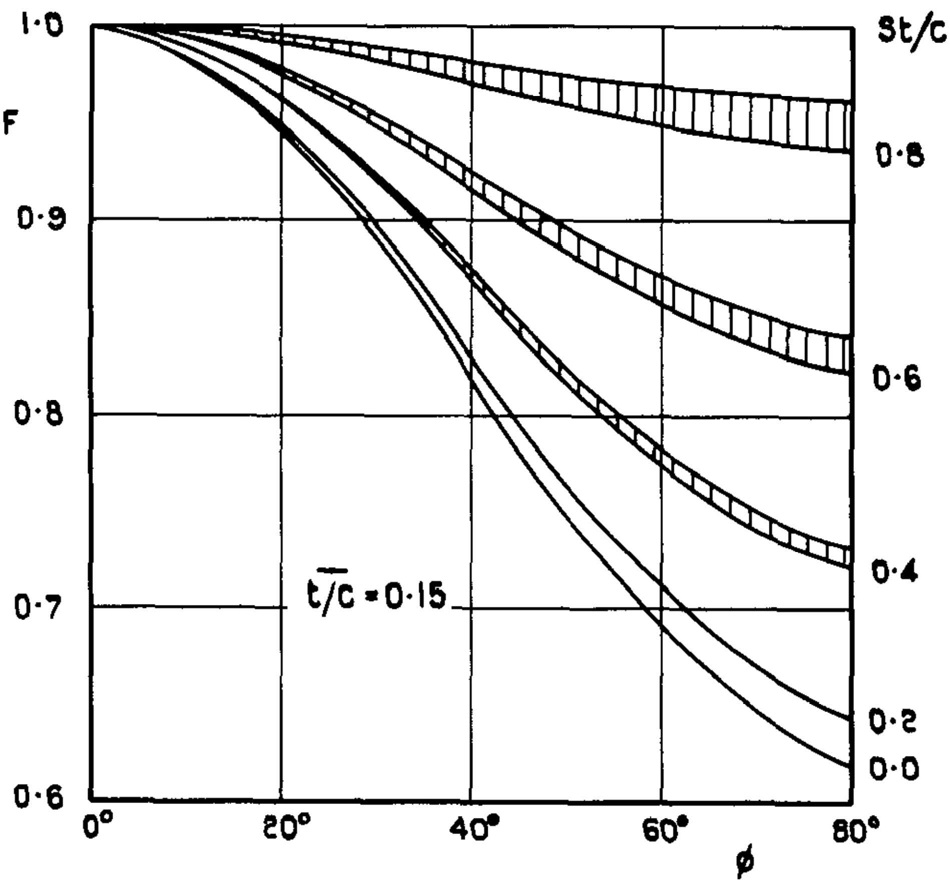


FIG.4(b) FACTOR FOR R.A.E. 101. IN CURVES JOINED BY SHADING THE UPPER IS FOR $R = 10^8$ AND THE LOWER FOR $R = 10^7$ SINGLE CURVES ARE FOR BOTH 10^7 AND 10^8.

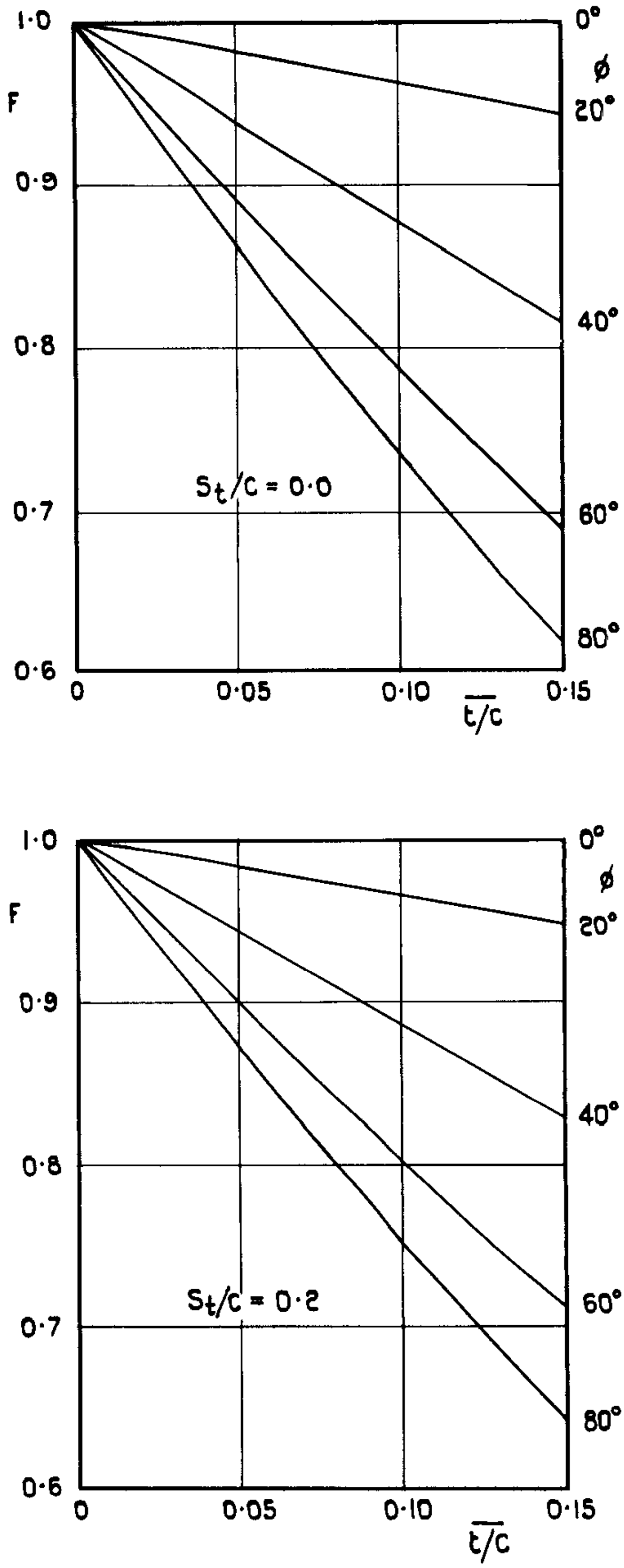
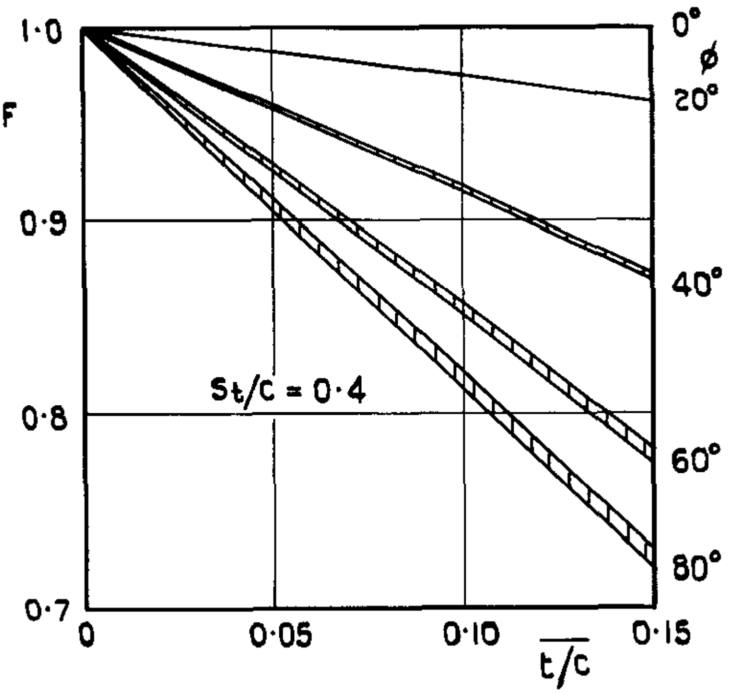


FIG. 5(a). FACTOR FOR R.A.E. 101. $R = 10^7$ AND 10^8.



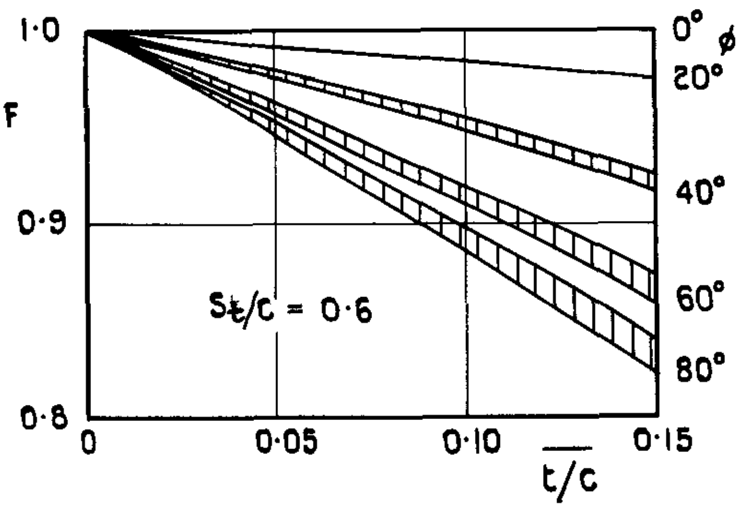


FIG.5(b). FACTOR FOR R.A.E. 101. IN CURVES JOINED BY SHADING THE UPPER IS FOR $R = 10^8$ AND THE LOWER FOR $R = 10^7$ SINGLE CURVES ARE FOR BOTH 10^7 AND 10^8.

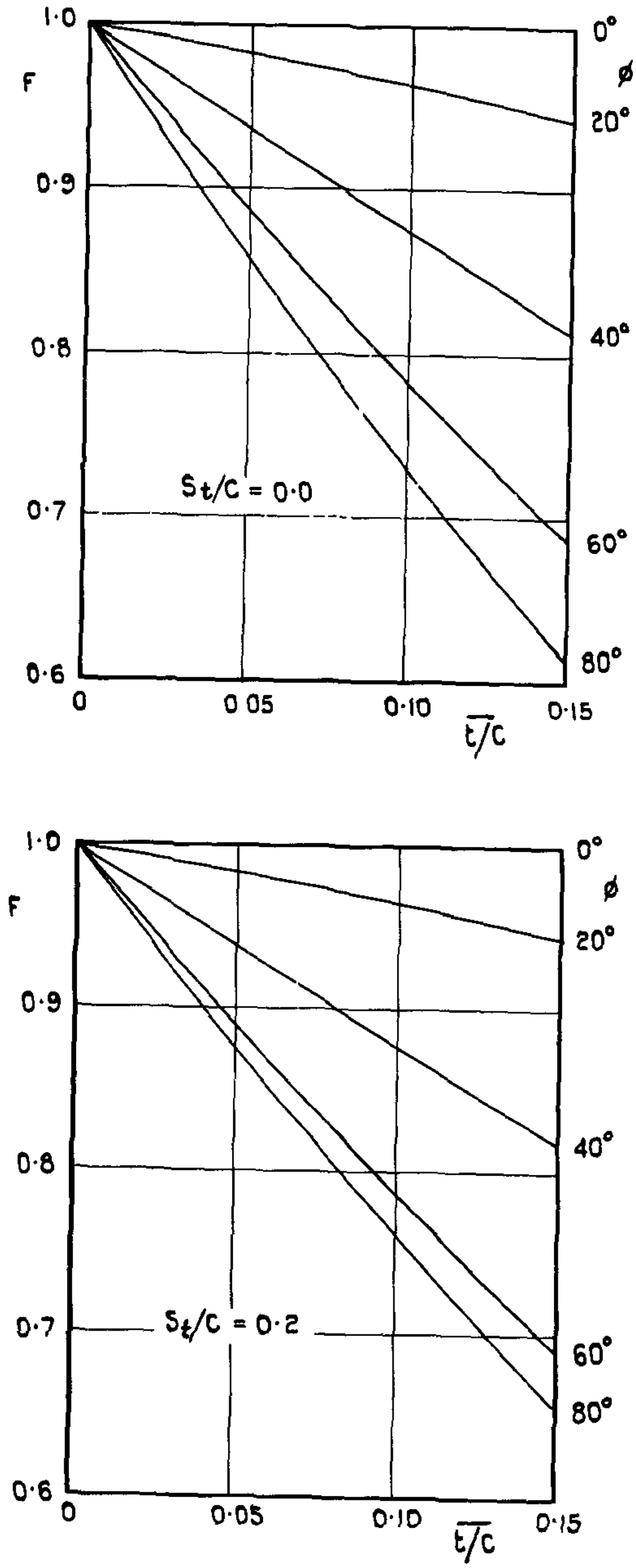


FIG.6(a). FACTOR FOR R.A.E. 104 . $R = 10^7$ AND 10^8.

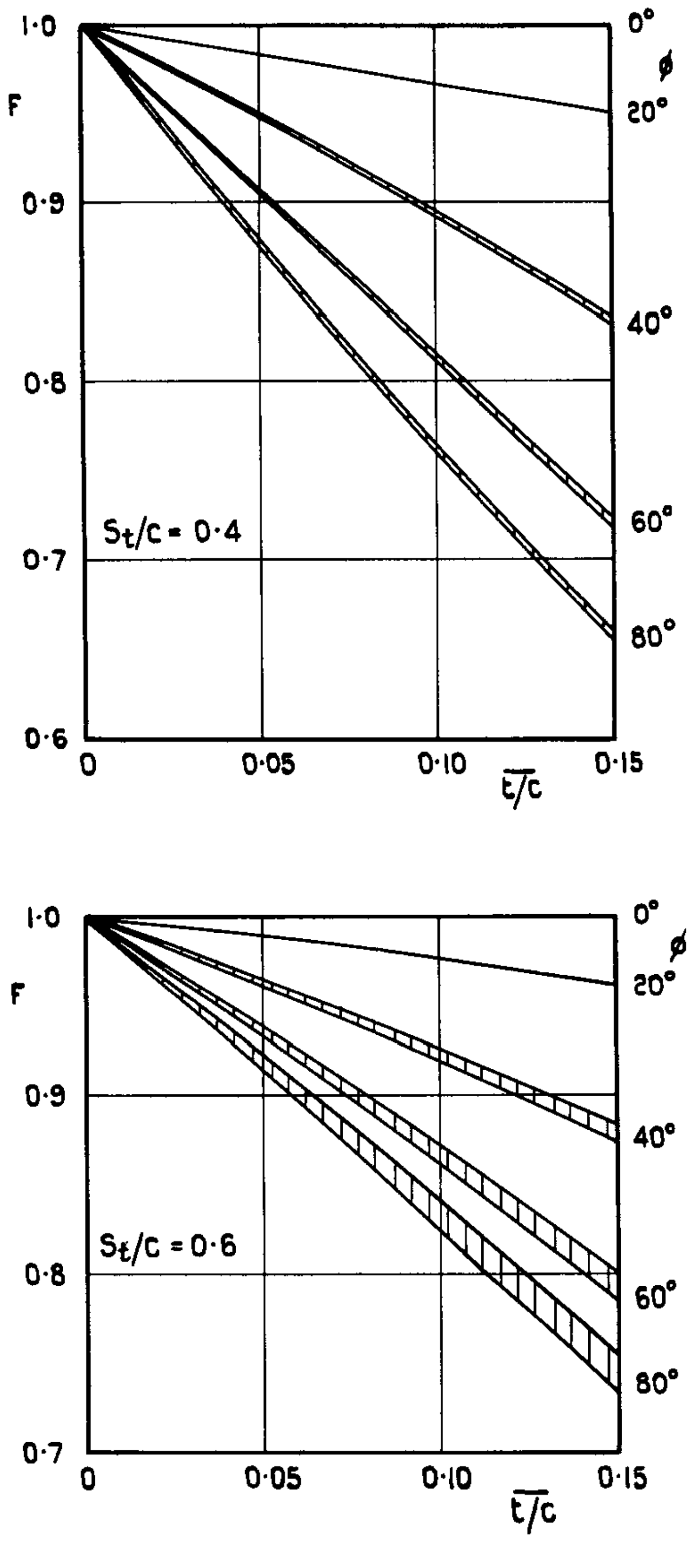


FIG.6 (b). FACTOR FOR R.A.E. 104. IN CURVES JOINED BY SHADING THE UPPER IS FOR $R = 10^8$ AND THE LOWER FOR $R = 10^7$. SINGLE CURVES ARE FOR BOTH 10^7 AND 10^8.

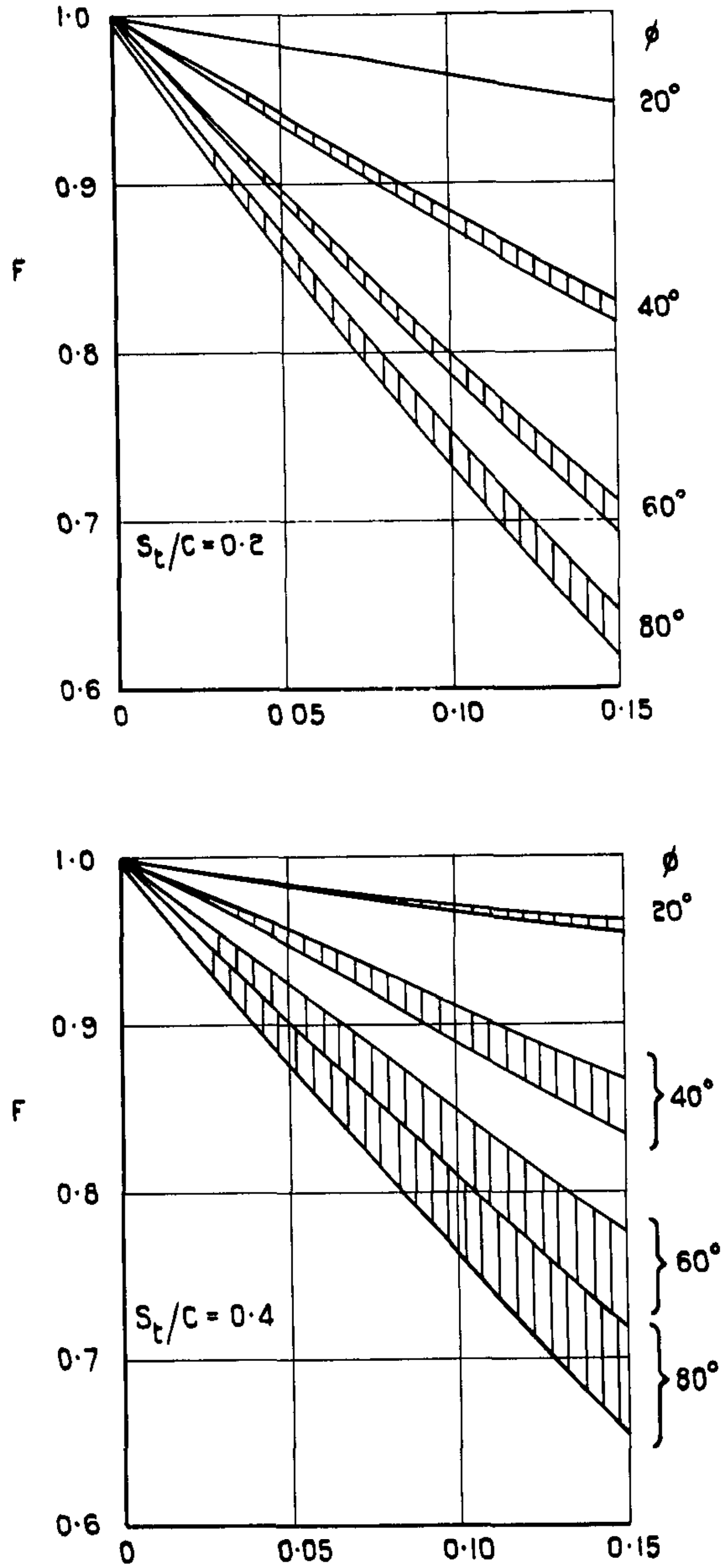


FIG.7. DEPENDENCE ON SECTION SHAPE $R=10^7$. WHERE CURVES ARE JOINED BY SHADING THE UPPER CURVE REFERS TO R.A.E.101. AND LOWER TO R.A.E. 104. CURVES ARE INDISTINGUISHABLE FOR $S_t/c = 0·0$.

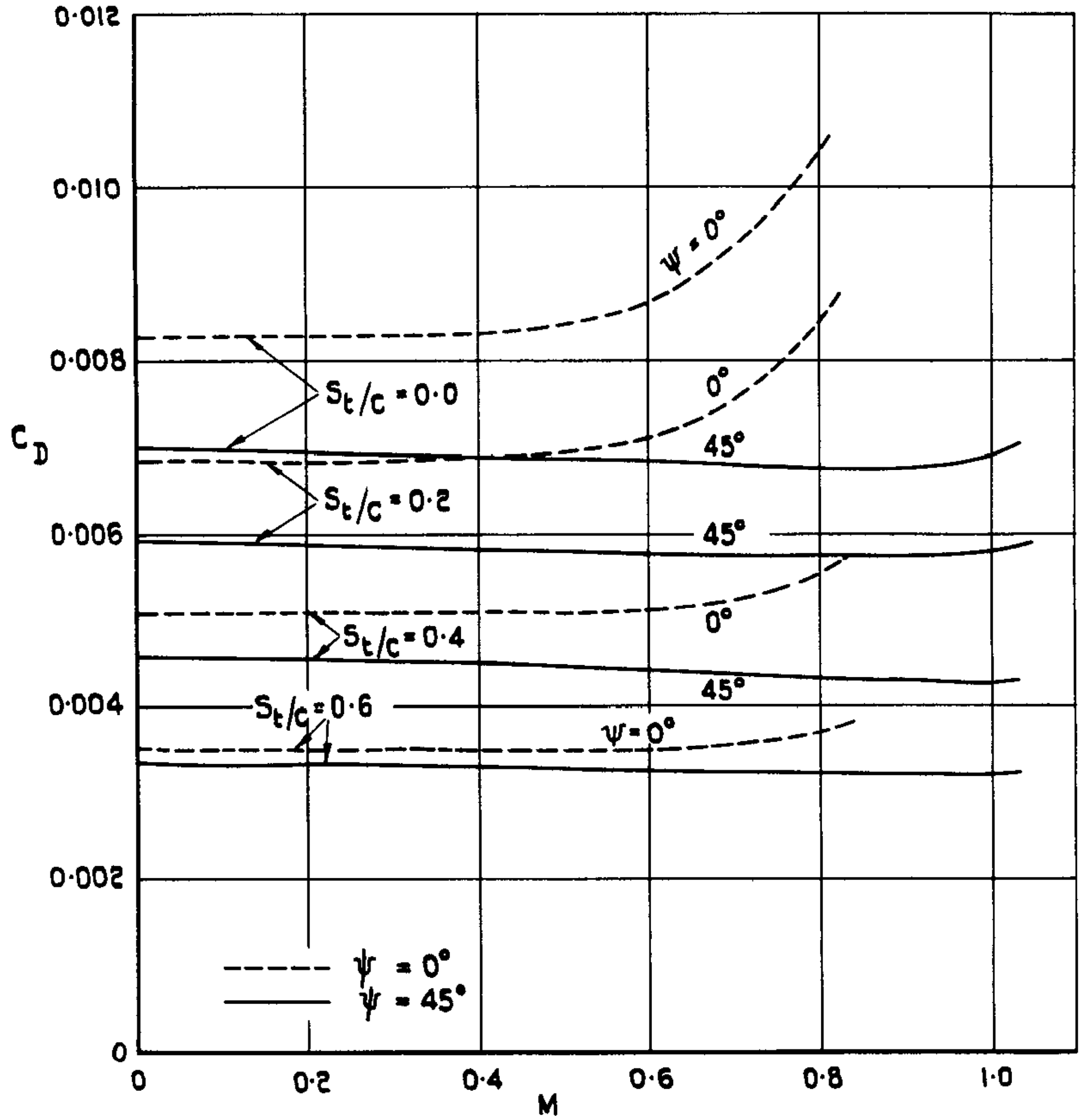


FIG.8. DRAG COEFFICIENT AS A FUNCTION OF MACH NUMBER.
R.A.E. 101, $t/c = 0{\cdot}12$, $R = 10^7$, AT ZERO LIFT.

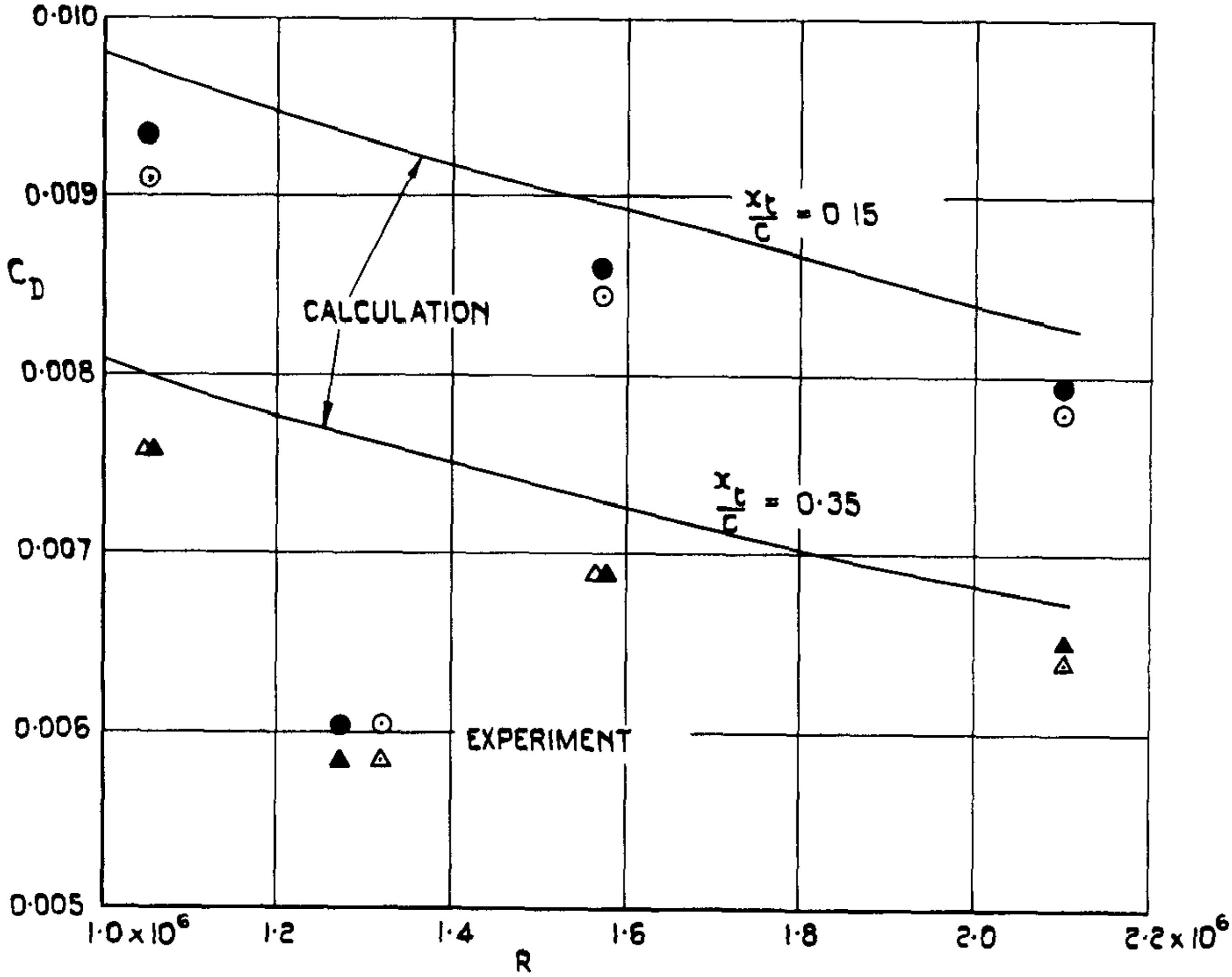


FIG.9. COMPARISON WITH WEBER AND BREBNER'S EXPERIMENTS. R.A.E. 101, $\overline{t/c} = 0{\cdot}17$, $\phi = 45°$.

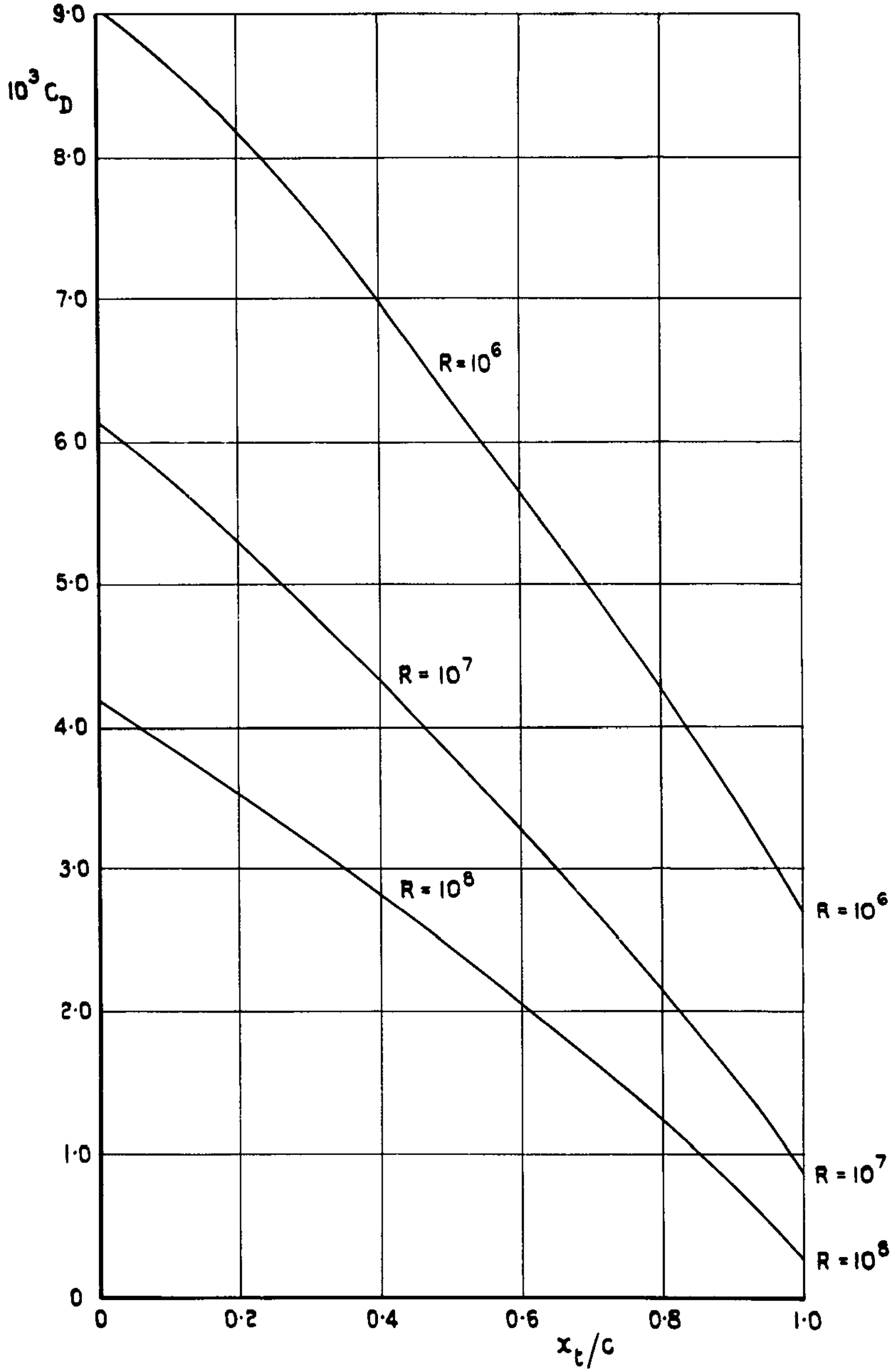


FIG.10. DRAG COEFFICIENT OF A FLAT PLATE. (BOTH SIDES).

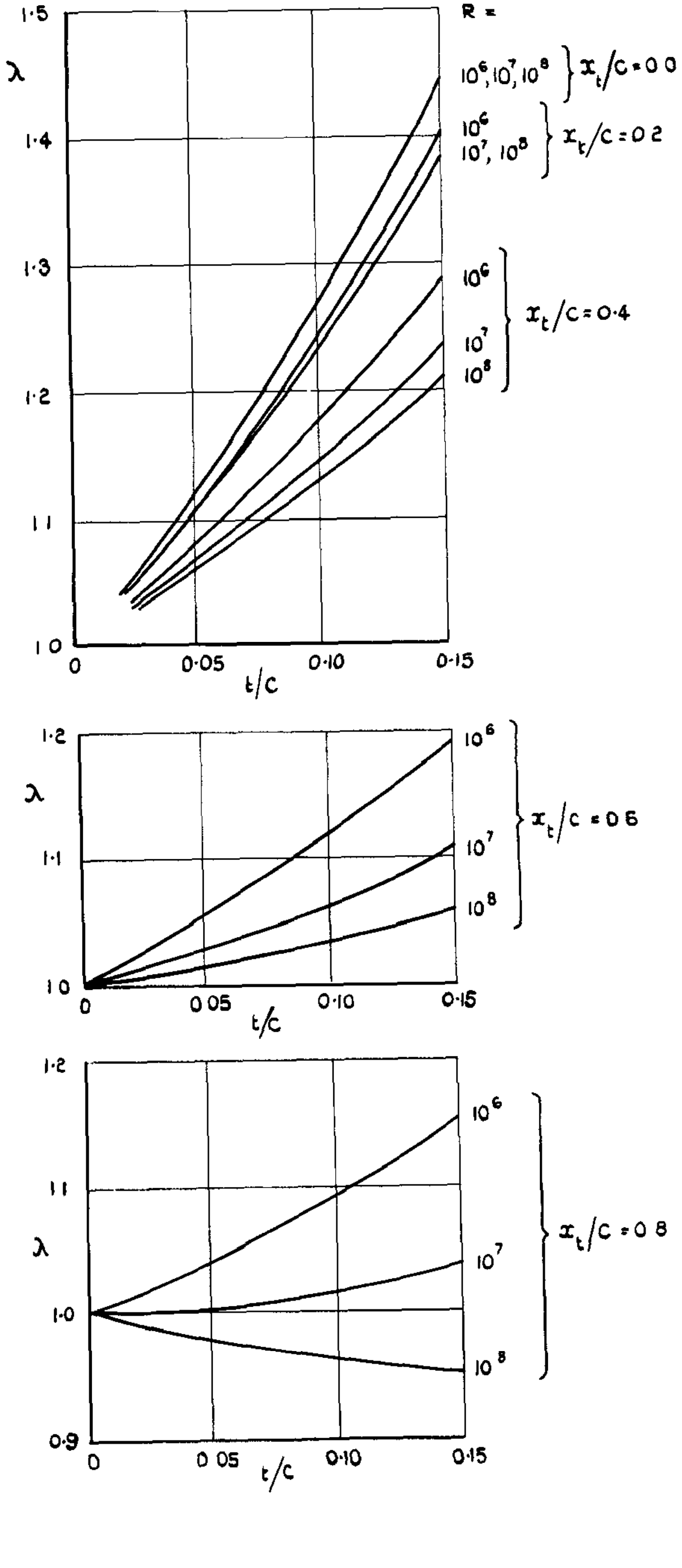


FIG. 11. FACTOR λ FOR R.A.E. 101. UNSWEPT.

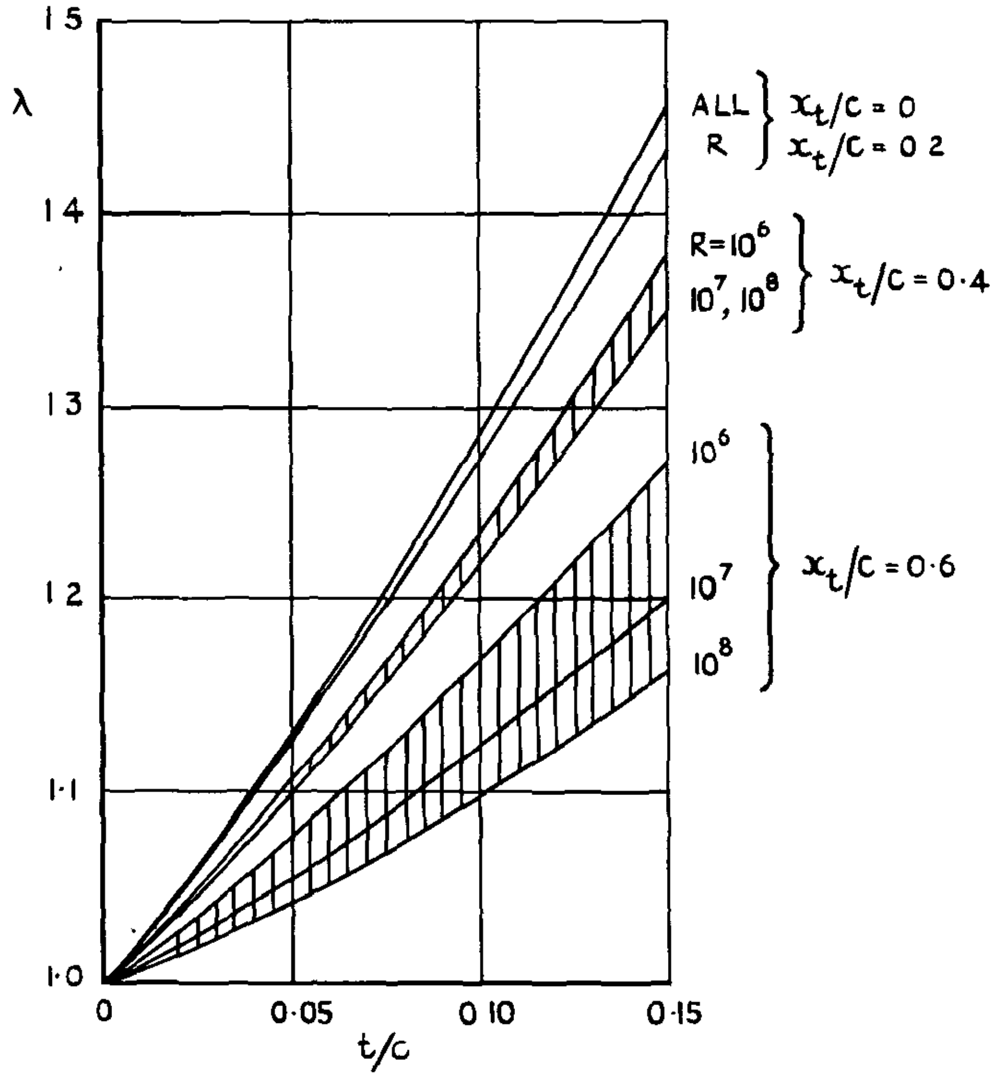


FIG. 12 FACTOR λ FOR RAE 104 UNSWEPT

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Cooke, J.C.

THE DRAG OF INFINITE SWEPT WINGS WITH AN ADDENDUM

The drag of an infinite swept wing is found in terms of the drag of a related unswept wing having the same relative position of transition. Results in incompressible flow are expressed in terms of a "sweep factor". Detailed calculations are made for wings of RAE 101 and 104 sections and the factor appears to have a reasonably universal character not very dependent on shape or Reynolds number if transition takes place early, but strongly dependent on thickness. Results are given as a series of curves and an empirical formula is given for the sweep factor in terms of thickness-chord ratio, angle of sweep and point of transition.

(Over)

DETACHABLE

A few results are given for compressible flow over an RAE 101 section at sweep angles of 0^{o} and 45^{o}; these show the effect of sweep in delaying the compressibility drag rise.

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